A new scheme for fault detection based on Optimal Upper Bound Interval Kalman Filter

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Outline

1. INTRODUCTION

- Adaptive Degrees of Freedom χ^2-statistic (ADFC) method :
  - + Passive approach,
  - + Adaptive threshold technique,
  - + Based on *Optimal Upper Bound Interval Kalman Filter* used as residual generator
- Optimal Upper Bound Interval Kalman Filter (OUBIKF) [Lu et al., 2019] :
  - + a Standard Kalman Filter (SKF) - based method,
  - + Linear Discrete-time System,
  - + Mixed uncertainty : stochastic or bounded uncertainties in system parameters
  - + Interval analysis.

2. PRELIMINARY

Preliminary (1/3)

- Positive semi-definite matrix : $M \succeq 0$
- M is called an *upper bound* of N , denoted by $N \preceq M$:

$$N \preceq M \qquad \Longleftrightarrow \qquad M - N \succeq 0$$

- Ω : non empty set of real square matrices.
  K is an *upper bound* of Ω, denoted $\Omega \preceq K$:

$$\Omega \preceq K \qquad \Longleftrightarrow \qquad M \preceq K, \ \forall M \in \Omega.$$

Preliminary (2/3)

- real interval matrix : $[X] = ([x_{ij}])$, $i = 1 : p$, $j = 1 : q$,
- $X = (x_{ij}) \in [X] \iff x_{ij} \in [x_{ij}],\ \forall i = 1 : p,\ \forall j = 1 : q,$
- $\sup([X]) \triangleq (\sup([x_{ij}])) \equiv \overline{X}$, $\inf([X]) \triangleq (\inf([x_{ij}])) \equiv \underline{X}$,
- $\mathrm{mid}([X]) \triangleq (\overline{X} + \underline{X})/2$, $\mathrm{rad}([X]) \triangleq (\overline{X} - \underline{X})/2$,
- $\mathrm{width}([X]) \triangleq \overline{X} - \underline{X}$,
- $S_+([X]) \triangleq \left\{X \in [X] : X = X^T \text{ and } X \succeq 0\right\}$,
- $BS_+([X]) \triangleq \left\{K : K = K^T,\ K \succeq 0,\ S_+([X]) \preceq K\right\}$: the set of symmetric positive semi-definite upper bounds of $S_+([X])$.
- Arithmetic operators $(+, -, \times, \div)$ and all other notions are used as defined in [Jaulin et al., 2001].

Preliminary (3/3)

OUBIKF : based on the following two theorems introduced in [Lu et al., 2019]

- **Assumptions** : $[M] = ([m_{ij}])$: symmetric and $S_+([M]) \neq \emptyset$.
- **Theorem 1 [Existence of Optimal upper bound]** :
  - $\alpha_* \triangleq \sup_{M \in S_+([M])} \{\lambda_{\max}(M)\} < \infty$,
  - $\alpha_* I$ is the optimal upper bound of $S_+([M])$ in the set $BS_+([M])$ in the sense of operator norm minimization,
  - α_* is called the **optimal value** of $BS_+([M])$.
- **Theorem 2 [Bounds of Optimal value α_*]** :
  Define $Max = (Max_{ij})$ as :

$$Max_{ij} = \begin{cases} \overline{m}_{ij} & , \text{ if } \mathrm{mid}([m_{ij}]) \geq 0 \\ \underline{m}_{ij} & , \text{ otherwise} \end{cases}$$

  then : $\alpha_* \leq \|Max\|_F$.

  In addition, if $Max \succeq 0$ then : $\lambda_{\max}(Max) \leq \alpha_* \leq \|Max\|_F$.

3. MAIN RESULTS

Formulation

- System under consideration :

$$\begin{cases} x_k = A_k x_{k-1} + B_k u_k + w_k \\ y_k = C_k x_k + D_k u_k + v_k + f_k^s \end{cases} , \quad k \in \mathbb{N}^*,$$

- **Assumptions (H)**
  - $w_k \sim \mathcal{N}(0, Q_k)$, $v_k \sim \mathcal{N}(0, R_k)$, $x_0 \sim \mathcal{N}(0, P_0)$,
  - For $F_k \in \{A_k, B_k, C_k, D_k, Q_k, R_k\}$, F_k : unknown, deterministic and belonging to known $[F]$ resp. ,
  - x_0, $\{w_1 : w_k\}$ and $\{v_1 : v_k\}$: mutually independent.

- $f_k^s \in \mathbb{R}^{n_y}$: sensor fault vector.
  + multiple faults : some (or all) sensors are faulty,
  + single fault : only one sensor is faulty.

  A sensor fault $\longleftrightarrow$ one component of f_k^s.

  $f_k^s = 0$: fault free case.

Principle of the method (1/2)

+ Compute $[r_k]$ using OUBIKF as a residual generator

+ Find Σ_k : $S_+([S_k]) \preceq \Sigma_k$, $\Sigma_k = a_k I$ $(a_k \in \mathbb{R}^{*+})$, where S_k : covariance of r_k, $S_k \in [S_k]$.

+ Use $U_k = \sup(\mathrm{abs}([r_k]^T \Sigma_k^{-1} [r_k])) = \sup(\mathrm{abs}([r_k]^T [r_k]/a_k))$ as the statistic for hypothesis testing.

  $U_k \approx \chi^2(\kappa_k n_y)$ where $\kappa_k = \sum_{i=1}^{n_y} \mathrm{width}([r_{k,i}])/n_y$: **adaptive amplifier coefficient (a.c.c.)**

$$\mathrm{abs}([a\,,b]) \triangleq \begin{cases} [\min(|a|,|b|)\,,\max(|a|,|b|)] & ,\ 0 \notin [a\,,b] \\ [0\,,\max(|a|,|b|)] & ,\ 0 \in [a\,,b]. \end{cases}$$

Principle of the method (2/2)

+ **adaptive threshold δ_k determined by** $\mathbb{P}(\chi^2(\kappa_k n_y) > \delta_k) = \alpha$ with a chosen significance level α.

+ **Fault detection test :**
  - (H_0) : $U_k \leq \delta_k$, no error occurred,
  - (H_1) : $U_k > \delta_k$, an error occurred.

+ **Adjustment procedure :**
  - In a window of size w :

$$\#\{\text{consecutive error occurrences}\} \leq w \quad \longrightarrow \quad \text{dismissed.}$$

  - all detection signals will be shifted to the left $\lfloor w/2 \rfloor$ steps ($\lfloor . \rfloor$ is the floor function).

Evaluation indicators

- N iterations
- **faults occur in a region $\mathcal{R} \subset \{1 : N\}$ with length l** $(0 \leq l \leq N)$
- $\mathcal{R}$: a range or union of ranges (called hereafter an *error range* for simplicity).

Define :

+ *Detection Rate* : $\mathrm{DR} = \dfrac{\sum_{k \in \mathcal{R}} \mathbb{I}(\pi_k = 1)}{l} \times 100\%,$
+ *No Detection Rate* : $\mathrm{NDR} = 100\% - \mathrm{DR},$
+ *False Alarm Rate* : $\mathrm{FAR} = \dfrac{\sum_{k \notin \mathcal{R}} \mathbb{I}(\pi_k = 1)}{N - l} \times 100\%,$
+ *Efficiency* : $\mathrm{EFF} = \mathrm{DR} - \mathrm{FAR}.$

4. APPLICATION

Bicycle vehicle model

- a nonlinear continuous-time model, discretized/linearized (more details can be found in [Fergani, 2014])

$$\begin{bmatrix} \dot{\beta}(t) \\ \ddot{\psi}(t) \end{bmatrix} = \begin{bmatrix} \frac{-C_f - C_r}{mv} & 1 + \mu \frac{-l_r C_r - l_f C_f}{mv^2} \\ \frac{-l_r C_r - l_f C_f}{I_z} & \frac{-l_f^2 C_f - l_r^2 C_r}{I_z v} \end{bmatrix} \begin{bmatrix} \beta(t) \\ \dot{\psi}(t) \end{bmatrix} + \begin{bmatrix} \frac{C_f}{mv} & 0 & 0 & 0 \\ \frac{l_f C_f}{I_z} & \frac{1}{I_z} & \frac{S_r R t_r}{2 I_z} & -\frac{S_r R t_r}{2 I_z} \end{bmatrix} \begin{bmatrix} \delta \\ M_{dz} \\ T_{b_{rl}} \\ T_{b_{rr}} \end{bmatrix} . \quad (1)$$

- state variable : the sideslip angle $\beta(k)$ and the vehicle yaw $\psi(k)$
- sampling time $T = 0.05s \longrightarrow A_d, B_d, C_d, D_d$: point matrices
- simulate interval matrix $[F]$ for $F \in \{A_d, B_d, C_d, D_d\}$ with radius chosen at random in $[0, 0.5]$

General settings

- $f_k^s = b.\mathbf{1}$, $\forall k \in \mathcal{R}$ with length $l = 50$,
- Process $L = 100$ times of simulations for each $b \in \{0 : 5 : 30\}$:
  - + *Scenario 1*. $\{x_k, y_k\}_{k=1:N}$: fixed, $\mathcal{R}$: random.
  - + *Scenario 2*. $\mathcal{R}$: fixed, $\{x_k, y_k\}_{k=1:N}$: resimulated.

  $\longrightarrow$ **Indicators are computed for each of L simulation times and their means are yielded afterward**.
- Comparison with the method proposed in [Raka and Combastel, 2013] (called method B in the next).

Simulation results (1/5)

Scenario 1 : considers the method performance in term of fault values b and the positions at which errors occur (in $\mathcal{R}$) w.r.t. a given measurement sample $\{y_k\}_{k \in 1:N}$.

Table 1. Fault detection for scenario 1 without the adjustment procedure

b	τ	DR%	NDR%	FAR%	EFF%
0	0	1.26	98.74	1.03	0.23
5	13.8	5.34	94.66	1.04	4.30
10	27.5	21.12	78.88	1.04	20.08
15	41.3	67.28	32.72	1.17	66.11
20	55.0	94.94	5.06	1.26	93.68
25	68.8	98.64	1.36	1.36	97.28
30	82.5	99.84	0.16	1.42	98.42

Simulation results (2/5)

Scenario 1 :

Table 2. Fault detection for scenario 1 with the adjustment procedure

b	τ	DR%	NDR%	FAR%	EFF%
0	0	0	100	0	0
5	13.8	3.30	96.70	0.01	3.29
10	27.5	17.70	82.30	0.02	17.68
15	41.3	63.54	36.46	0.05	63.49
20	55.0	96.36	3.64	0.06	96.30
25	68.8	99.96	0.04	0.15	99.81
30	82.5	100	0	0.38	99.62

Simulation results (3/5)

Scenario 2 : show the effects of different measurement samples $\{y_k\}_s$, $k = 1 : N$, $s = 1 : L$, on the fault detection procedure for a given error range $\mathcal{R}$. These effects come from random noises existing inside of y_k.

Table 3. Fault detection for scenario 2 without the adjustment procedure

b	τ	DR%	NDR%	FAR%	EFF%
0	0	3.34	96.66	1.90	1.44
5	13.8	3.08	96.92	2.36	0.72
10	27.5	19.24	80.76	2.41	16.83
15	41.3	82.48	17.52	2.18	80.30
20	55.0	94.74	5.26	2.48	92.26
25	68.8	98.66	1.34	2.28	96.38
30	82.5	99.88	0.12	2.37	97.51

Simulation results (4/5)

Scenario 2 :

Table 4. Fault detection for scenario 2 with the adjustment procedure

b	τ	DR%	NDR%	FAR%	EFF%
0	0	2.8	97.20	1.44	1.36
5	13.8	2.16	97.84	1.84	0.32
10	27.5	14.06	85.94	1.97	12.09
15	41.3	82.42	17.58	1.62	80.80
20	55.0	96.96	3.04	1.82	95.14
25	68.8	99.8	0.20	1.71	98.10
30	82.5	100	0	1.89	98.11

Simulation results (5/5)

Comparison with method B

The method B consists in :

- applying interval observer for linear continuous time system with additive and multiplicative disturbances
- computing adaptively upper bounds (ub_t) and lower bounds (lb_t) of residuals r_t,
- fault detection rule : a fault is detected if $0 \notin [\mathrm{lb}_t, \mathrm{ub}_t]$.

Table 5. Method B of [Raka and Combastel, 2013] versus the Proposed FD scheme for $b = 20$.

	DR%	NDR%	FAR%	EFF%
Method B	8.14	91.86	5.64	2.50
Proposed method	98	2	0.12	97.88

5. CONCLUSION AND PERSPECTIVE

1. A **new scheme for fault detection** combining :
   - OUBIKF
   - adaptive degrees of freedom χ^2-statistics hypothesis test method.
2. **simulations results** highlight the potential of this approach.
3. the **modified EFF index** : $\text{EFF} = c_1.\text{DR} - c_2.\text{FAR}$, $c_1, c_2 \in [0, 1]$:
   $\longrightarrow$ control the importance of the two indexes DR and FAR .
4. **potential perspective** :
   - investigating the possibility of adjusting the a.a.c. κ_k according to different purposes of fault detection.
   - extending to other types of fault, e.g. actuator fault.

$\longrightarrow$ the great flexibility of this method by adjusting tuning factors makes it suitable to multiple applications.

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THANKS FOR YOUR ATTENTION

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Q & A !