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Remaining Useful Life Prognostics for Deteriorating Feedback Control Systems Using Stochastic Diffusion Process

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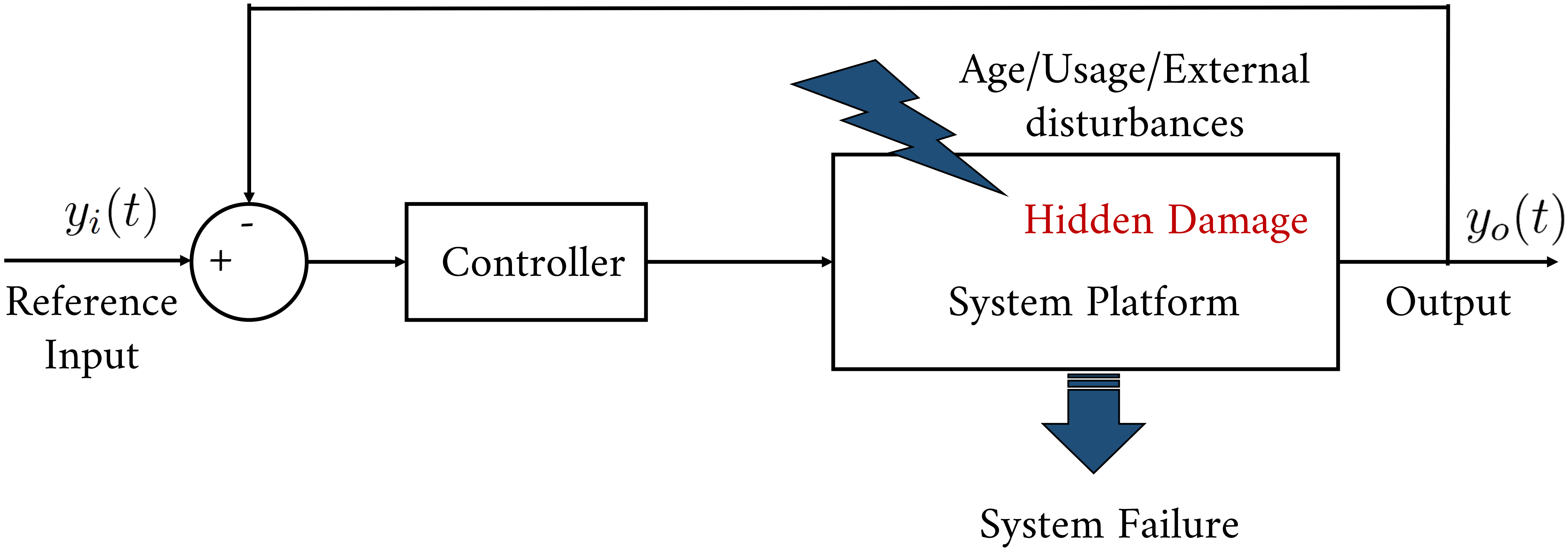
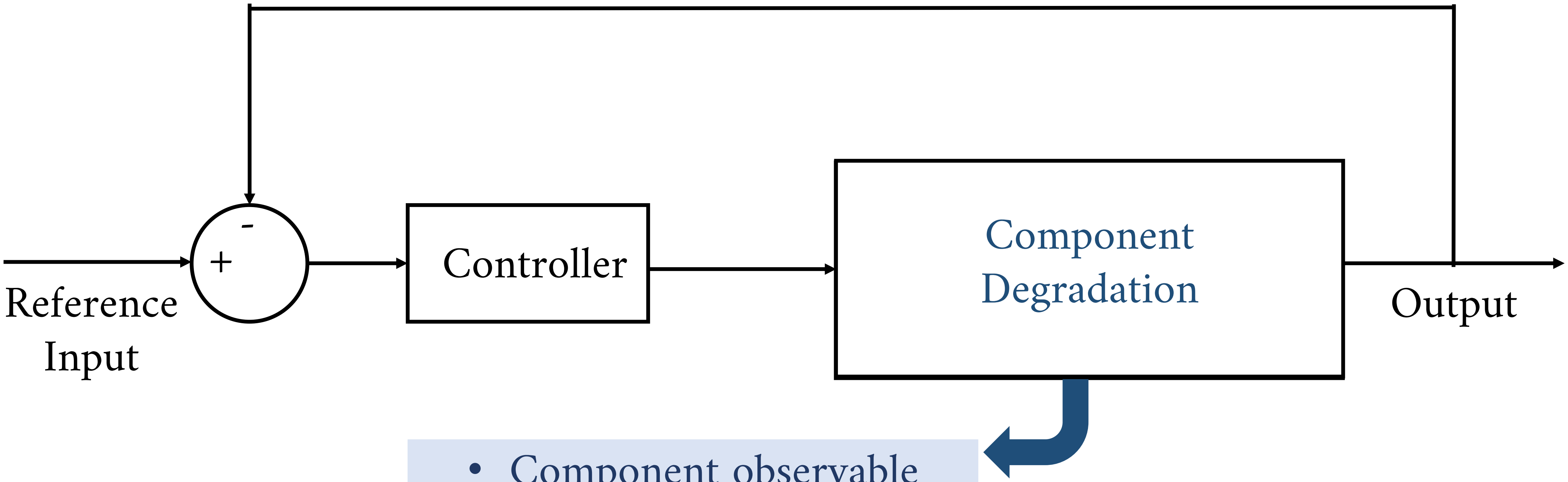
- A deteriorating feedback control system



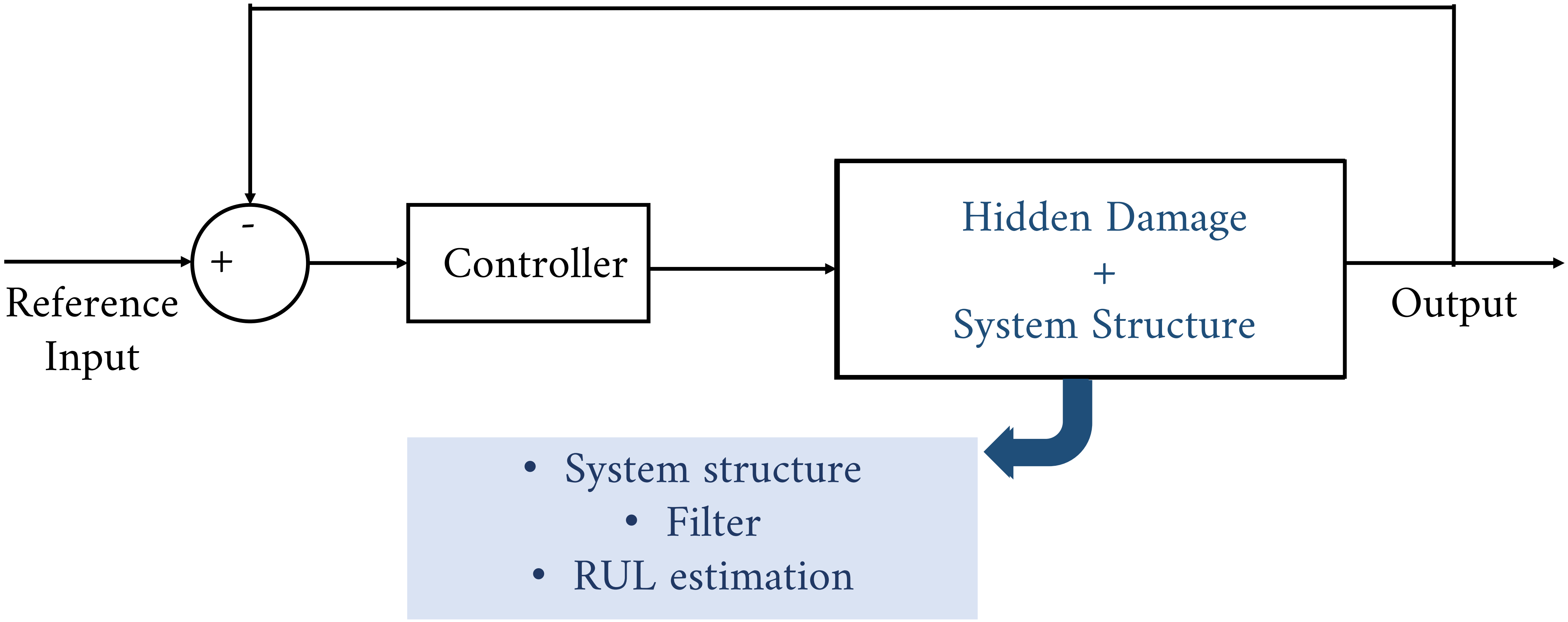
- Remaining useful life (RUL)

- Remaining useful life (RUL) Component-level

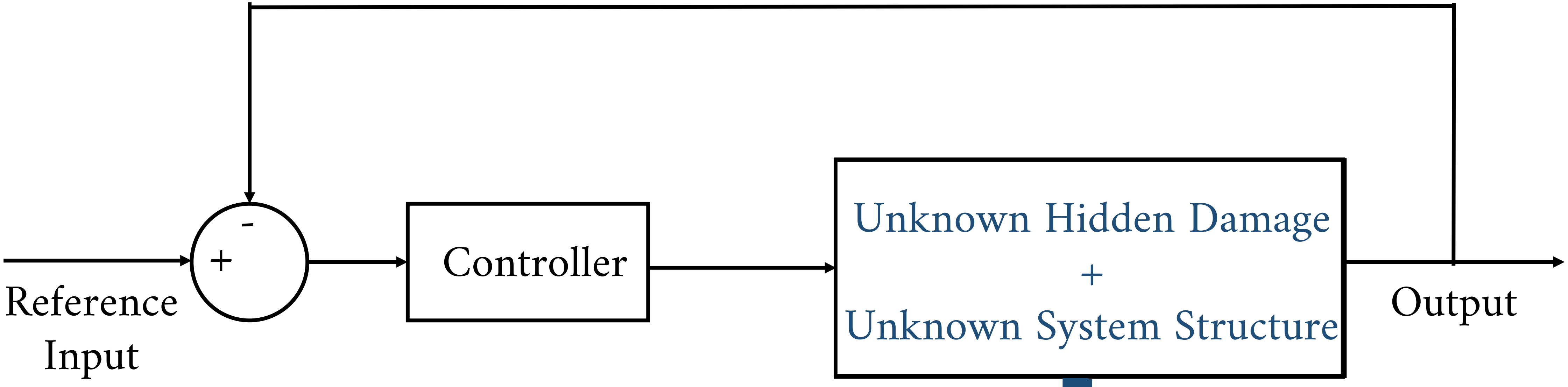


- Component observable
- Stochastic process modeling
- Probability calculation

- Remaining useful life (RUL) ~~Co… …evel~~ Physical-based



- Remaining useful life (RUL) Time-consuming



- Input-Output
- Maximum gain
- Learning RUL

❖ Degradation modeling of a deteriorating feedback control system with its RUL prognosis

- System-level degadation index
- degradation modeling

- Probability density function (PDF)/
  Cumulative density function (CDF) calculation of RUL

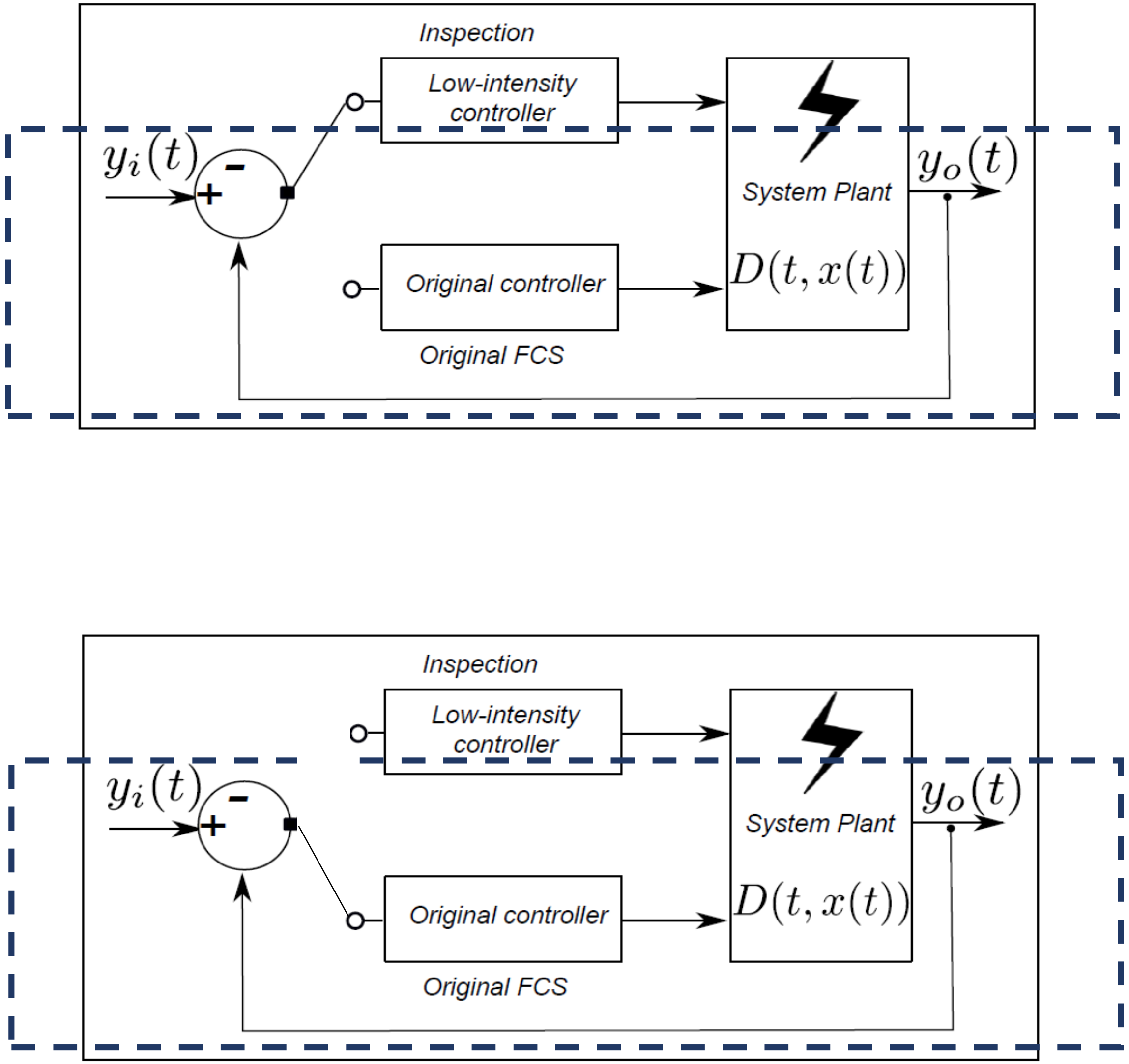


Fig. 1.: Inspection scheme.

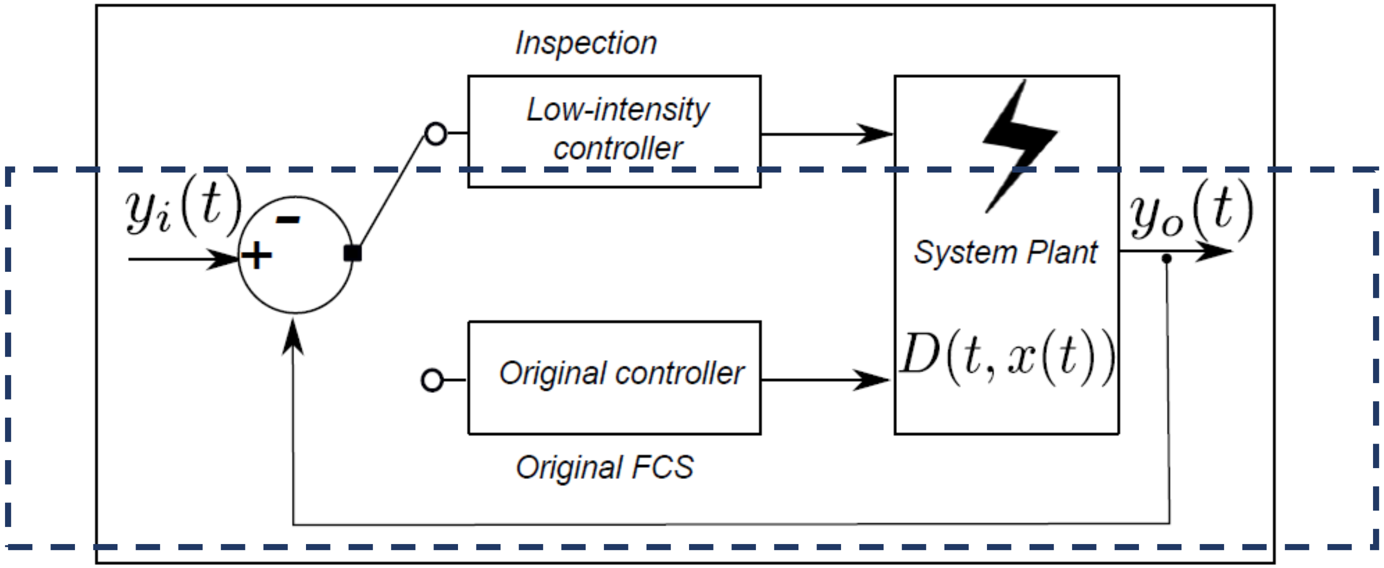


Fig. 1.: Inspection scheme.

- Degradation index construction only from system input and output
- Degradation modeling by stochastic differential process with its RUL estimation via PDF/CDF approximation methods

- System

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(D(t, \mathbf{x}(t)))\mathbf{x}(t) + \mathbf{B}(D(t, \mathbf{x}(t)))u(t), \\ y_o(t) = \mathbf{C}(D(t, \mathbf{x}(t)))\mathbf{x}(t), \end{cases}$$

- Hidden damage

$$D(t, \mathbf{x}(t)) = D(0, \mathbf{x}(0)) + \int_0^t \mu(r, \mathbf{x}(r))dr + \int_0^t \sigma(r, \mathbf{x}(r))dB(r).$$

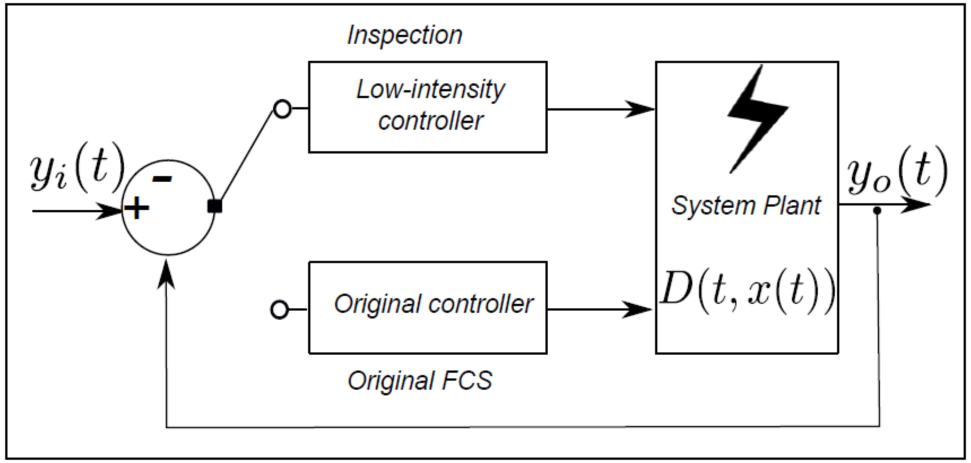


Fig. 1.: Inspection scheme.

$$H(s) = \frac{Y_o(s)}{Y_i(s)},$$

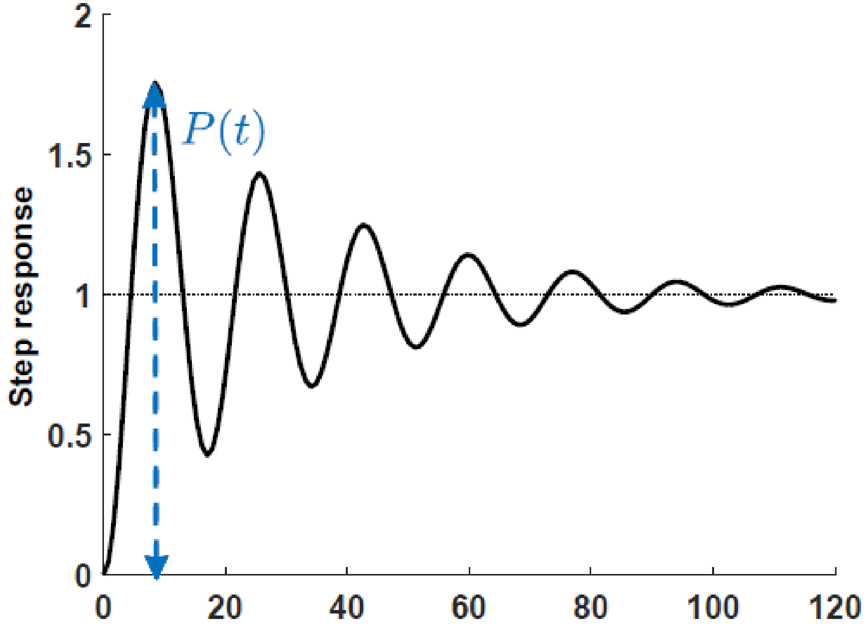


Fig. 2.: $P(t)$ from the step response of the transfer function $H(s)$.

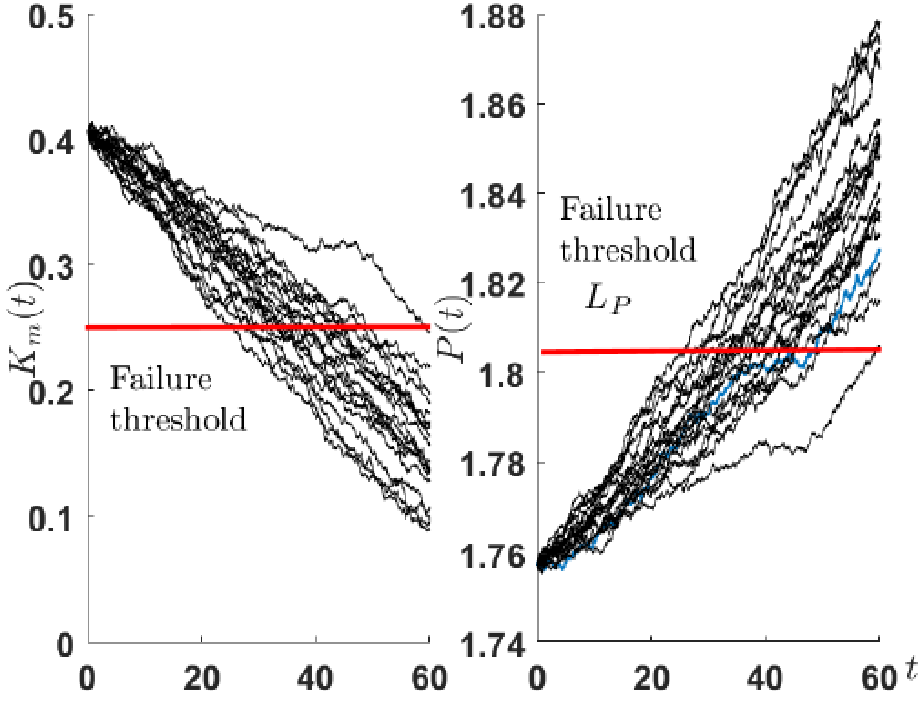


Fig. 3.: $K_m(t)$ and $P(t)$ in $[0, 60]$.

- Stochastic differential process (SDP)

$$dX(t) = \mu(t, X(t))dt + \sigma(t, X(t))dB(t)$$

$$\mu(t, X(t)) = a_{t_m}t^m + \cdots + a_{t_1}t + a_{X_m}X^m + \cdots + a_{X_1}X + a_c$$
$$\sigma(t, X(t)) = b_{t_m}t^m + \cdots + b_{t_1}t + b_{X_m}X^m + \cdots + b_{X_1}X + b_c,$$

- Maximum likelihood estimation

- SDP to Standard Brownian Motion (SBM)

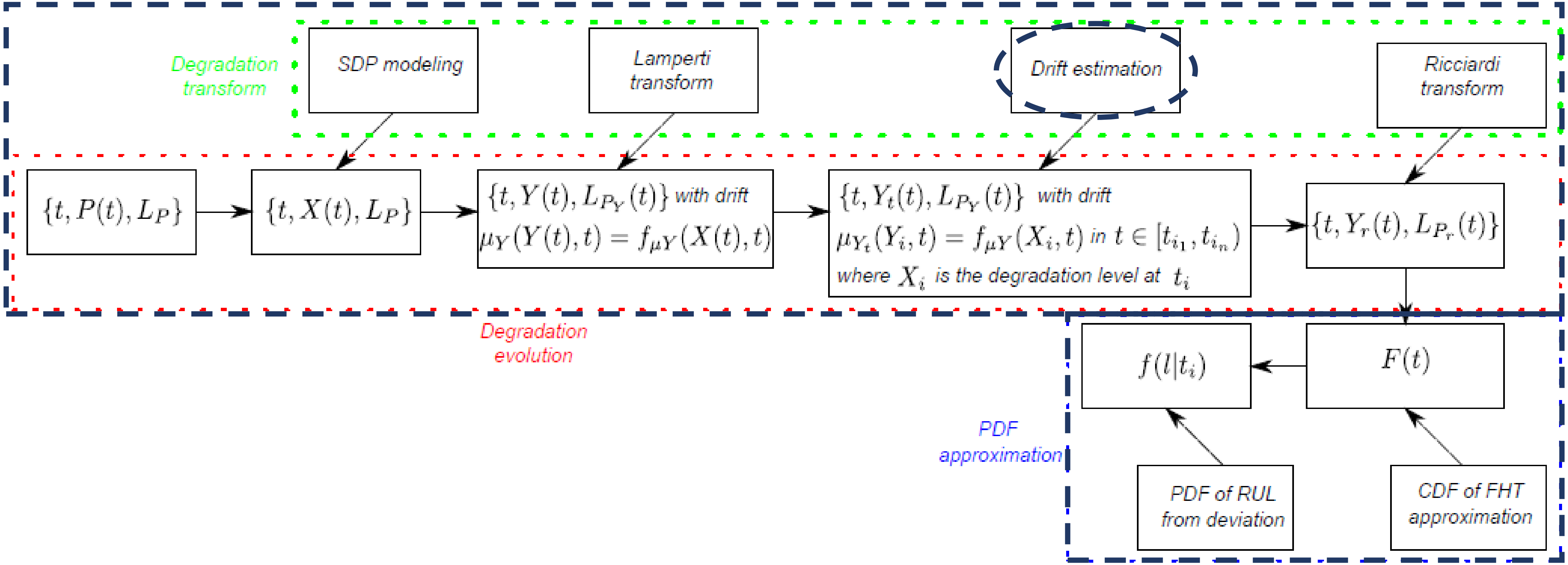


FIG. 6: The frame of degradation process transformation and its RUL estimation.

- Lamperti Transform

$$Y(t) := \varphi(t, X(t)) := \int \frac{1}{\sigma(t,x)} du \Big|_{x=X(t)}, \qquad dY(t) = \mu_Y(t, Y(t))dt + dB(t)$$

$$L_{P_Y}(t) = \varphi(t, L_P) := \int \frac{1}{\sigma(t,x)} dx \Big|_{x=L_P},$$

$$Y_c := Y(t_c) = \int_\xi^x \frac{1}{\sigma(t,u)} du \Big|_{x=X_c:=X(t_c)} \qquad dY_t(t) = \mu_{Y_t}(t, Y_c)dt + dB(t)$$

- Ricciardi Transform

$$\tilde{X}(\tilde{t}) \triangleq \psi(t,y) = e^{-\frac{1}{2}\int_0^t c_2(\tau)d\tau} y - \frac{1}{2}\int_0^t c_1(\tau) e^{-\frac{1}{2}\int_0^\tau c_2(r)dr} d\tau \qquad L_{f_R}(t) = \psi(t, L_{f_L}(t)).$$

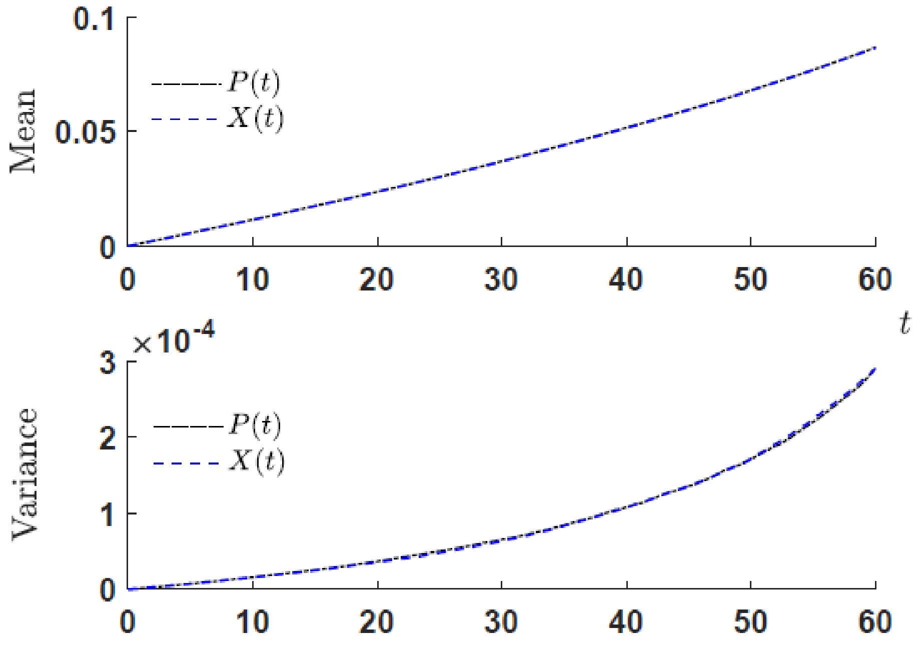
$$\tilde{t} \triangleq \phi(t) = \int_0^t e^{-\int_0^\tau c_2(r)dr} d\tau,$$

$$\mu_Y(t, Y(t)) = \frac{1}{2}\left(c_1(t) + \int_z^y c_2(t)dy\right)$$

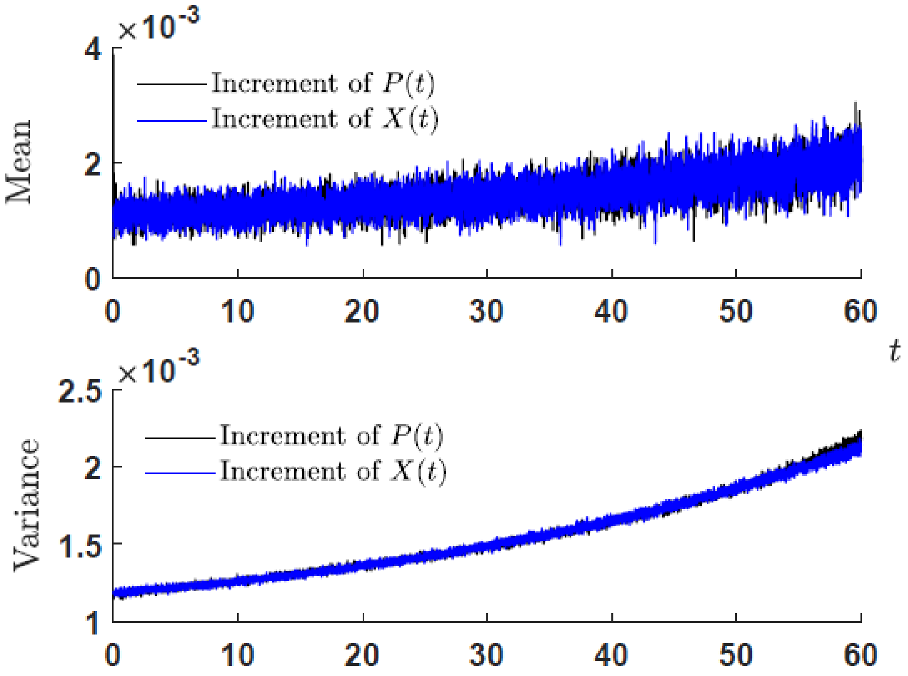
- CDF approximation of SBM with curvy boundary : PDF via deviation

$$F(t) = \Phi\left(\frac{-\left(L_{P_Y}(t) - \int_0^t \mu_Y(\tau, Y_i)d\tau - L_{P_Y}(t)t + \mu_Y(t, Y_i)t\right) + \left(\mu_Y(t, Y_i) - L'_{P_Y}(t)\right)t}{\sqrt{t}}\right)$$
$$+ \exp\left(2(\mu_Y(t, Y_i) - L'_{P_Y}(t))\left(L_{P_Y}(t) - \int_0^t \mu_Y(\tau, Y_i)d\tau - L_{P_Y}(t)t + \mu_Y(t_0, Y_i)t0\right)\right)$$
$$\cdot \Phi\left(\frac{-\left(L_{P_Y}(t) - \int_0^t \mu_Y(\tau, Y_i)d\tau - L_{P_Y}(t)t + \mu_Y(t, Y_i)t\right) - \left(\mu_Y(t, Y_i) - L'_{P_Y}(t)\right)t}{\sqrt{t}}\right)$$

$$F(t) = \Phi\left(\frac{-\left(L_{P_Y}(t) - \int_0^t \mu_Y(\tau, Y_i)d\tau - L'_{P_Y}(t_i) + \int_0^{t_i} \mu_Y(\tau, Y_i)d\tau - L'_{P_Y}(t)t + \mu_Y(t, Y_i)t\right) + \left(\mu_Y(t, Y_i) - L'_{P_Y}(t)\right)t}{\sqrt{t - t_i}}\right)$$
$$+ \exp\left(2\left(\mu_Y(t, Y_i) - L'_{P_Y}(t)\right)\left(L_{P_Y}(t) - \int_0^t \mu_Y(\tau, Y_i)d\tau - L'_{P_Y}(t_i) + \int_0^{t_i} \mu_Y(\tau, Y_i)d\tau - L'_{P_Y}(t)t + \mu_Y(t, Y_i)t\right)\right)$$
$$\cdot \Phi\left(\frac{-\left(L_{P_Y}(t) - \int_0^t \mu_Y(\tau, Y_i)d\tau - L'_{P_Y}(t_i) + \int_0^{t_i} \mu_Y(\tau, Y_i)d\tau - L'_{P_Y}(t)t + \mu_Y(t, Y_i)t\right) - \left(\mu_Y(t, Y_i) - L'_{P_Y}(t)\right)t}{\sqrt{t - t_i}}\right)$$



(a) The fitting of $P(t)$.



(b) The fitting of $\Delta P(t)$.

Fig. 4.: Model $P(t)$ by $X(t)$, $t \in [0, 60]$.

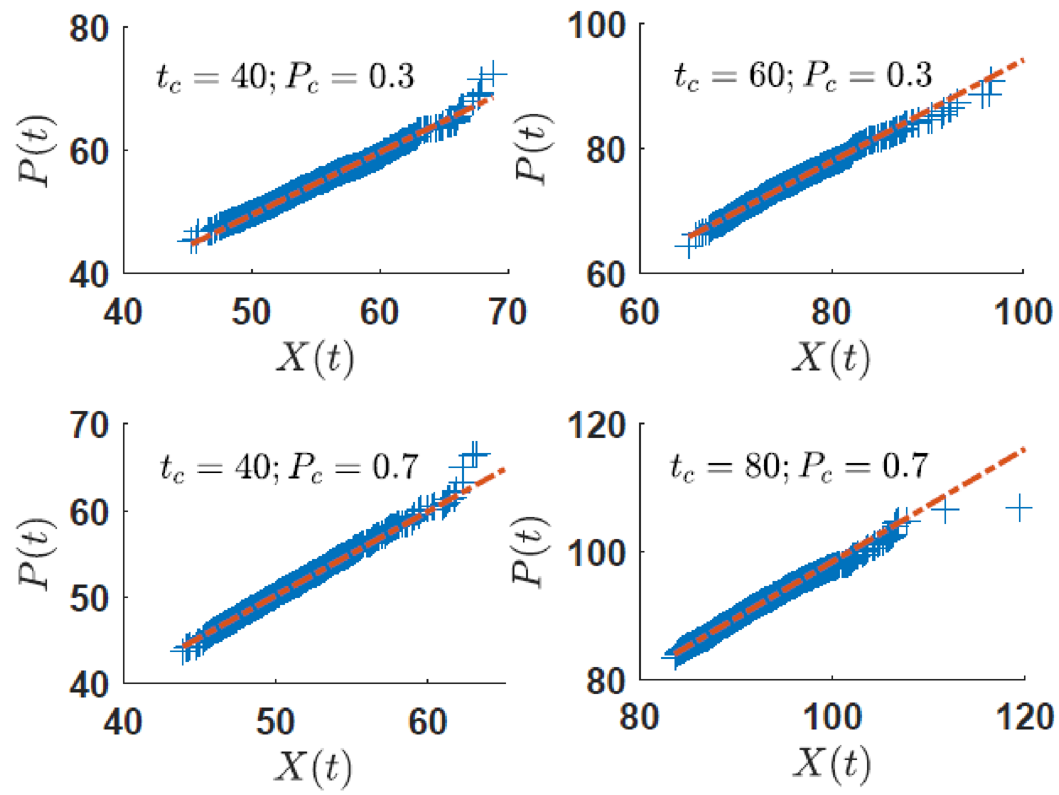
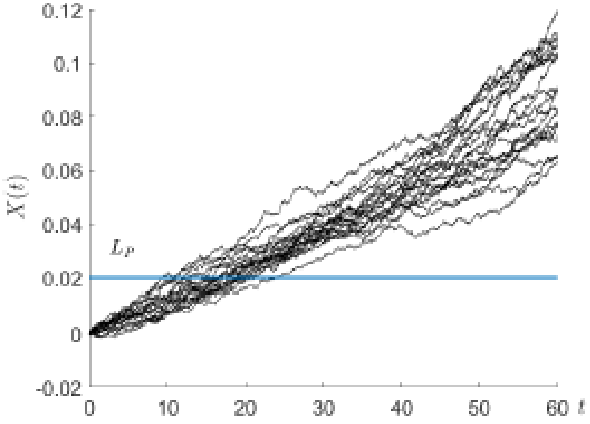
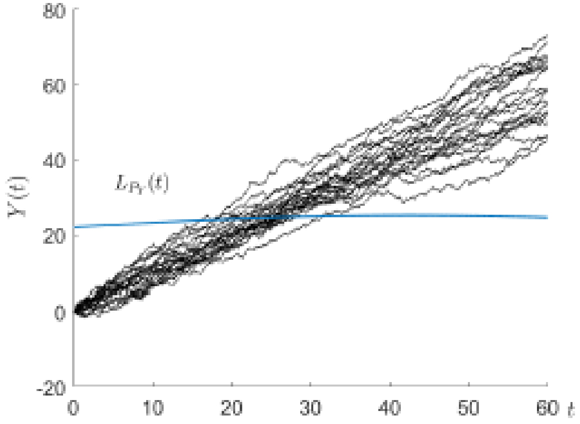


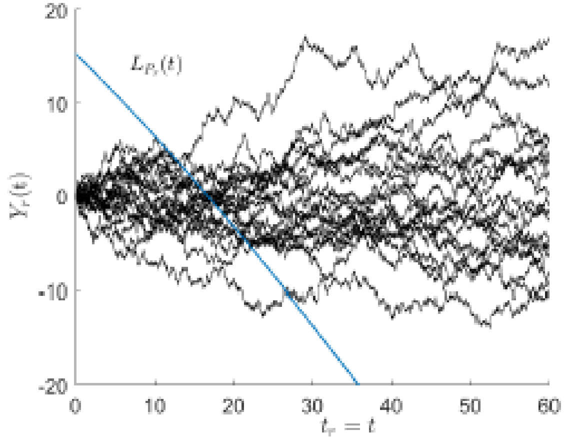
Fig. 5.: Quantile-Quantile plots between $P(t)$ and $X(t)$ under $L_f = P_c + 0.02$ at different groups of inspection dates.



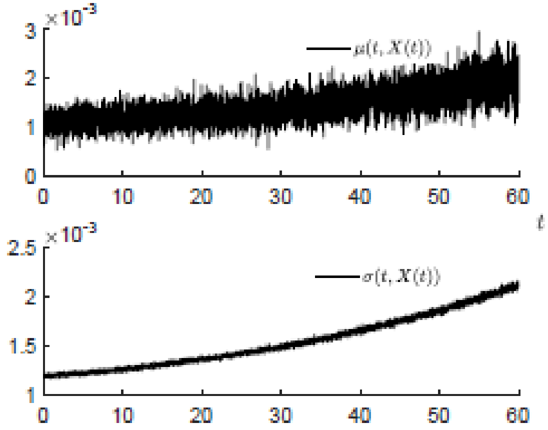
(a) $X(t)$ in $[0, 60]$.



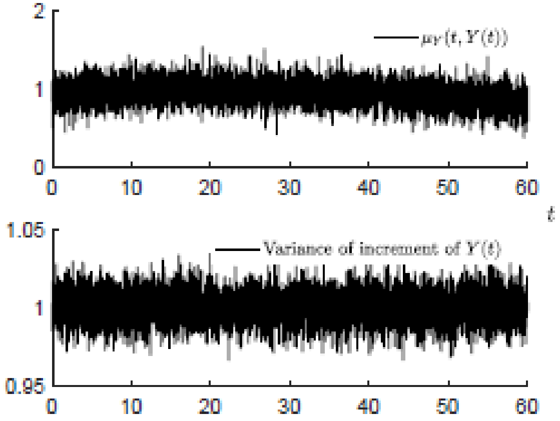
(b) $Y(t)$ in $[0, 60]$.



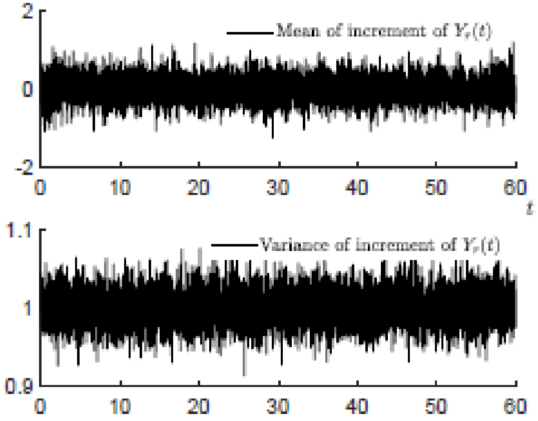
(c) $Y_r(t)$ in $[0, 60]$.



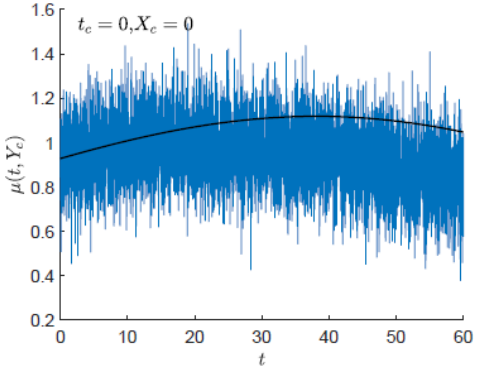
(d) Mean and Variance of $X(t)$.



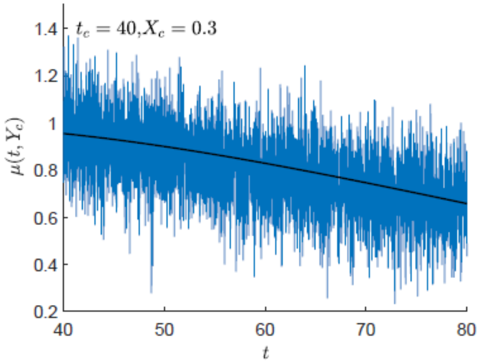
(e) Mean and Variance of $Y(t)$.



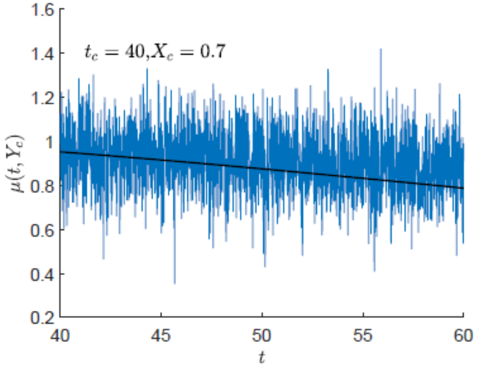
(f) Mean and Variance of $Y_r(t)$.



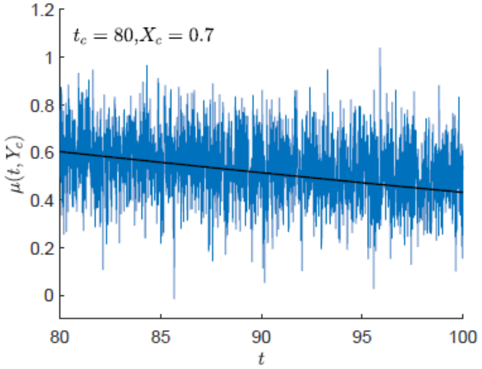
(a) $t_c = 40,\ X_c = 0.03.$



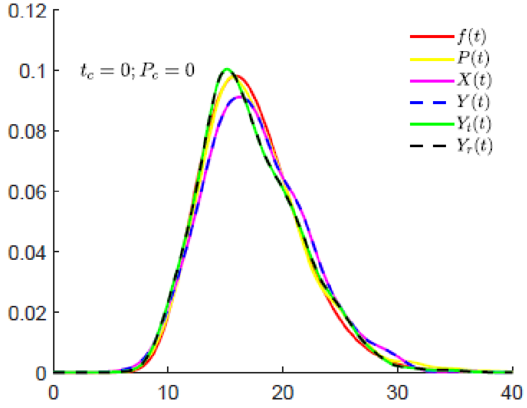
(b) $t_c = 60,\ X_c = 0.03.$



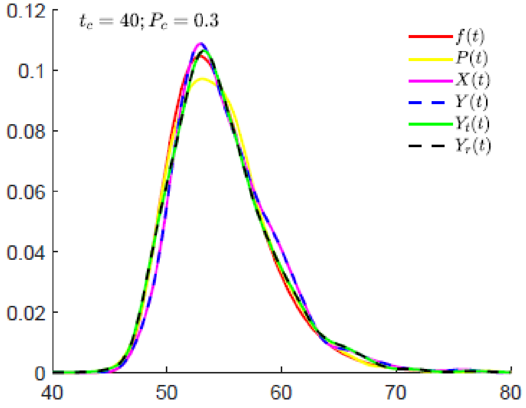
(c) $t_c = 40,\ X_c = 0.07.$



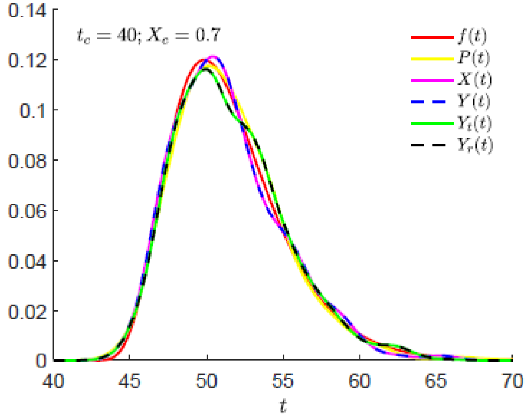
(d) $t_c = 80,\ X_c = 0.07.$



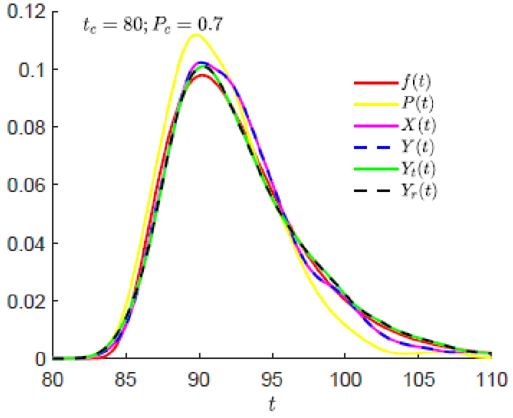
(a) FHT at $t_c = 0,\ P_c = 0.$



(b) FHT at $t_c = 40,\ P_c = 0.3.$



(c) FHT at $t_c = 40,\ P_c = 0.7.$



(d) FHT at $t_c = 80,\ P_c = 0.7.$

- Low-intensity controller → Input—Output →Degradation index construction
- Degradation modeling via SDP with drift and diffusion (polynomial of time and state, respectively)
- Lamperti transform + Ricciardi transoform →SBM
- CDF approximation of SBM with curvy boundary →PDF of SDP

Thanks!

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