Control allocation of deteriorating over-actuated system

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Congrès annuel de la SAGIP
Sûreté / Surveillance / Supervision
May 2025

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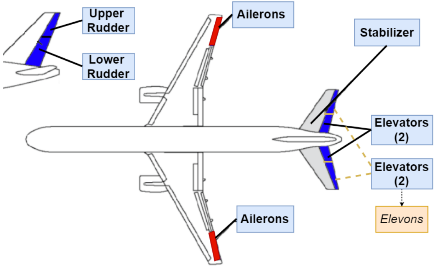
Over-actuated system

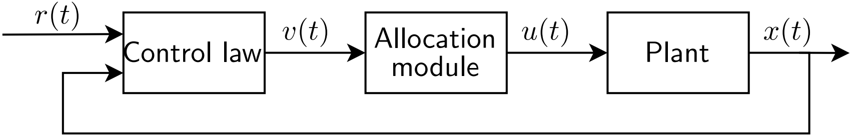
- Definition: More actuators than necessary to control its degrees of freedom.
- Key characteristics:
  - Redundancy
  - Fault tolerance
  - Performance optimization
- Typical applications:
  - Aircraft with multiple control surfaces
  - Electric vehicles with independent motors on each wheel
- Key challenges:
  - How to distribute control effort among redundant actuators?
  - How to adapt the control strategy as the system deteriorates?



Example of actuator redundancy in aircraft[1]

[1] Reconfigurable Dynamic Control Allocation with SDRE As a FTFC for NASA GTM Design.

Control allocation



The structure of an over-actuated system with a control allocation module

$$\dot{x}(t) = Ax(t) + B_u u(t) \quad \Rightarrow \quad \begin{array}{ll} ① & \dot{x}(t) = Ax(t) + B_v v(t), \\ ② & v(t) = Bu(t), \quad B = \left(B_v^T B_v\right)^{-1} B_v^T B_u \end{array}$$

- $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$
- $A \in \mathbb{R}^{n\times n}$, $B_u \in \mathbb{R}^{n\times m}$
- $v(t) \in \mathbb{R}^k$, $k < m$
- $B_v \in \mathbb{R}^{n\times k}$, $B \in \mathbb{R}^{k\times m}$

Control allocation solver

A weighted least squares (WLS) optimization approach:

$$u = \underset{\underline{u} \leq u \leq \bar{u}}{\arg\min} \gamma \left\| W_v (\overset{①}{B} u - v) \right\|^2 + \left\| W_u \left(u \overset{②}{-} u_d \right) \right\|^2 ,$$

①: Tracks the virtual control as closely as possible.

②: Minimizes actuator effort to save energy.

Motivation: Actuators degrade during operation $\rightarrow$ redistribute control effort based on health.

Goal: Design a degradation-based weighting matrix W_u: prioritize healthy actuators, limit degraded ones $\rightarrow$ extend system lifetime.

Plant modeling

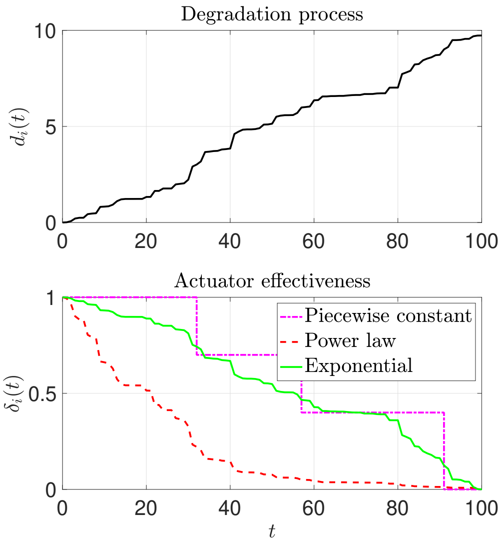
System dynamics	$\dot{x}(t) = Ax(t) + B_u\Delta(t)u(t)$
Degradation model	$d_i(t_{d+1}) = d_i(t_d) + \alpha_i \int_{t_d}^{t_{d+1}} \lvert u_i'(t)\rvert \, dt + \beta_i \int_{t_d}^{t_{d+1}} u_i^2(t) \, dt + \sigma_i \Delta B$
Effectiveness model	$\Delta(t) = \operatorname{diag}\left(\delta_1(t), \cdots, \delta_m(t)\right), \delta_i(t) = f\left(d_i(t_d)\right), 0 \leq \delta_i \leq 1, t_d \leq t \leq t_{d+1}$
Weighting matrix	$W_u(t) = \operatorname{diag}\left(W_{u1}(t), \cdots, W_{um}(t)\right), W_{ui}(t) = g\left(d_i(t_d)\right), t_d \leq t \leq t_{d+1}$

Effectiveness model

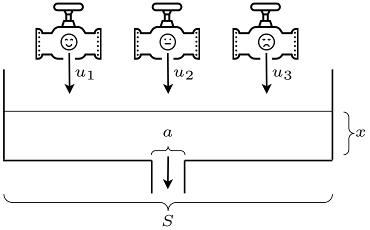
- Piecewise constant

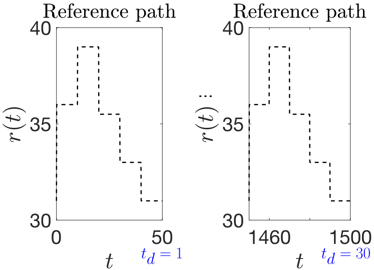
$$\delta_i\left(d_i\right) = \begin{cases} 1, & 0 \leq d_i < \tau_1 \\ \delta_2, & \tau_1 \leq d_i < \tau_2 \\ \vdots & \\ \delta_N, & \tau_{N-1} \leq d_i < d_{\max} \\ 0, & d_i = d_{\max} \end{cases}$$

- Continuous
  - Power law $\delta_i = (1 - \frac{d_i}{d_{\max}})^p$;
  - Exponential $\delta_i = e^{-qd_i}$;
  - Etc.



Design of experiments





System dynamics $\rho S\dot{x} = -\rho a\sqrt{2gx} + B_u \Delta(t) u$

Virtual controller $v(t) = K_p \cdot \left(e(t) + \frac{1}{T_i} \cdot \int_0^t e(z)dz + T_d \cdot \frac{de(t)}{dt} \right)$

Degradation model

$$d_i(t_{d+1}) = d_i(t_d) + \alpha_i \int_{t_d}^{t_{d+1}} |u_i'(t)|\, dt + \beta_i \int_{t_d}^{t_{d+1}} u_i^2(t)\, dt + \sigma_i \Delta B$$

Effectiveness model

- Piecewise constant

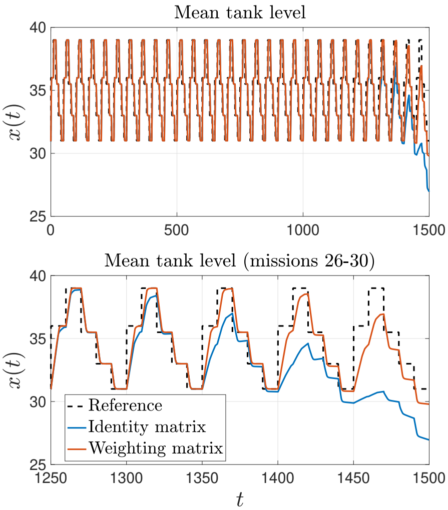
$$\delta_i(t) = \begin{cases} 1, & d_i(t) \le 0.3 d_{\max} \\ 0.9, & 0.3 d_{\max} < d_i(t) \le 0.5 d_{\max} \\ 0.6, & 0.5 d_{\max} < d_i(t) \le 0.8 d_{\max} \\ 0.4, & 0.8 d_{\max} < d_i(t) \le 0.99 d_{\max} \\ 0, & d_i(t) = d_{\max} \end{cases}$$

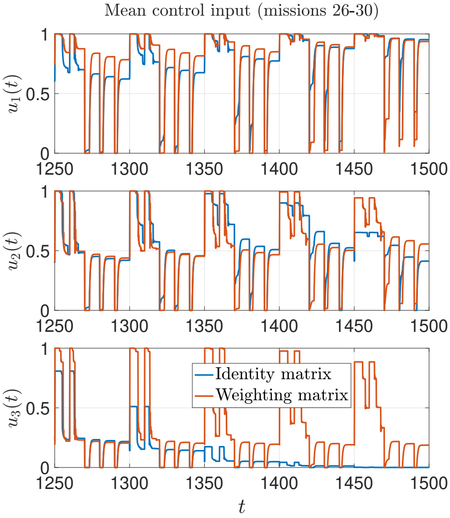
- Continuous $\delta_i(t) = (1 - d_i(t_d)/d_{max})^{0.8}$

Weighting matrix

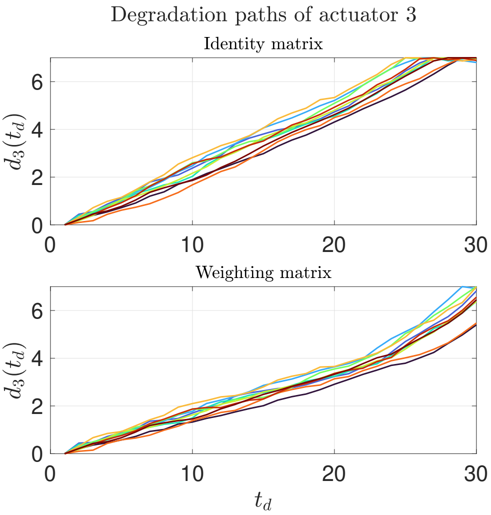
$$W_u(t) = \operatorname{diag}\left(e^{-\tilde{d}_1(t)}, \ldots, e^{-\tilde{d}_m(t)} \right), \tilde{d}_i(t_d) = \max_i d_i(t_d) - d_i(t_d)$$

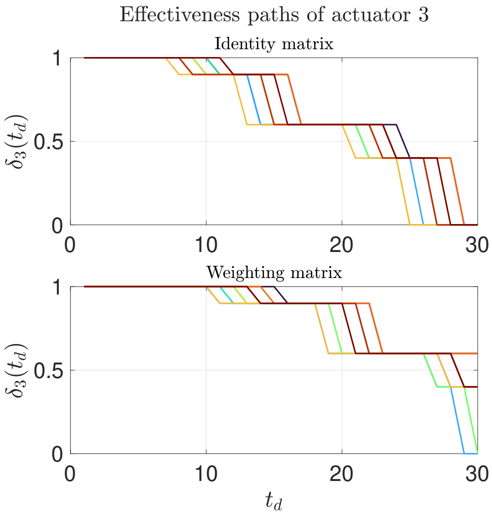
System behaviour (Piecewise constant effectiveness model)



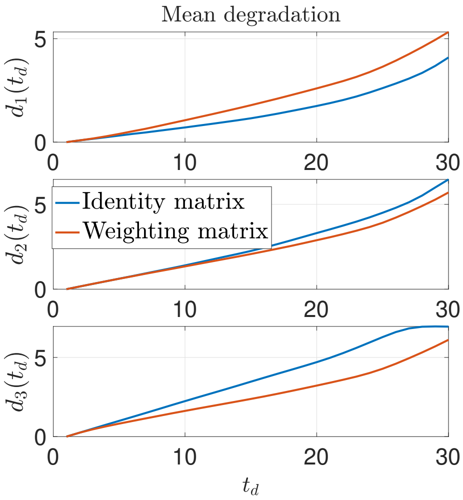


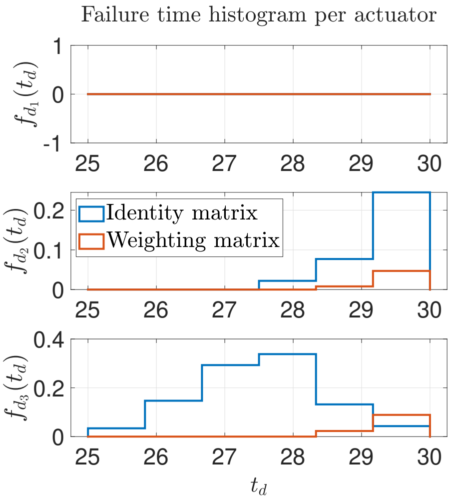
Sampled paths of actuator 3 (Piecewise constant effectiveness model)



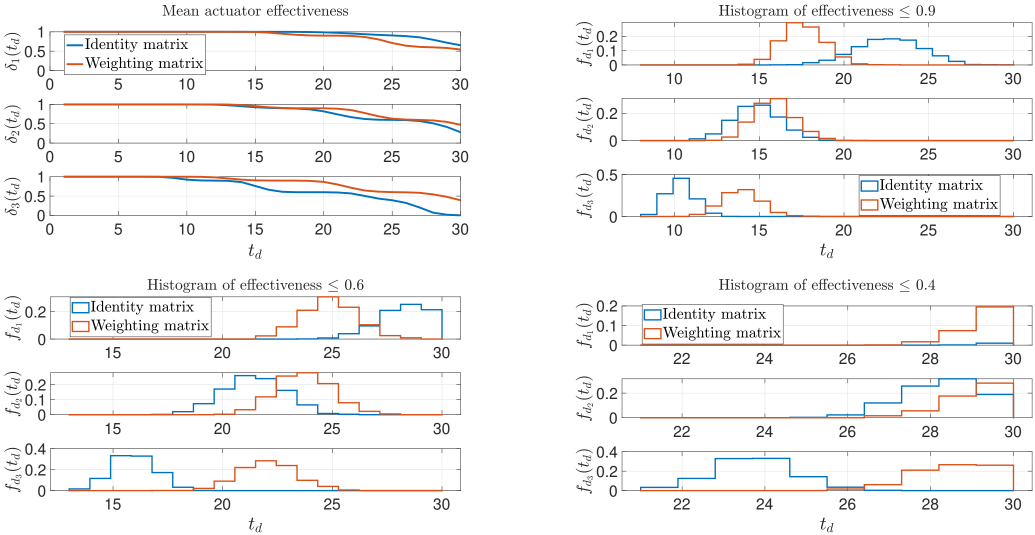


Degradation at component level (Piecewise constant effectiveness model)





Loss of effectiveness (Piecewise constant effectiveness model)

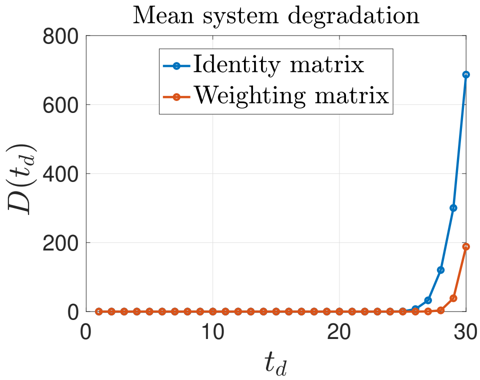


Degradation at system level (Piecewise constant effectiveness model)

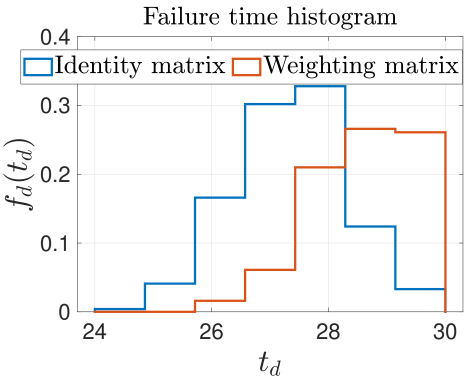
Degradation index

$$D(t_d) = \sum_{j=1}^{J} \sum_{t=t_j+0.75T_j}^{t_j+T_j} |x(t) - r(t)|,$$

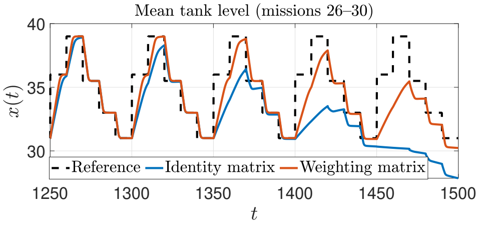
- J: number of reference step transitions
- T_j: duration of each step transition
- $t_j = \sum_{l=1}^{J} T_l t_d + \sum_{l=1}^{j-1} T_l$: start time of step transition j

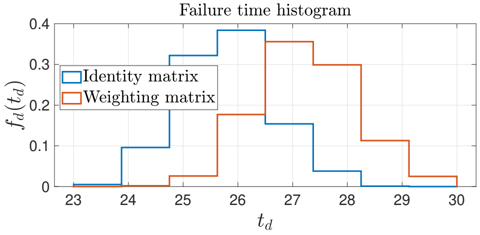


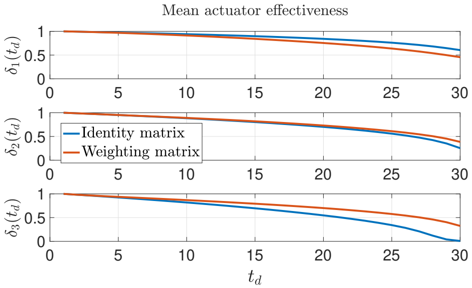
Failure threshold $D_L = 0.1$

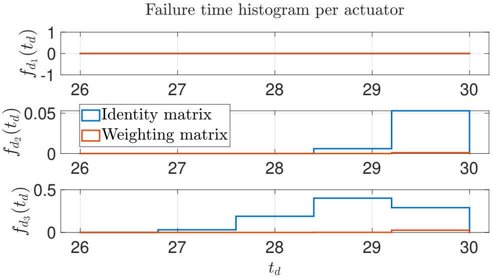


Case of continuous effectiveness model









Takeaways

- Dual time scales are considered for dynamics and degradation;
- Both piecewise constant and continuous effectiveness models are analyzed;
- By using the degradation-based weighting strategy:

$$\begin{cases} \text{prioritize healthier actuators} \\ \text{limit more degraded ones} \end{cases} \Rightarrow \text{extend system lifespan.}$$

Future work

- Quantify degradation at component level;
- Infer degradation at system level.

Thank you for your attention!