On Unknown Input Observers for LTI and LPV systems

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Plan

Plan

- Consider the discrete-time LTI system:

$$x_{k+1} = Ax_k + Bu_k + Ed_k \quad (1)$$

$$y_k = Cx_k \quad (2)$$

with:

- $d_k \in \mathbb{R}^{n_d}$: Unknown Input vector
- $u_k \in \mathbb{R}^{n_u}$: Known Input vector
- $y_k \in \mathbb{R}^{n_y}$: Measured output vector
- $x_k \in \mathbb{R}^{n}$: Unmeasured state vector
- A, B, E and C are known real matrices with appropriate dimensions

Objective

Estimate asymptotically the state $x(t)$ from only the knowledge of u_k and y_k and the model in the presence of unknwon inputs d_k.

Problem Statement

- Consider the discrete-time LTI system:

$$
\begin{aligned}
x_{k+1} &= Ax_k + Bu_k + Ed_k \quad (3)\\
y_k &= Cx_k \quad (4)
\end{aligned}
$$

Classical Unknown Input Observer

$$
\begin{aligned}
z_{k+1} &= Nz_k + Gu_k + Ly_k \quad (5)\\
\hat{x}_k &= z_k - Hy_k \quad (6)
\end{aligned}
$$

Find the matrices N, G, L and H in order to:

1. decouple the state estimation error from d_k
2. ensure $e_k \to 0$ when $k \to +\infty$

Problem Statement

- Consider the discrete-time LTI system:

$$
\begin{aligned}
x_{k+1} &= Ax_k + Bu_k + Ed_k && (7)\\
y_k &= Cx_k && (8)
\end{aligned}
$$

Classical Unknown Input Observer

$$
\begin{aligned}
z_{k+1} &= Nz_k + Gu_k + Ly_k && (9)\\
\hat{x}_k &= z_k - Hy_k && (10)
\end{aligned}
$$

The state estimation error dynamics is:

$$
e_{k+1} = Ne_k + \underbrace{(PA - LC - N - NHC)}_{=0} x_k + \underbrace{(PB - G)}_{=0} u_k + \underbrace{PE}_{=0} d_k
$$

$P = I + HC$

Classical Unknown Input Observer

$$\begin{aligned} N &= PA - LC - NHC \\ PB - G &= 0 \\ PE &= 0 \\ P &= I + HC \end{aligned}$$

Necessary and Sufficient existence conditions (Darouach et al 1994, IEEE TAC)

1. $rank(CE) = rank(E)$
2. All invariant zeros of the triplet (A, C, E) are located inside the unit circle (Stable invariant zeros)

Problem

1. If the internal dynamics are slow $\rightarrow$ the error dynamics is considerably affected

- Consider the discrete-time LTI system:

$$
\begin{aligned}
x_{k+1} &= Ax_k + Bu_k + Ed_k &(11)\\
y_k &= Cx_k &(12)
\end{aligned}
$$

Objective

How to estimate asymptotically the state x_k and enhance the convergence rate in the presence of stable but sable invariant zeros

- Consider the discrete-time LTI system:

$$x_{k+1} = Ax_k + Bu_k + Ed_k \quad (13)$$
$$y_k = Cx_k \quad (14)$$

Asymptotic Decoupling notion

The condition $(I + HC)E = 0$ is replaced by $(I + M_k C)E \to 0$ when $k \to +\infty$

Assumptions

- $rank(CE) = rank(E)$
- The pair (A, C) is observable
- d_k is bounded

$\to$ How to do it in order to preserve the observability at least in a short interval time?

$\to$ Solution: **Asymptotic decoupling**

Ichalal and Mammar IEEE TAC 2020 (Continuous-time), IEEE Control Letters 2020, IEEE CDC 2019 (Discrete-Time)

- The proposed LTV observer has the form:

$$z_{k+1} = N_k z_k + G_k u_k + L_k y_k \quad (15)$$

$$\hat{x}_k = z_k - M_k y_k \quad (16)$$

The matrices N_k, G_k, L_k and M_k are time-dependent matrices which will be defined later in order to ensure asymptotic convergence of the state estimation error $e_k = x_k - \hat{x}_k$.

Choice of H_k

The matrix H_k is chosen in such a way that when $k \to +\infty$, it converges to M and defined by

$$M_k = (1 - f(k))\, H = \left(1 - \rho\alpha^k\right) H \quad (17)$$

$f(k)$ is a time decreasing function which converges to zero (i.e. $0 < \alpha < 1$).
H is computed in the same way as in the classical UIO.

- The state estimation error is defined by the equation

$$e_k = x_k - \hat{x}_k \tag{18}$$
$$= \underbrace{(I + M_k C)}_{P_k} x_k - z_k \tag{19}$$

- its dynamics obeys to the following difference equation

$$\begin{aligned} e_{k+1} &= (P_{k+1}A - N_k P_k - L_k C)\, x_k + (P_{k+1}B - G_k)\, u_k \\ &+ P_{k+1}Ed_k + N_k e_k \end{aligned} \tag{20}$$

- Under the conditions
  1. $P_k = I + M_k C$
  2. $P_{k+1}A - N_k P_k - L_k C = 0$
  3. $P_{k+1}B - G_k = 0$

  are satisfied $\forall k$, the state estimation error dynamics is reduced to

$$e_{k+1} = N_k e_k + S_k d_k \tag{21}$$

- State estimation error dynamics is:

$$e_{k+1} = N_k e_k + S_k d_k \tag{22}$$

where:

- $S_k = P_{k+1} E$
- $N_k = P_{k+1} A - K_k C$
- $K_k = L_k + N_k M_k$
- $P_{k+1} = I + HC - \rho\alpha\alpha^k HC$

$$\begin{aligned} \lim_{k \to +\infty} S_k &= \lim_{k \to +\infty} (I + HC)E - \rho\alpha\alpha^k HCE & (23) \\ &= \lim_{k \to +\infty} (-\rho\alpha\alpha^k HCE) = 0 & (24) \end{aligned}$$

- Error dynamics:

$$e_{k+1} = N_k e_k + S_k d_k \tag{25}$$

- Polytopic transformation:

$$e_{k+1} = \sum_{i=1}^{2} h_i(k)((\mathcal{A}_i - T_i C)e_k + \mathcal{S}_i d_k) \tag{26}$$

where

$$\mathcal{A}_1 = A + HCA - \rho\alpha HCA, \quad \mathcal{A}_2 = A + HCA \tag{27}$$

$$\mathcal{S}_1 = -\rho\alpha HCE, \quad \mathcal{S}_2 = 0 \tag{28}$$

Theorem

Given positive scalars μ and η such that $0 < \eta < \frac{1}{1+\mu}$. For $\tau_1 = \eta(1+\mu)$ and $\tau_2 = \eta$, if there exist symmetric and positive definite matrices $X \in \mathbb{R}^{n\times n}$ and $G_i \in \mathbb{R}^{n\times n}$, gains matrices $\bar{T}_i \in \mathbb{R}^{n\times n_y}$, $i = 1, 2$ and a positive scalar γ such that the following LMIs hold

$$\begin{bmatrix} (\tau_1 - 1)X & \mathcal{A}_1^T G_1^T - C^T \bar{T}_1^T & \Gamma_1 \\ G_1\mathcal{A}_1 - \bar{T}_1 C & X - 2G_1 & 0 \\ \Gamma_1^T & 0 & \Omega_1 \end{bmatrix} < 0 \tag{29}$$

$$\begin{bmatrix} (\tau_2 - 1)X & \mathcal{A}_2^T G_2^T - C^T \bar{T}_2^T \\ G_2\mathcal{A}_2 - \bar{T}_2 C & X - 2G_2 \end{bmatrix} < 0 \tag{30}$$

$$\Gamma_1 = (\mathcal{A}_1^T G_1^T - C^T \bar{T}_1^T) S_1, \quad \Omega_1 = 2S_1^T G_1 S_1 - \gamma S_1^T S_1 \tag{31}$$

then the state estimation error converges asymptotically towards zero.

$$A = \begin{bmatrix} 0.5 & -0.3847 & 0.7036 \\ 0 & 0.7 & 0.5468 \\ 0 & 0 & -0.8 \end{bmatrix}, E = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.3518 & -0.2734 & 0.5 \end{bmatrix}$$

The pair (A, C) is observable and the invariant zeros of the system are located at $z_1 = 0.995$ and $z_2 = 0.999$.

$$H = -E(CE)^{-1} = \begin{bmatrix} 0 & 0 & -2 \end{bmatrix}^T$$

The function f_k is defined by $f_k = 0.993^k$ according to Definition **??**. The parameter η should be chosen such that

$$\max_{i=1,2}(|z_i|) < 1 - \eta < 1$$

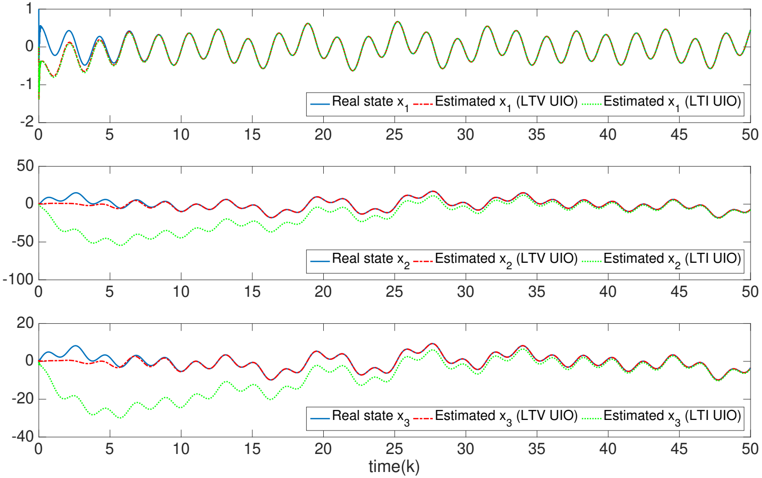

Figure: Comparison of state estimations: Proposed LTV UIO and classical LTI UIO

Simulation results

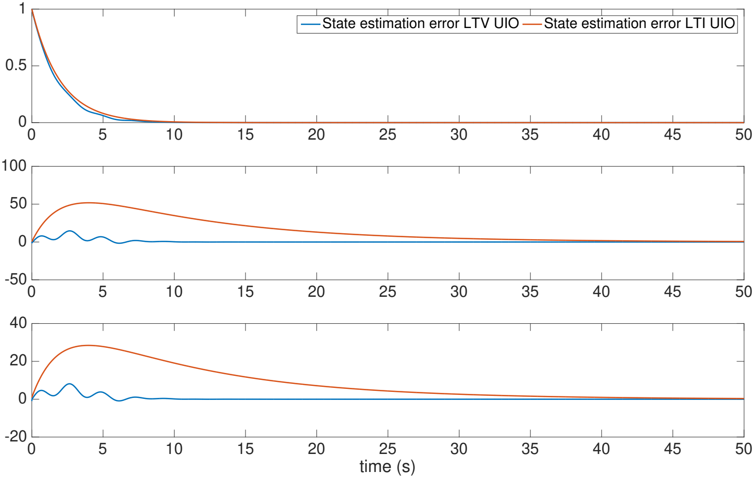

Figure: Comparison of state estimation errors: Proposed LTV UIO and classical LTI UIO

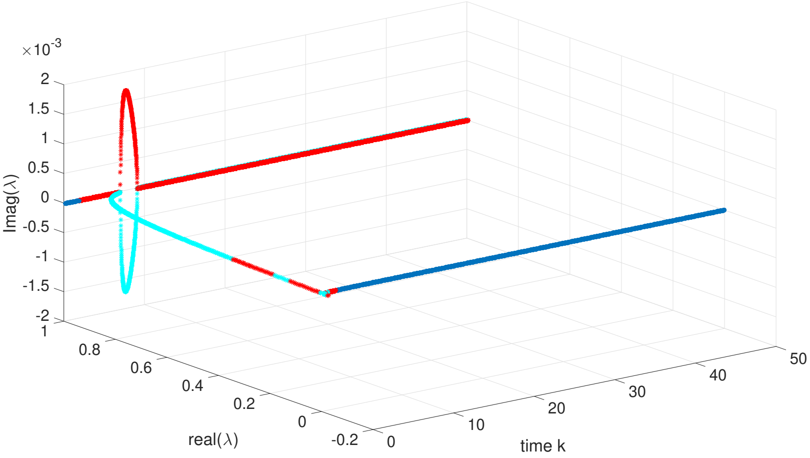

Figure: Time evolution of the eigenvalues of the matrix N_k

Consider the Linear model of a DC Motor described by:

$$\dot{x} = Ax + Ed, \quad y = Cx$$

$$A = \begin{bmatrix} -\frac{R}{L} & 0 & -\frac{a}{L} \\ 0 & 0 & 1 \\ \frac{b}{m} & -\frac{k}{m} & -\frac{f}{m} \end{bmatrix}, E = \begin{bmatrix} -\frac{1}{L} \\ 0 \\ 0 \end{bmatrix}$$

and $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$.
The existence conditions of the classical design are satisfied and the invariant zeros are $-0.05 \pm i9.999$.

Simulation results: Continuous-time case

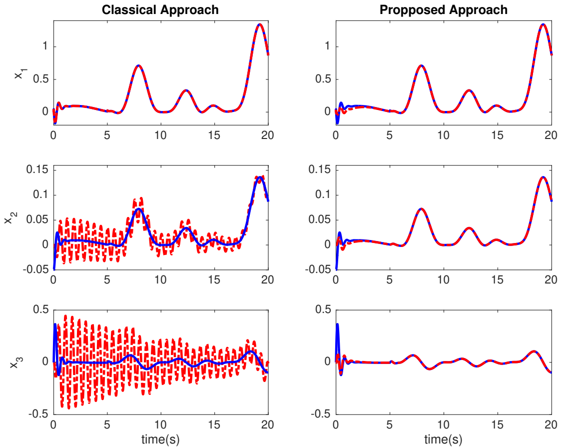

Figure: Real states (blue) and estimations (red)

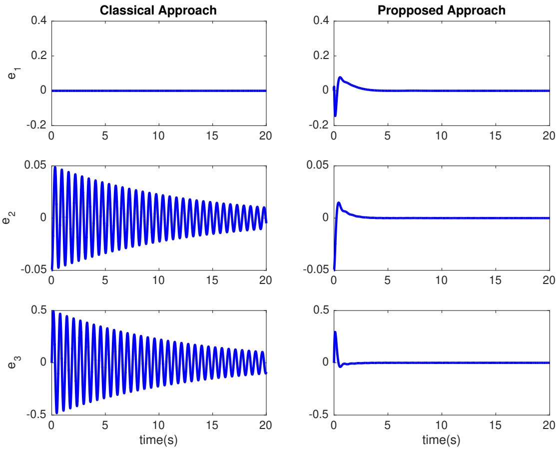

Figure: State estimation error comparison

Plan

Problem statement

LPV system affected by Unknown Inputs

$$\dot{x} = A(\rho)x + B(\rho)u + E(\rho)d, \quad y = C(\rho)x$$

Classical approach

- Polytopic transformation at the beginning

$$\dot{x} = \sum_{i=1}^{r} h_i(\rho)\left(A_i x + B_i u + E_i d\right), \quad y = Cx$$

- The UIO uses the same polytopic form:

$$\dot{z} = \sum_{i=1}^{r} h_i(\rho)\left(N_i x + G_i u + L_i d\right), \quad \hat{x} = z - Hy$$

Existing solutions

- Replacing E_i by E^* solution to $\min_i \|E^* - E_i\|_F$ (Rodriguez and Theilliol 2005)
- $E(\rho)d = E_1\bar{E}(\rho)d = E_1 f$ (Alwi and Edwards 2014)
- $rank(C[E_1 ... E_r]) = rank([E_1 ... E_r])$ (Marx and Ragot 2007, Chadli and Karimi 2013)

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State estimation error dynamics $e = x - \hat{x}$

$$\dot{e} = \sum_{i=1}^{r} h_i(\rho)\left(N_i e + \underbrace{(PA_i - L_i C - N_i P)}_{=0} x + \underbrace{(PB_i - G_i)}_{=0} u + \underbrace{PE_i}_{=0} d\right)$$

LPV UIO: Motivating example

Consider the LPV system affected by the unknown input d :

$$\begin{cases} \dot{x}(t) = \begin{pmatrix} 0 & \rho(t) \\ 1-\rho(t) & -3 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t) + \begin{pmatrix} \rho(t) \\ \rho(t)+1 \end{pmatrix} d(t) \\ y(t) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(t) \end{cases}$$

with a bounded parameter: $2 \leq \rho(t) \leq 4$

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with a bounded parameter: $2 \leq \rho(t) \leq 4$
By using the polytopic transformation, $\rho(t) \in [2\ 4]$, it becomes:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} h_i(\rho(t))(A_i x(t) + B_i u(t) + E_i d(t)) \\ y(t) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(t) \end{cases}$$

where the matrices are defined by:

$$A_1 = \begin{pmatrix} 0 & 4 \\ -3 & -3 \end{pmatrix},\ A_2 = \begin{pmatrix} 0 & 2 \\ -1 & -3 \end{pmatrix},\ B_1 = B_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ E_1 = \begin{pmatrix} 4 \\ 5 \end{pmatrix},\ E_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

LPV UIO: Motivating example

$$\dot{e} = \sum_{i=1}^{r} h_i(\rho)\left(N_i e + (PA_i - L_i C - N_i P)\,x + (PB_i - G_i)u - PE_i d\right)$$

$$PE_i = (I + HC)E_i = 0 \Rightarrow E_i = -HCE_i, \quad i = 1, 2$$

$$N_i = PA_i - L_i C - N_i HC$$

$$PB_i = G_i$$

$$\Rightarrow \dot{e} = \sum_{i=1}^{r} h_i(\rho) N_i e$$

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$$\dot{e} = \sum_{i=1}^{r} h_i(\rho)\left(N_i e + (PA_i - L_i C - N_i P)\,x + (PB_i - G_i)u - PE_i d\right)$$

$$PE_i = (I + HC)E_i = 0 \Rightarrow E_i = -HCE_i, \quad i = 1, 2$$

$$N_i = PA_i - L_i C - N_i HC$$

$$PB_i = G_i$$

$$\Rightarrow \dot{e} = \sum_{i=1}^{r} h_i(\rho) N_i e$$

Focus on the decoupling condition: Find H such that $E_i = -HCE_i$, i=1,2

- $rang(E_i) = rang(CE_i)$ is not sufficient
- $rang\left([E_1 \;\; E_2]\right) = rang\left(C\,[E_1 \;\; E_2]\right) \Rightarrow E_i = -HCE_i$, is not satisfied
- The UIO does not exist
- The idea is to postpone the polytopic transformation at the end of the design and ensure
  $E(\rho(t)) = -H(?)CE(\rho(t))$ instead of $E_i = -HCE_i$
  (Ichalal IEEE TIE'15, IFAC ICONS'16, / Marx AUTOMATICA'19)

LPV UIO: Proposed solution

- LPV system affected by Unknown Inputs:

$$\begin{cases} \dot{x} = A(\rho)x + B(\rho)u + E(\rho)d \\ y = C(\rho)x \end{cases}$$

- Assumption : $\rho \in \Omega_\rho$ et $\dot{\rho} \in \Omega_{\dot{\rho}}$ known (measured) and bounded
- UIO:

$$\begin{cases} \dot{z} = N(\rho, \dot{\rho})z + G(\rho)u + L(\rho, \dot{\rho})y \\ \hat{x} = z - H(\rho)y \end{cases}$$

  - Decoupling condition $rang(C(\rho)E(\rho)) = rang(E(\rho)), \forall \rho \in \Omega_\rho$

Extensions

- Relative degree $1 < r \leq n$: Ichalal and Mammar IEEE TIE 2015
- Decay rate enhancement (stable but slow internal dynamics): Ichalal and Mammar IEEE TAC 2019 (Asymptotic Decoupling: $\lim_{t \to +\infty} (I_n + H(\rho, \beta)C(\rho)) = 0$ instead of $(I_n + H(\rho, \beta)C(\rho)) = 0$.)
- Taking into account errors in the estimation of parameter time derivatives OR unavailable time derivatives - IEEE CDC 2019 - Ichalal and Guerra
- Parameters depending partially on unmeasured state variables - IEEE CDC 2019 - Ichalal and Guerra

LPV UIO: example

- Consider the LPV system with parameter varying output matrix:

$$\begin{cases} \dot{x}(t) = \begin{pmatrix} 0 & \rho(t) \\ 1-\rho(t) & -3 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t) \begin{pmatrix} \rho(t) \\ \rho(t)+1 \end{pmatrix} d(t) \\ y(t) = \begin{pmatrix} \rho(t) & 0 \end{pmatrix} x(t) \end{cases}$$

where the parameter is bounded: $2 \leq \rho(t) \leq 4$.

- UI decoupling with constant a matrix H is not possible
  - ▶ The classical LPV UIO does not exist

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  where the parameter is bounded: $2 \leq \rho(t) \leq 4$.

- UI decoupling with constant a matrix H is not possible
  - ▶ The classical LPV UIO does not exist

- The condition $rank\,(E(\rho)) = rank\,(C(\rho)E(\rho))$ is satisfied

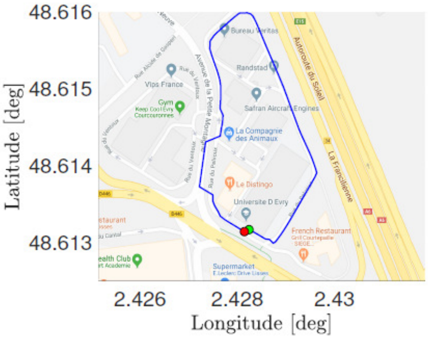
  $$rank\,(E(\rho)) = 1 \text{ et } rank\,(C(\rho)E(\rho)) = rank\,\left(\rho^2(t)\right) = 1$$

  - ▶ avec $H(\rho) = \left[-\frac{1}{\rho(t)} \quad -\frac{1+\rho(t)}{\rho^2(t)}\right]^T$

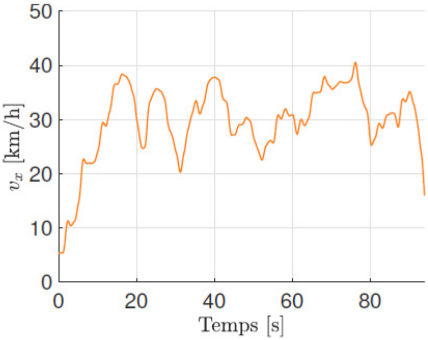
  The LPV UIO can be constructed

LPV UIO: Application to Motorcycle lateral dynamics

Figure: Experimental Platform



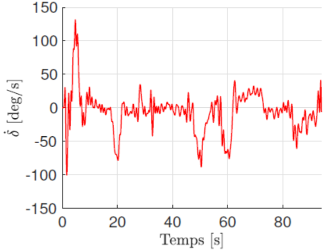
(a) Trajectoire du véhicule



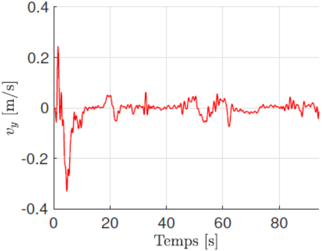
(b) Vitesse longitudinale

Figure: Experimentation site and longitudinal velocity (Dabladji (2015) and Damon (2018))

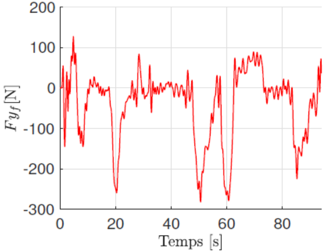
LPV UIO: Application to Motorcycle lateral dynamics



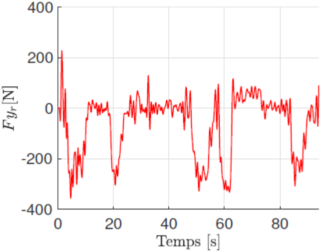
(a) Vitesse de l'angle de braquage estimée



(b) Vitesse latérale estimée

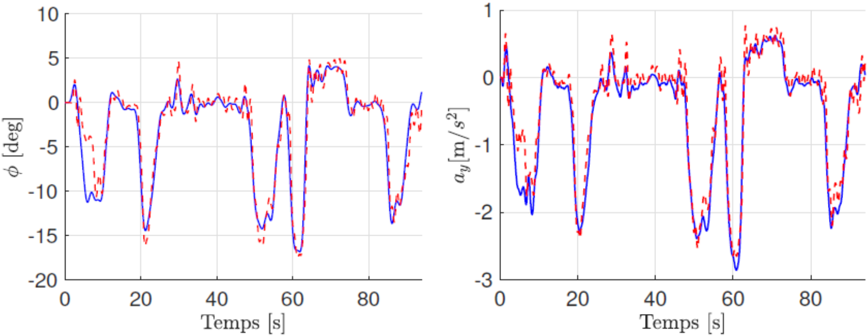


(c) Force latérale estimée du pneumatique avant



(d) Force latérale estimée du pneumatique arrière

Figure: Estimated states



(a) Angle de roulis mesuré (bleu) et estimé (rouge)

(b) Accélération latérale mesurée (bleu) et estimée (rouge)

Figure: Validation

Plan

Conclusion and perspectives

- New LTV Unknown Input Observer for LTI systems
  - Asymptotic Decoupling Approach: convergence rate enhancement
  - LMI stability conditions for design
- UIO for LPV systems with smooth parameters
  - Postponing the polytopic transformation

Thank you for your attention
Questions?