Robust Actuator Fault Diagnosis for LPV systems: Application to Quadrotor

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Journées Nationales Automatique de la SAGIP

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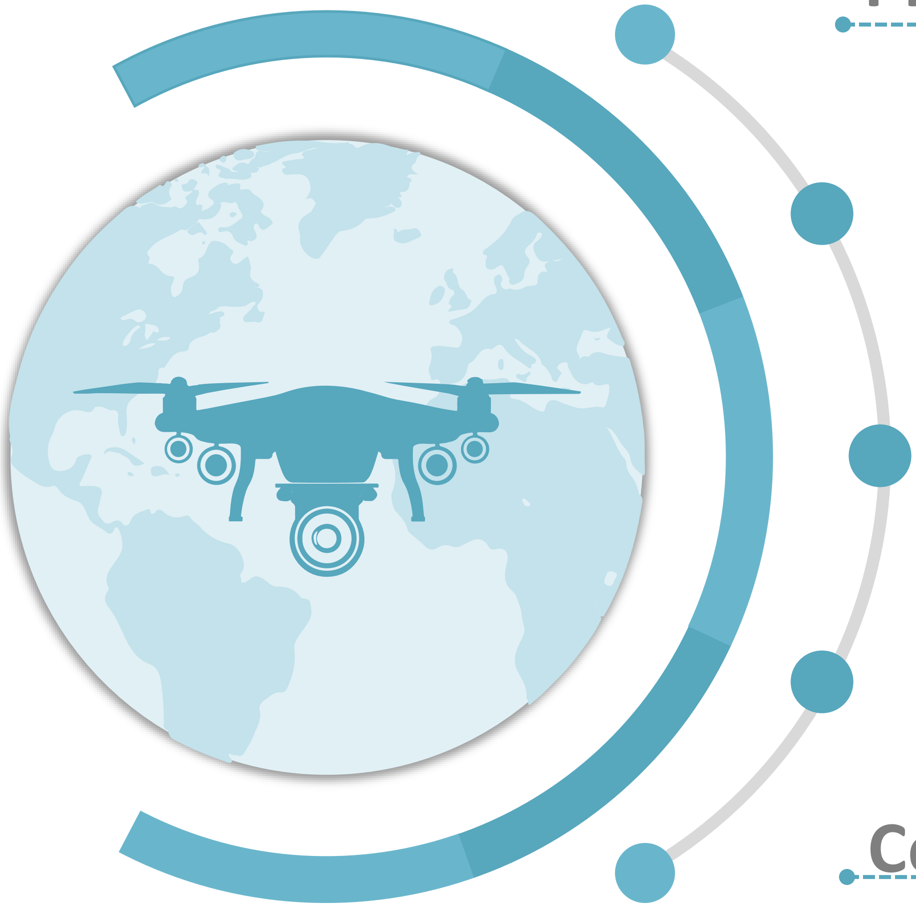
Presentation outlines



Problem statement

Proposed solution

Solution synthesis

Results

Conclusions & Perspectives

Problem statement

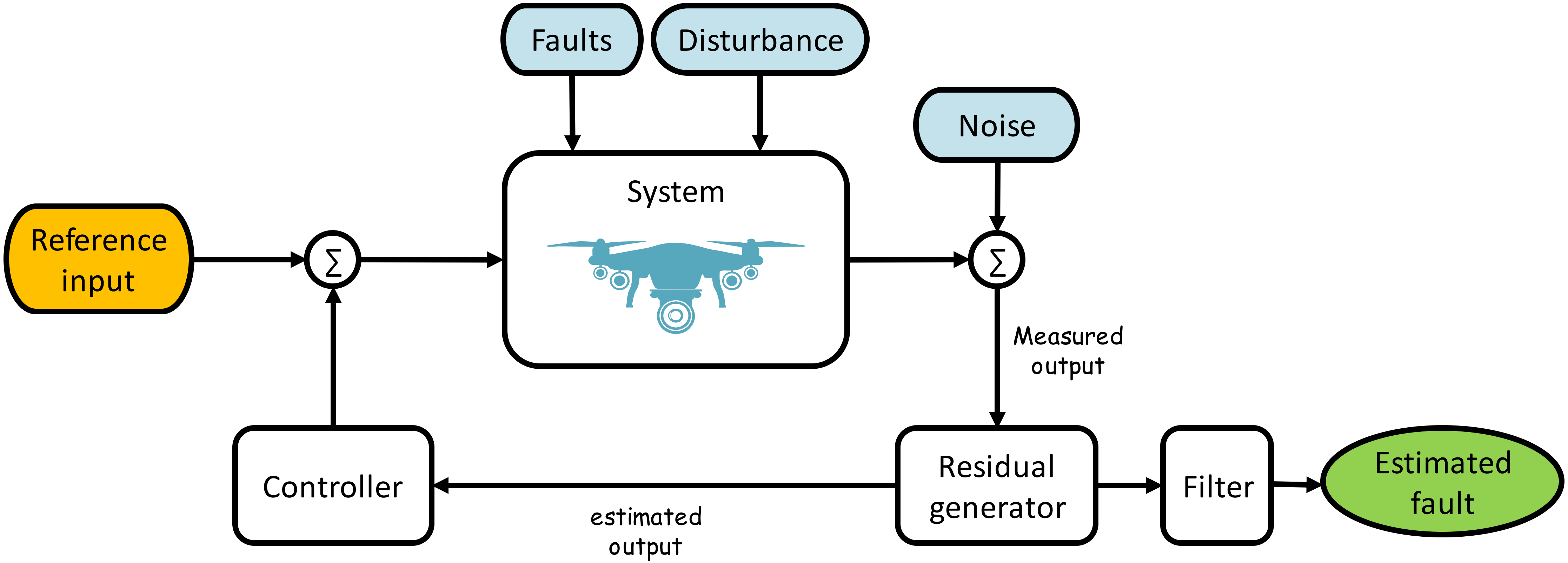
- Proposed solution
- Solution synthesis
- Results
- Conclusions & Perspectives

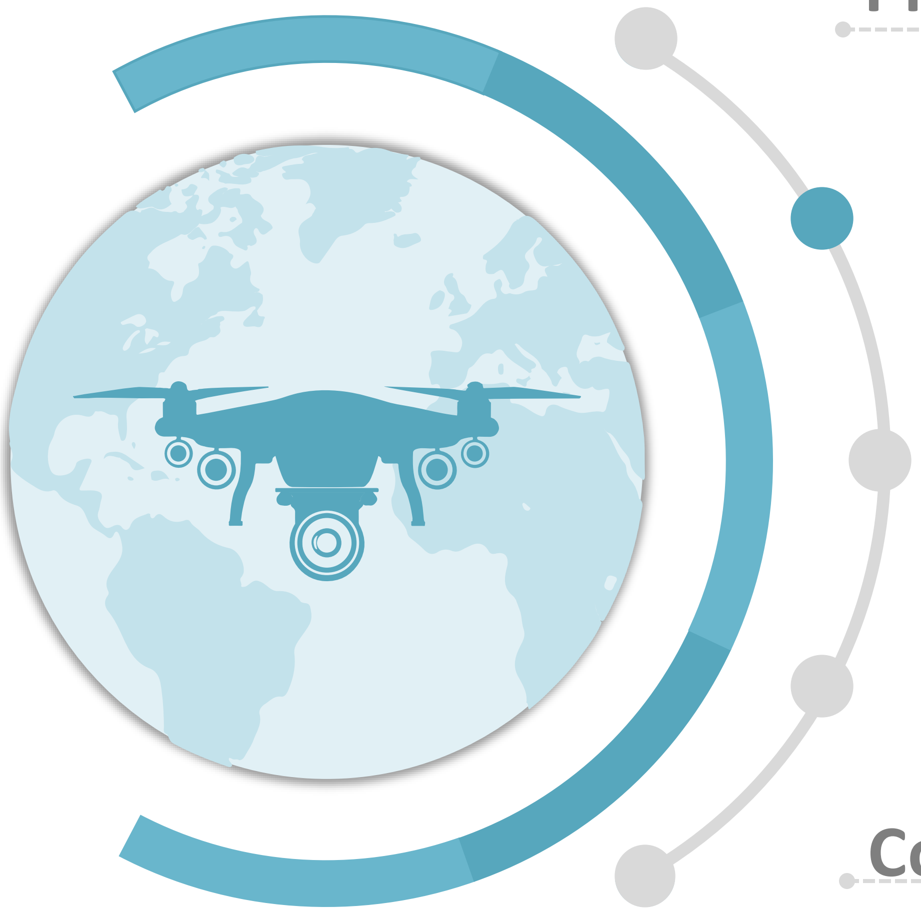
Problem statement

- **FTC importance for automated systems**
- **Why quadrotor?**

Problem statement

❑ FDD scheme





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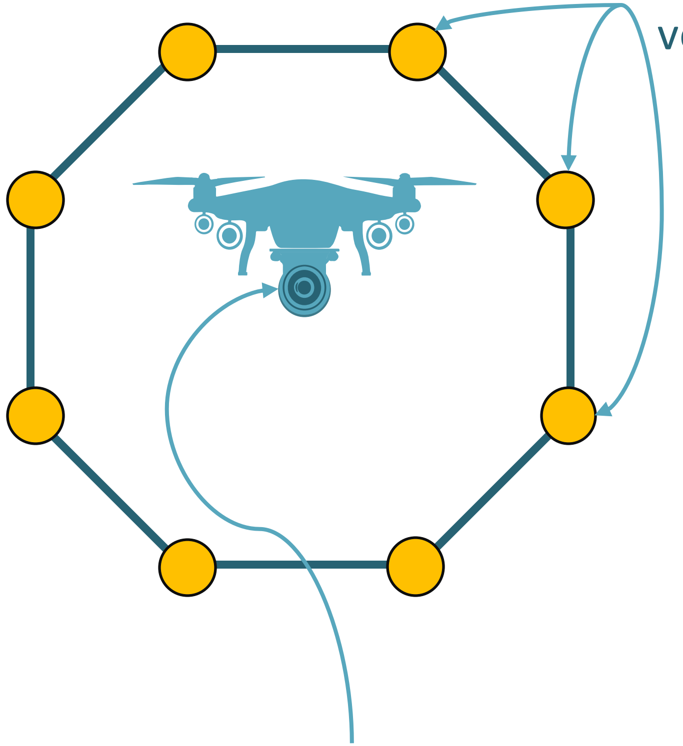
❑ Why LPV modeling?

LPV model:

$$\begin{cases} \dot{x}(t) = A\big(\rho(t)\big)\, x(t) + B\big(\rho(t)\big)\, u(t) + F\big(\rho(t)\big)\, f(t) + E\big(\rho(t)\big)\, d(t) \\ y(t) = C\big(\rho(t)\big)\, x(t) + D\big(\rho(t)\big)\, u(t) \end{cases}$$

where

$$M\big(\rho(t)\big) = \sum_{k=1}^{w} \mu_k\big(\rho(t)\big)\, M_k\,, \qquad w\!: no.of\ vertices$$



Proposed solution

❑ Robust H_∞ control

Control law:

$$u(t) = -K(\rho(t))\,\hat{x}(t) + V(\rho(t))\,\eta\,(t)$$

Lyapunov inequality:

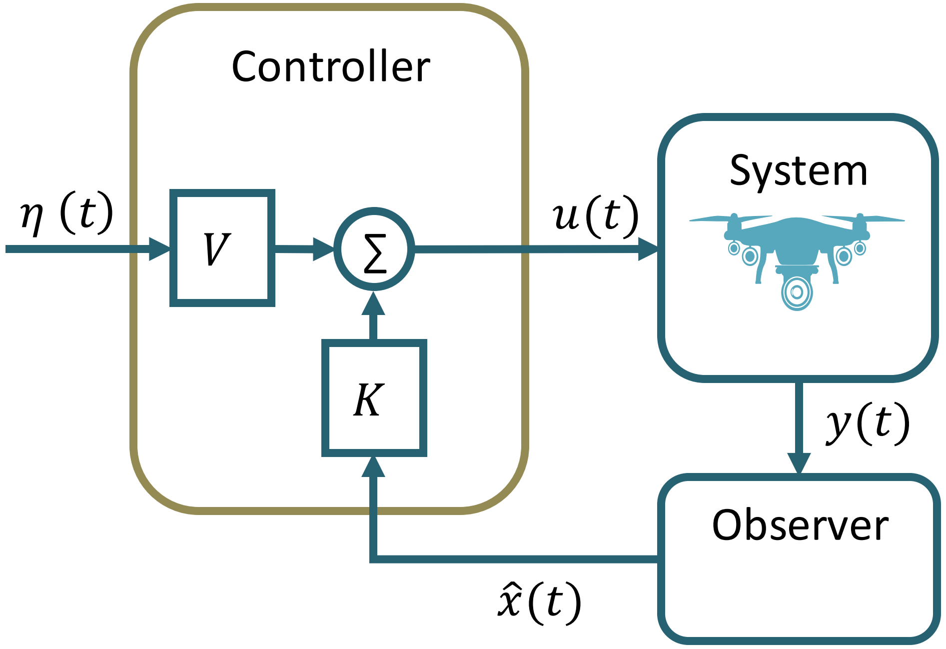
$$(A_k - B_k\,K_k)^T\,P + P(A_k - B_k\,K_k) + 2\,\zeta_c\,P \le 0$$

Bounded real lemma:

$$\begin{pmatrix} (A_k - B_k\,K_k)^T\,P + P(A_k - B_k\,K_k) & P\,B_k & C_k^T \\ B_k^T\,P & -\gamma_c\,I & D_k^T \\ C_k & D_k & -I \end{pmatrix}$$

DC gain

$$V_k = -(C_k\,(A_k - B_k\,K_k)^{-1}\,B_k)^{-1}$$



Proposed solution

Classical H_- / H_∞ observer

Observer scheme:

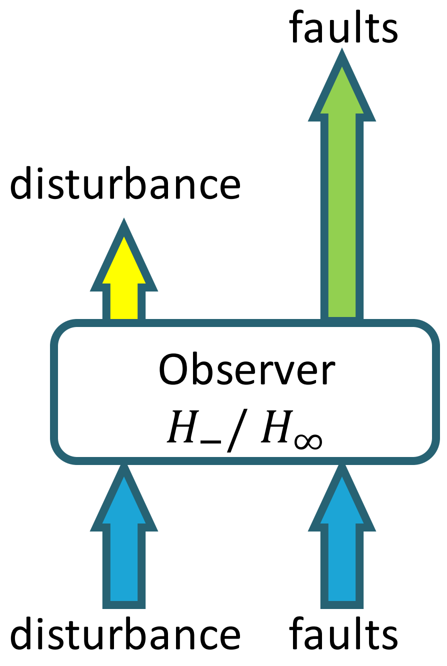
$$\begin{cases} \dot{\hat{x}}(t) = A_\rho\, \hat{x}(t) + B_\rho\, u(t) + L_\rho(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C_\rho \hat{x}(t) + D_\rho u(t) \\ r(t) = y(t) - \hat{y}(t) \end{cases}$$

The system:

$$\begin{cases} \dot{x}(t) = A_\rho x(t) + B_\rho\, u(t) + F_\rho\, f(t) + E_\rho\, d(t) \\ y(t) = C_\rho\, x(t) + D_\rho\, u(t) + J_\rho f(t) + H_\rho d(t) \end{cases}$$

Resulting error dynamics:

$$\begin{cases} \dot{e}(t) = (A_\rho - L_\rho\, C_\rho)e(t) + (F_\rho - L_\rho\, J_\rho)f(t) + (E_\rho - L_\rho\, H_\rho)d(t) \\ r(t) = C_\rho e(t) + J_\rho f(t) + H_\rho d(t) \end{cases}$$



Proposed solution

❑ The new approach

The system:

$$\begin{cases} \dot{x}(t) = A_\rho x(t) + B_\rho\, u(t) + F_\rho\, f(t) + E_\rho\, d(t) \\ y(t) = C_\rho\, x(t) + D_\rho\, u(t) \end{cases}$$

Differentiated output: $\tilde{y}(t)$

$$\boxed{\tilde{y}(t)} \leftarrow \boxed{\frac{d}{dt}} \leftarrow \boxed{y(t)}$$

System classification according to the relative degree

$\boldsymbol{\lambda_f < \lambda_d}$

$$\tilde{y}\,(t) = C_{\bar{\rho}}\, x(t) + B_{\bar{\rho}}\, u(t) + R_{\bar{\rho}}\, f(t)$$

$\boldsymbol{\lambda_f = \lambda_d}$

$$\tilde{y}\,(t) = C_{\bar{\rho}}\, x(t) + B_{\bar{\rho}}\, u(t) + R_{\bar{\rho}}\, f(t) + D_{\bar{\rho}}\, d(t)$$

$\boldsymbol{\lambda_f > \lambda_d}$

$$\tilde{y}\,(t) = C_{\bar{\rho}}\, x(t) + B_{\bar{\rho}}\, u(t) + R_{\bar{\rho}}\, f(t) + D_{\bar{\rho}}\, \bar{d}(t)$$

Proposed solution

❑ The new approach

The proposed residual generator:

$$\begin{cases} \dot{\hat{x}}(t) = A_\rho(t)\,\hat{x}(t) + B_\rho\, u(t) + L_{1\bar{\rho}}\big(y(t) - \hat{y}(t)\big) + L_{2\bar{\rho}}\big(\tilde{y}(t) - \hat{\tilde{y}}(t)\big) \\ \hat{y}(t) = C_\rho \hat{x}(t) \\ \hat{\tilde{y}}(t) = C_{\bar{\rho}}\,\hat{x}(t) + B_{\bar{\rho}}\, u(t) \\ r(t) = M_{\bar{\rho}}\big(\tilde{y}(t) - \hat{\tilde{y}}(t)\big) \end{cases}$$

Reference residual signal:

$$r_r(t) = Q\, f(t)$$

Virtual residual signal:

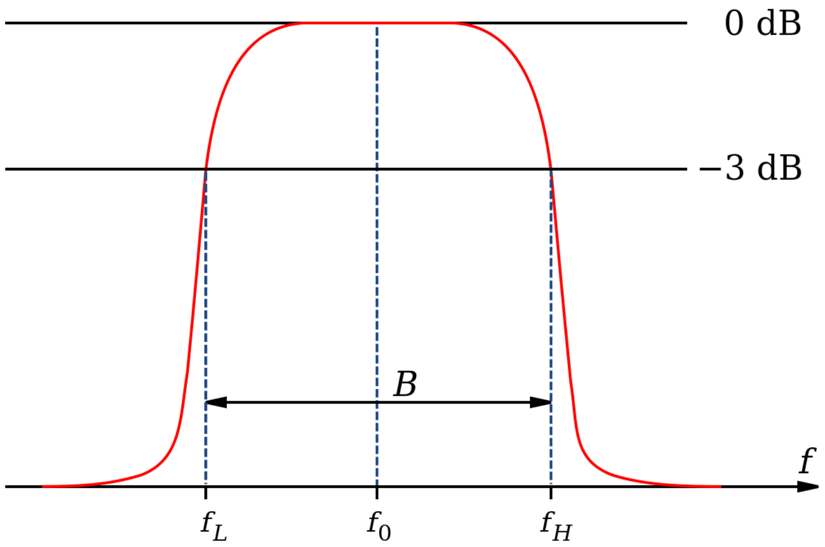
$$r_e(t) = r(t) - r_r(t)$$

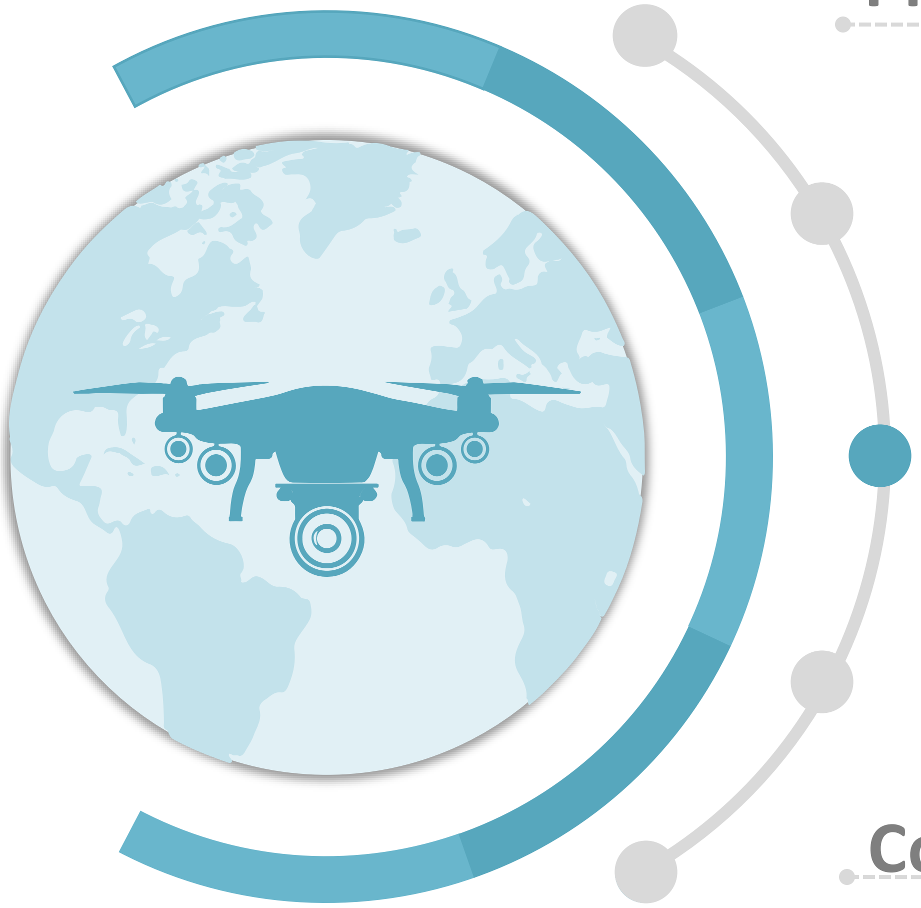
Proposed solution

❑ The new approach

Post bandpass filter

$$\begin{cases} \dot{x}_h(t) = A_h\, x_h(t) + B_h\, r_e(t) \\ r_f(t) = C_h\, x_h(t) + D_h\, r_e(t) \end{cases}$$





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Solution synthesis

❑ Applying for system of case 2 ($\lambda_f = \lambda_d$)

Error dynamics become

$$\dot{e}(t) = (A_\rho - L_{1\bar{\rho}} C_\rho - L_{2\bar{\rho}} C_{\bar{\rho}})\, e(t) + (E_\rho - L_{2\bar{\rho}} D_{\bar{\rho}})\, d(t) + (F_\rho - L_{2\bar{\rho}} R_{\bar{\rho}})\, f(t)$$

$$r_e(t) = M_{\bar{\rho}}\, C_{\bar{\rho}}\, e(t) + M_{\bar{\rho}} D_{\bar{\rho}}\, d(t) + \left(M_{\bar{\rho}} R_{\bar{\rho}} - Q\right) f(t)$$

Theorem (1): Exact residual convergence

$$r_r(t) = Q\, f(t) \rightarrow r_{e(t)} = 0$$

Condition to satisfy: $rank\left(\begin{bmatrix} C_{\bar{\rho}} & R_{\bar{\rho}} & D_{\bar{\rho}} \\ 0 & Q & 0 \end{bmatrix}\right) = rank\left(\begin{bmatrix} C_{\bar{\rho}} & R_{\bar{\rho}} & D_{\bar{\rho}} \end{bmatrix}\right)$

The gains $M_{\bar{\rho}} = (0 \quad Q \quad 0)(C_{\bar{\rho}} \quad R_{\bar{\rho}} \quad D_{\bar{\rho}})^{-1}$, $L_{1\bar{\rho}}$, $L_{2\bar{\rho}}$ *can be chosen freely*

Solution synthesis

❑ Applying for system of case 2 ($\lambda_f = \lambda_d$)

Error dynamics become

$$\dot{e}(t) = (A_\rho - L_{1\bar{\rho}} C_\rho \ - L_{2\bar{\rho}} C_{\bar{\rho}})\, e(t) + (E_\rho - L_{2\bar{\rho}} D_{\bar{\rho}})\, d(t) + (F_\rho - L_{2\bar{\rho}} R_{\bar{\rho}})\, f(t)$$

$$r_e(t) = M_{\bar{\rho}} C_{\bar{\rho}}\, e(t) \ + M_{\bar{\rho}} D_{\bar{\rho}}\, d(t) \ + \left(M_{\bar{\rho}} R_{\bar{\rho}} \ - Q\right) f(t)$$

Theorem (2): Asymptotic residual convergence

$$\begin{cases} \lim\limits_{t \to \infty} r(t) = Q\, f\,, & d = 0 \\ \|r(t) \ - \ Q\, f(t)\|_2 \leq \gamma\, \|d(t)\|_2\,, & d \neq 0 \end{cases}$$

Condition to satisfy: $rank\left(R_{\bar{\rho}}\right) = n_f, \quad rank\left(\begin{bmatrix} F_\rho \\ R_{\bar{\rho}} \end{bmatrix}\right) = rank\ \left(F_\rho\right)$

The gains $M_{\bar{\rho}} = Q R_{\bar{\rho}}^{-1}$, $L_{2\bar{\rho}} = F_\rho\, R_{\bar{\rho}}^{-1}$, $L_{1\bar{\rho}}$ *is chosen to minimize disturbance effect.*

Solution synthesis

- Applying for system of case 2 ($\lambda_f = \lambda_d$)

Error dynamics become

$$\dot{e}(t) = (A_\rho - L_{1\overline{\rho}} C_\rho - L_{2\overline{\rho}} C_{\overline{\rho}})\, e(t) + (E_\rho - L_{2\overline{\rho}} D_{\overline{\rho}})\, d(t) + (F_\rho - L_{2\overline{\rho}} R_{\overline{\rho}})\, f(t)$$

$$r_e(t) = M_{\overline{\rho}} C_{\overline{\rho}}\, e(t) + M_{\overline{\rho}} D_{\overline{\rho}}\, d(t) + \left(M_{\overline{\rho}} R_{\overline{\rho}} - Q\right) f(t)$$

Worst case H_- / H_∞ technique

The gains $M_{\overline{\rho}}, L_{2\overline{\rho}}, L_{1\overline{\rho}}$ are chosen to minimize disturbance effect and maximize fault effect such that:

$$\left\|T_{rd}\right\|_\infty \leq \gamma$$

$$\left\|T_{rf}\right\|_\infty \geq \beta$$

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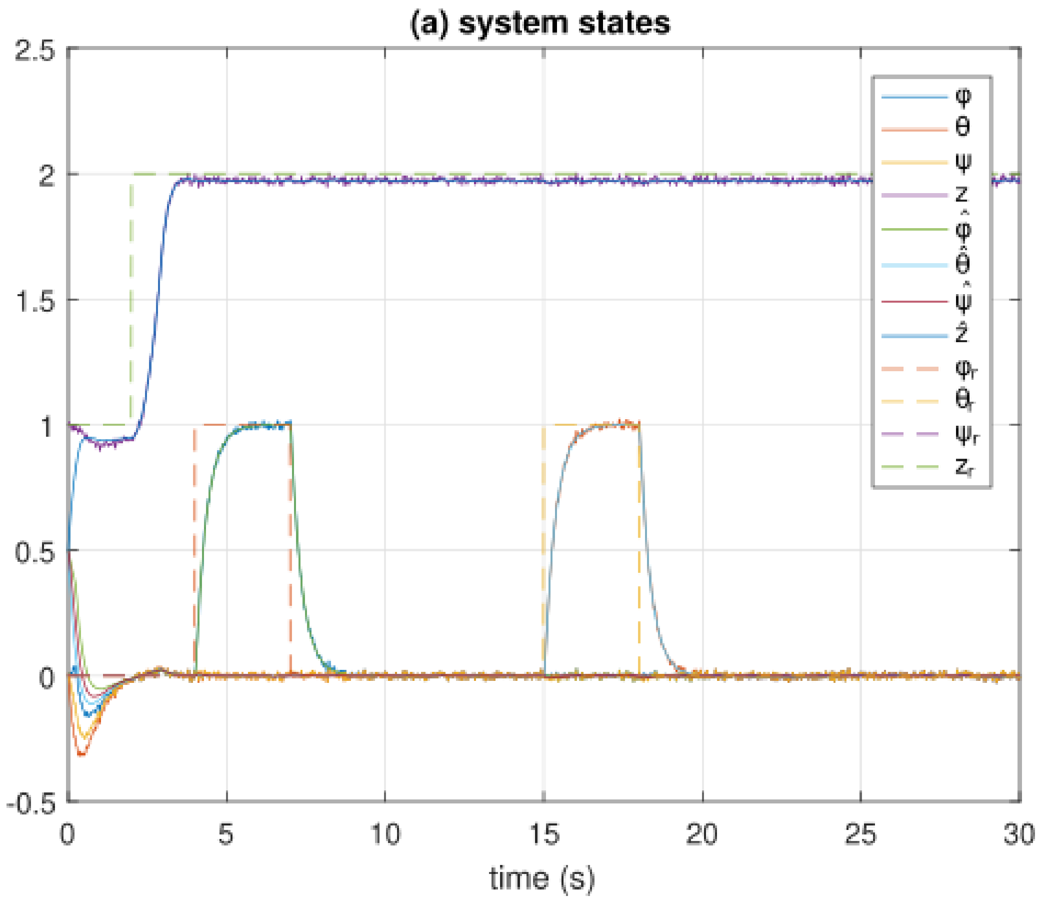
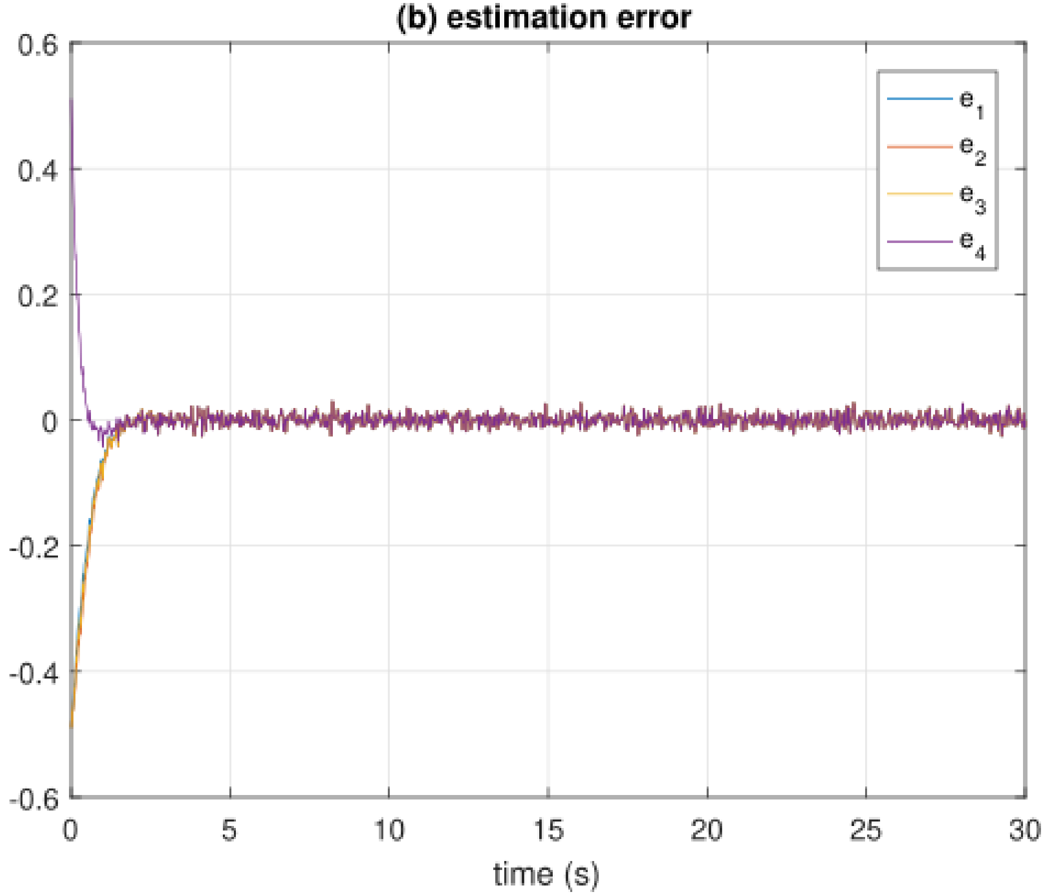
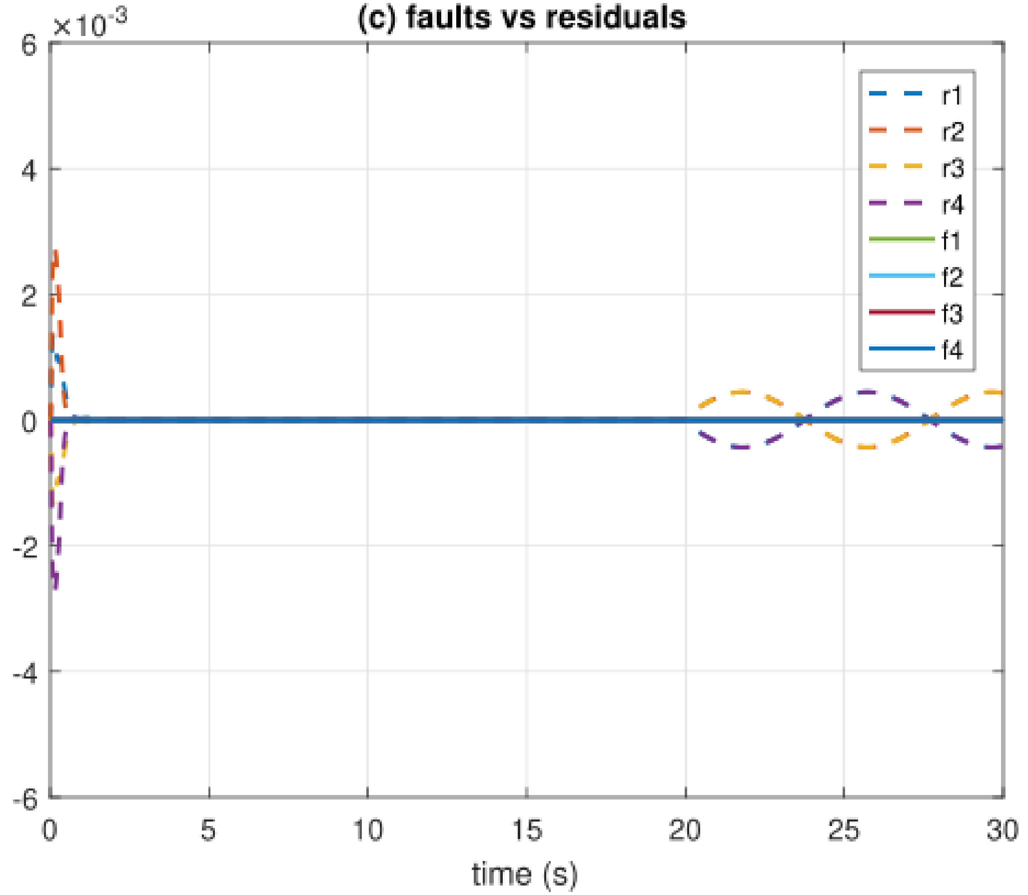
- Reference tracking using H_∞ control



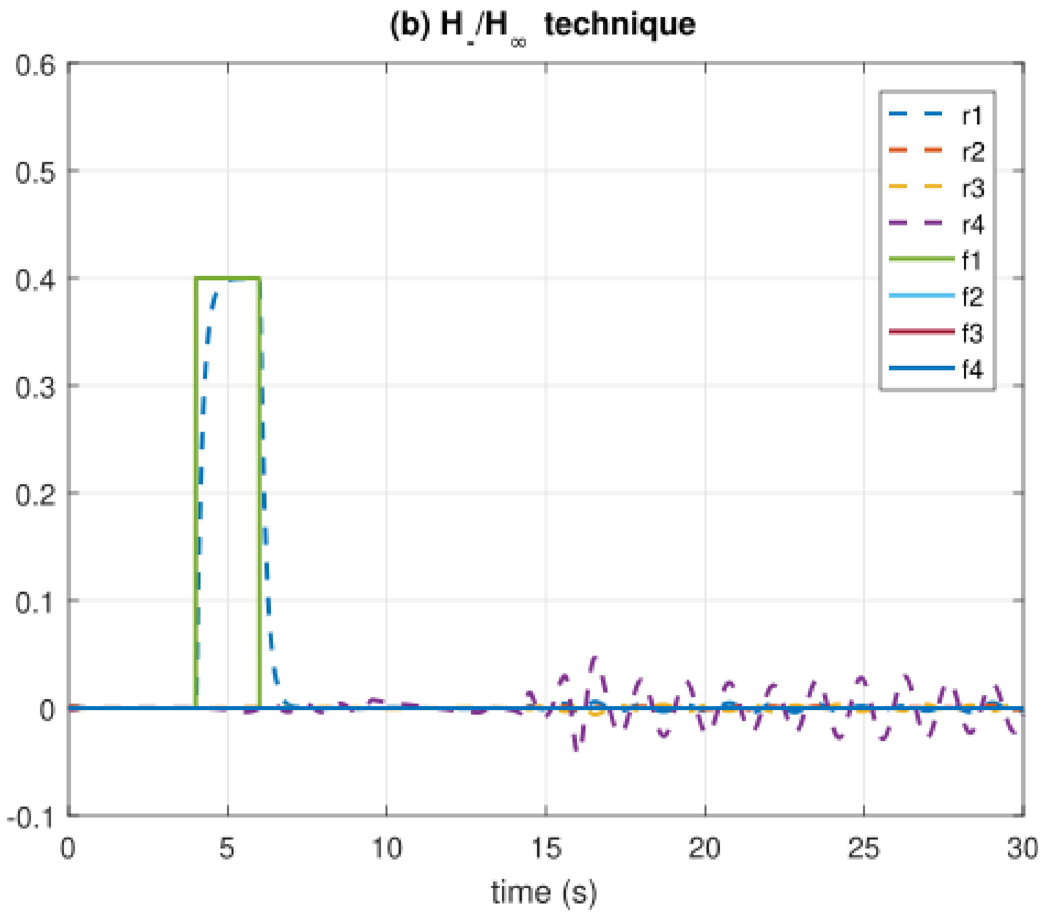
Results

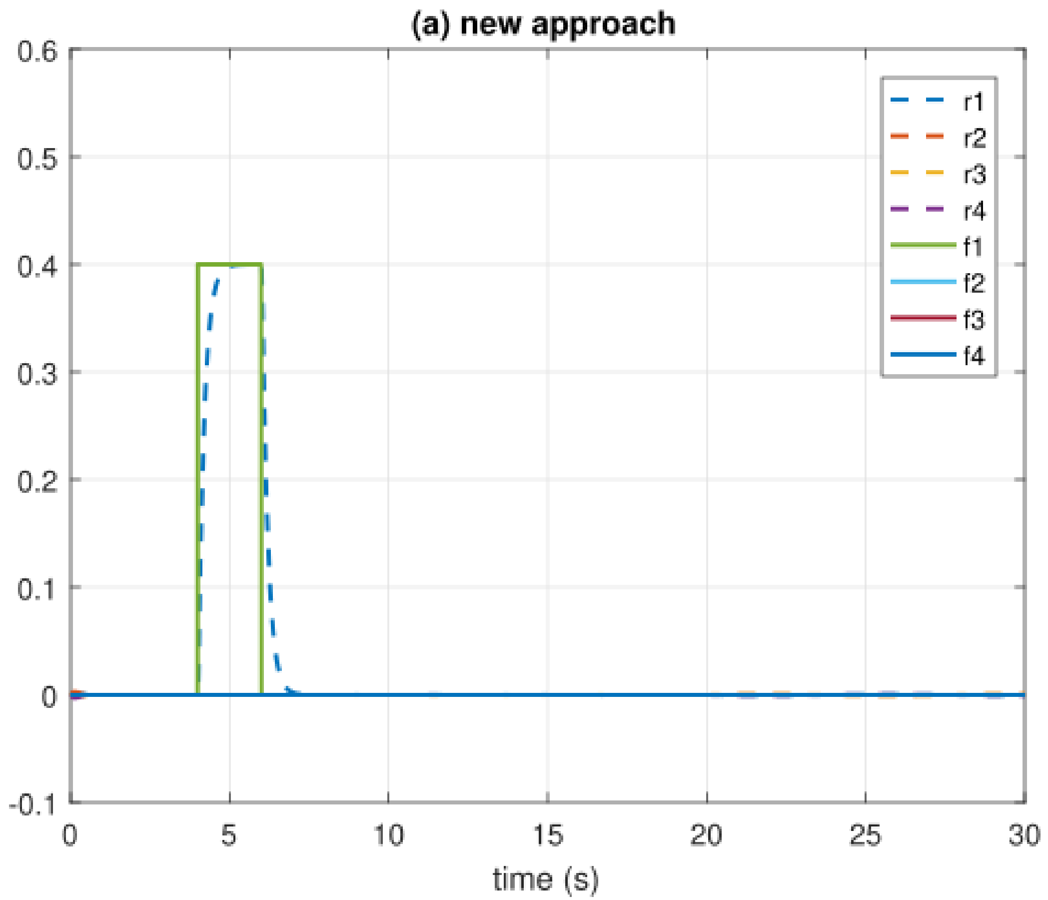
❑ Residual generator in fault free case





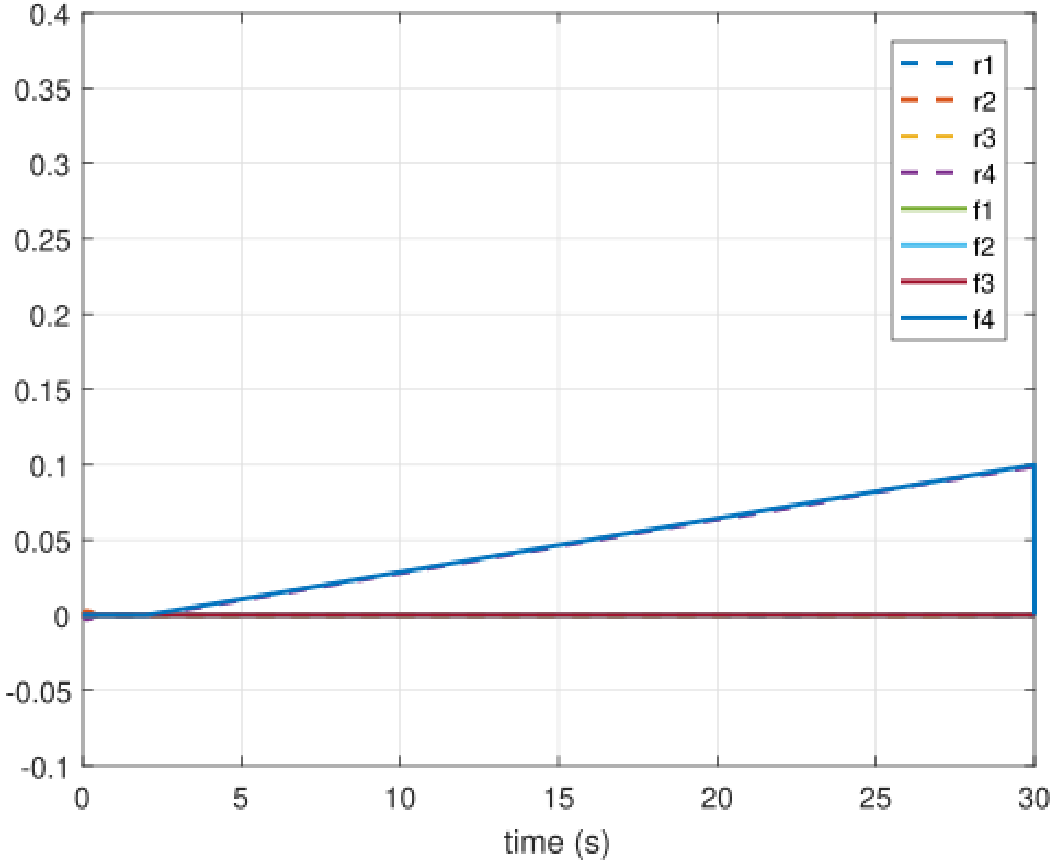
New approach vs H_- / H_∞ technique





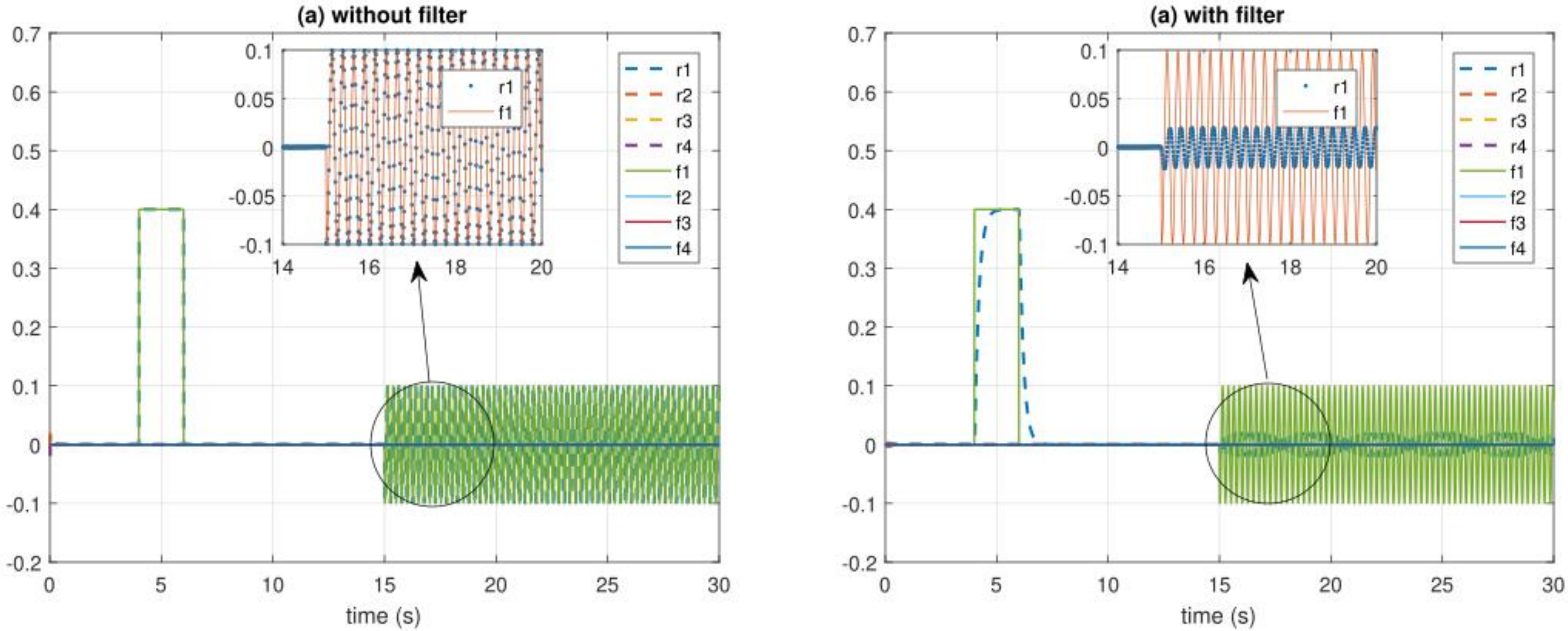
Results

❑ Battery level estimation



Results

❑ Post bandpass filter importance



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Conclusions

- LPV modeling gives more realistic and dynamic system model.
- Robustness of the controller is very essential feature.
- The H_{-}/H_{∞} observer is enhanced greatly using the auxiliary output.
- Some structural properties of the system may simplify fault estimation.
- A post bandpass filter can improve the obtained residual signal.

Perspectives

- ❑ Fault compensation in case of partial loss of actuators efficiency.
- ❑ Controller reconfiguration in case of one or more actuators failure.
- ❑ Applying the developed algorithms in real time on a drone.

THANK YOU

Questions?