

# Disturbance observer-based control of longitudinal vehicle platoon

Rafael N Silva

Thesis directors: Luciano Frezzato, Thierry Marie Guerra

Thesis co-supervisors: Anh-Tu Nguyen, Fernando Souza

# MOTIVATION

---

# Intelligent and Automatic Transportation

---

- Intelligent vehicles is considered as a promising solution to improve traffic flow and reduce the risk of accidents (FENG *et al.*, 2019).
- In the context of platooning, the inter-vehicular distances are controlled using automatic speed and distance controllers (MAHFOUZ *et al.*, 2023)



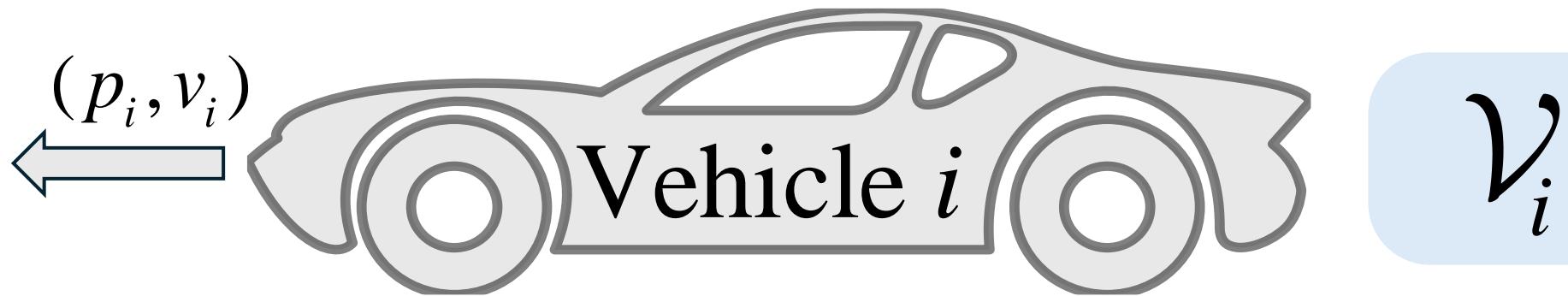
A group of self-driving cars successfully formed a platoon during a recent field test at Aberdeen Proving Ground, 2017, [https://www.volpe.dot.gov/sites/volpe.dot.gov/files/pictures/Cadillacs\\_platooning\\_dry-FHWA-500px.jpg](https://www.volpe.dot.gov/sites/volpe.dot.gov/files/pictures/Cadillacs_platooning_dry-FHWA-500px.jpg)

# LONGITUDINAL VEHICLE PLATOONING

---

**General problem**

# Vehicle dynamics And Measures



## Vehicle Longitudinal dynamics

$$\dot{p}_i = v_i$$

$$\dot{v}_i = \frac{1}{\mathbf{W}_i} (\mathbf{R}_{h,i} T_i - \mathbf{m}_i \mathbf{g} F_{r,i} - \mathbf{B}_i v_i - \mathbf{C}_i v_i^2)$$

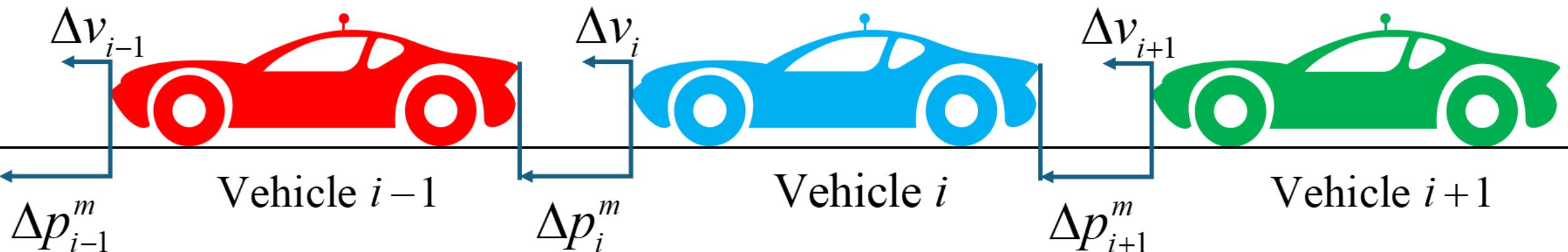
$$\dot{T}_i = -\frac{1}{\rho_i} T_i + \frac{1}{\rho_i} u_{e,i}$$

$$\mathbf{W}_i = \frac{(\mathbf{m}_i \mathbf{h}_{w,i}^2 + \mathbf{J}_{r,i} + \mathbf{J}_{f,i}) \mathbf{R}_{g,i}^2 + \mathbf{J}_{e,i}}{\mathbf{h}_{w,i}^2 \mathbf{R}_{g,i}^2}$$

$$\mathbf{R}_{h,i} = (\mathbf{h}_{w,i} \mathbf{R}_{g,i})^{-1}.$$

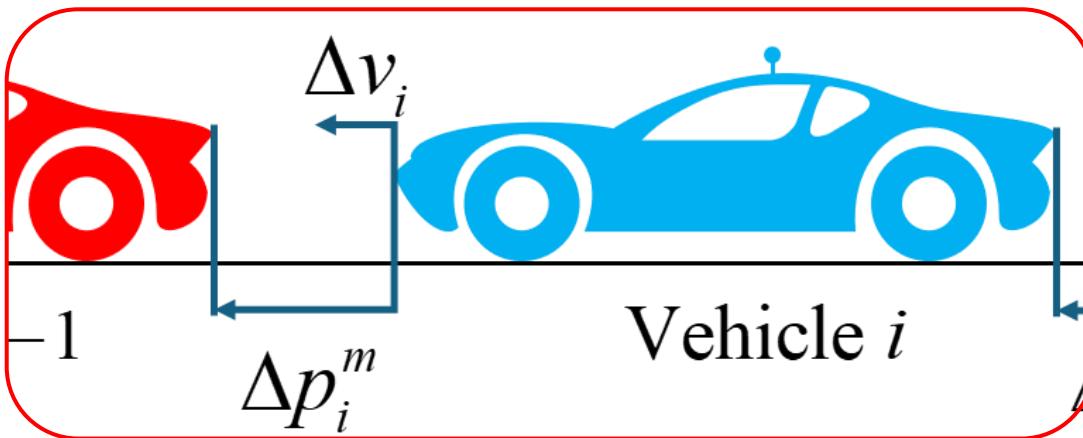
# Adaptive Cruise Control (ACC)

- ❑ Vehicles are equipped with sensor to provide measurements about distance, velocity and acceleration



# Adaptive Cruise Control (ACC)

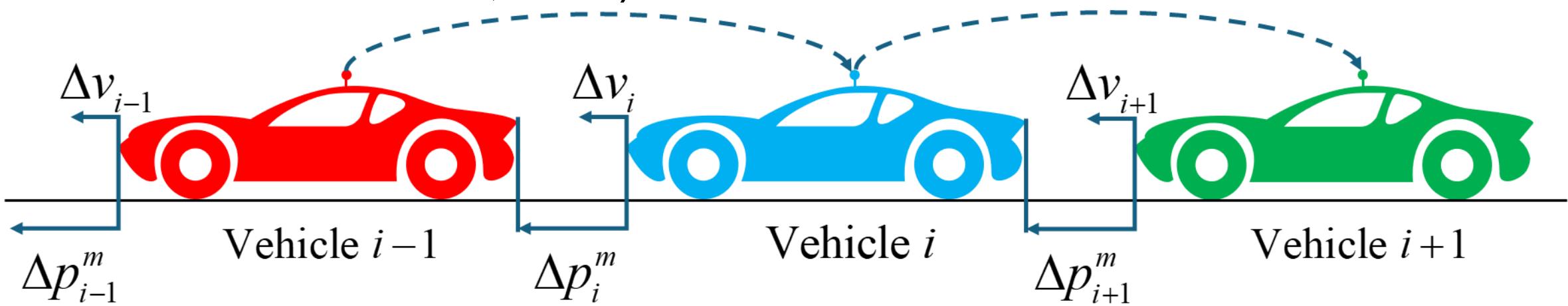
- ❑ Vehicles are equipped with sensor to provide measurements about distance, velocity and acceleration



- Inter vehicular distance:  $\Delta p_i^m = p_{i-1} - p_i - L_{c,i}$
- Velocity difference:  $\Delta v_i = v_{i-1} - v_i$
- Acceleration and velocity:  $(a_i, v_i)$

# Cooperative ACC

- Cooperative ACC introduces communication devices for vehicles to share information
  - Improves platoon performance (DARBHA *et al.*, 2019, GUANETTI *et al.*, 2018)



# Problem of Interest

---

- Designing a control law for vehicles platoons equipped with Cooperative ACC to:
  - Individual stability
  - Achieve vehicle formation ensuring a safe distance
- String stability
  - Ensure that perturbations will no be amplified in the platoon
- Designing an ETC to achieve efficient communication

# CONTROL AND DOB DESIGN

---

# Vehicle Dynamics

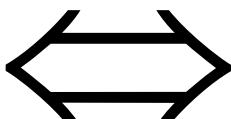
---

- Through classical feedback linearization procedure, we can write the vehicle dynamics as

$$\dot{p}_i = v_i$$

$$\dot{v}_i = \frac{1}{W_i} \left( \mathbf{R}_{h,i} T_i - \mathbf{m}_i \mathbf{g} F_{r,i} - \mathbf{B}_i v_i - \mathbf{C}_i v_i^2 \right)$$

$$\dot{T}_i = -\frac{1}{\rho_d} T_i + \frac{1}{\rho_i} u_{e,i}$$



$$\dot{p}_i = v_i$$

$$\dot{v}_i = a_i$$

$$\dot{a}_i = f_i(v_i, a_i) + b_i u_i$$

$\mathcal{V}_i$

# Vehicle Dynamics

- We can rewrite the vehicle dynamics as

$v_i$

$$\dot{a}_i = f_i^n(v_i, a_i) + b_i^n u_{e,i} - d_i$$

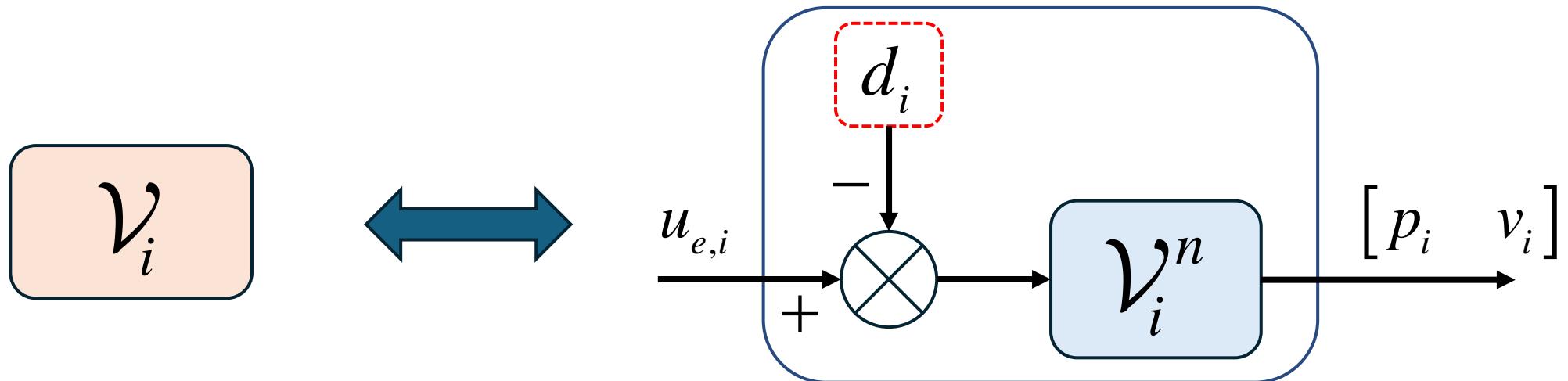
$$d_i = \underbrace{f_i(v_i, a_i) - f_i^n(v_i, a_i)}_{\Delta f_i} + \underbrace{(b_i - b_i^n) u_{e,i}}_{\Delta b_i}$$

- Uncertain:  $\{f_i(v_i, a_i), b_i\}$
- Known (nominal):  $\{f_i^n(v_i, a_i), b_i^n\}$

# Vehicle Dynamics

- We can rewrite the vehicle dynamics as

$$\dot{v}_i = f_i^n(v_i, a_i) + b_i^n u_{e,i} - d_i$$



# Feedback linearization

- ☐ Based on the vehicle dynamics

$\mathcal{V}_i$

$$\dot{a}_i = f_i^n(v_i, a_i) + b_i^n u_{e,i} - d_i$$

We consider a feedback linearizing control law

$$u_{e,i} = \frac{1}{b_i^n} \left( -f_i^n(v_i, a_i) - \frac{1}{\rho_d} a_i + \frac{1}{\rho_d} u_i + \hat{d}_i \right)$$

$\hat{d}_i$  Is an estimation of  $d_i$

$u_i$  Is a new control input

# Feedback linearization

- ☐ Based on the vehicle dynamics

$\mathcal{V}_i$

$$\dot{a}_i = f_i^n(v_i, a_i) + b_i^n u_{e,i} \quad - d_i$$

We consider a feedback linearizing control law

$$u_{e,i} = \frac{1}{b_i^n} \left( -f_i^n(v_i, a_i) - \frac{1}{\rho_d} a_i + \frac{1}{\rho_d} u_i + \hat{d}_i \right)$$

$\hat{d}_i$  Is an estimation of  $d_i$

$u_i$  Is a new control input

# Vehicle Dynamics

---

## □ Linearized dynamics

$$\dot{p}_i = v_i$$

$$\dot{v}_i = a_i$$

$$\dot{a}_i = -\frac{1}{\rho_d} a_i + \frac{1}{\rho_d} u_i$$

Estimation error:

$$e_{d,i} = \hat{d}_i - d_i$$

$$+ \hat{d}_i - d_i$$

# Vehicle Dynamics

## □ Linearized dynamics

$$\begin{aligned}\dot{p}_i &= v_i \\ \dot{v}_i &= a_i \\ \dot{a}_i &= -\frac{1}{\rho_d} a_i + \frac{1}{\rho_d} u_i + \cancel{\hat{d}_i} - \cancel{d_i}\end{aligned}$$

- Vehicles parameters without uncertainties

$$e_{d,i} = \hat{d}_i - d_i = 0$$

Homogeneity

- All vehicles have the same  $\rho_d$

# Vehicle Dynamics

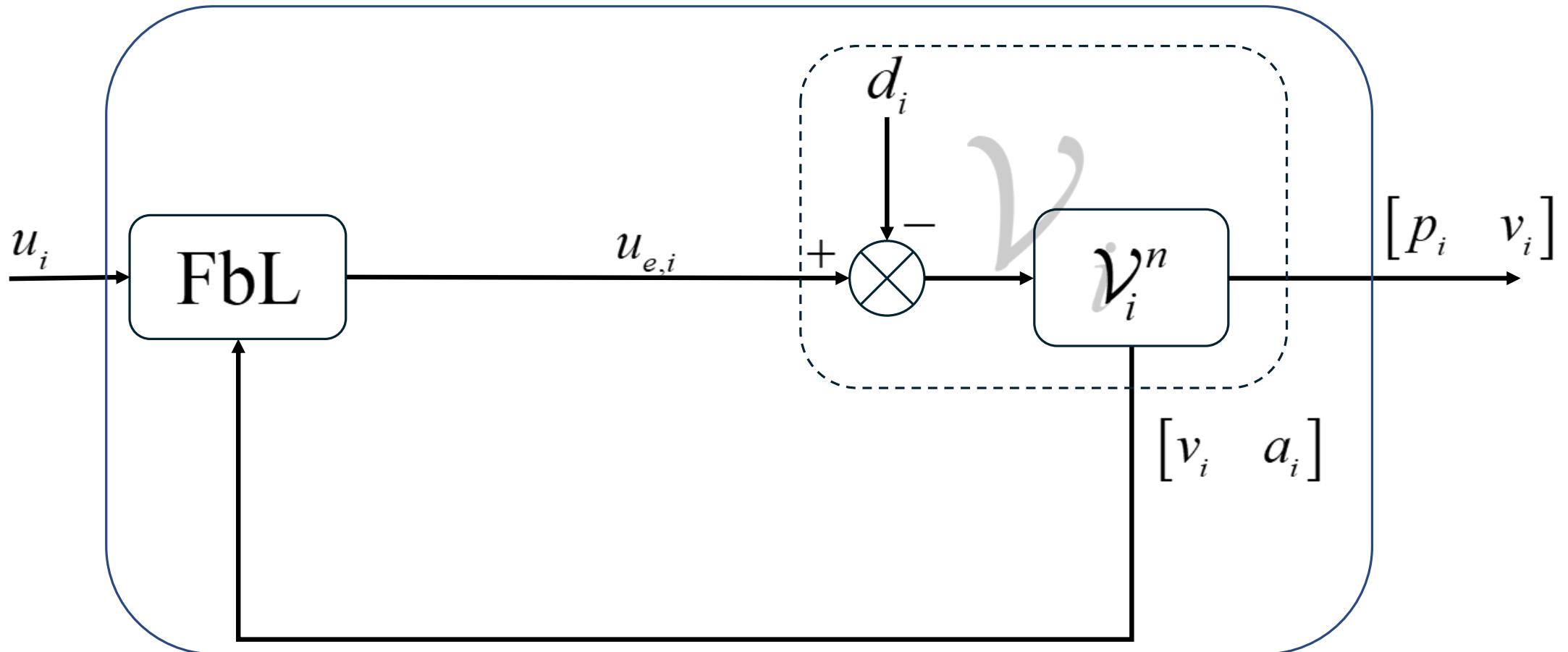
## □ Linearized dynamics

$$\begin{aligned}\dot{p}_i &= v_i \\ \dot{v}_i &= a_i \\ \dot{a}_i &= -\frac{1}{\rho_d} a_i + \frac{1}{\rho_d} u_i + \hat{d}_i - d_i\end{aligned}$$

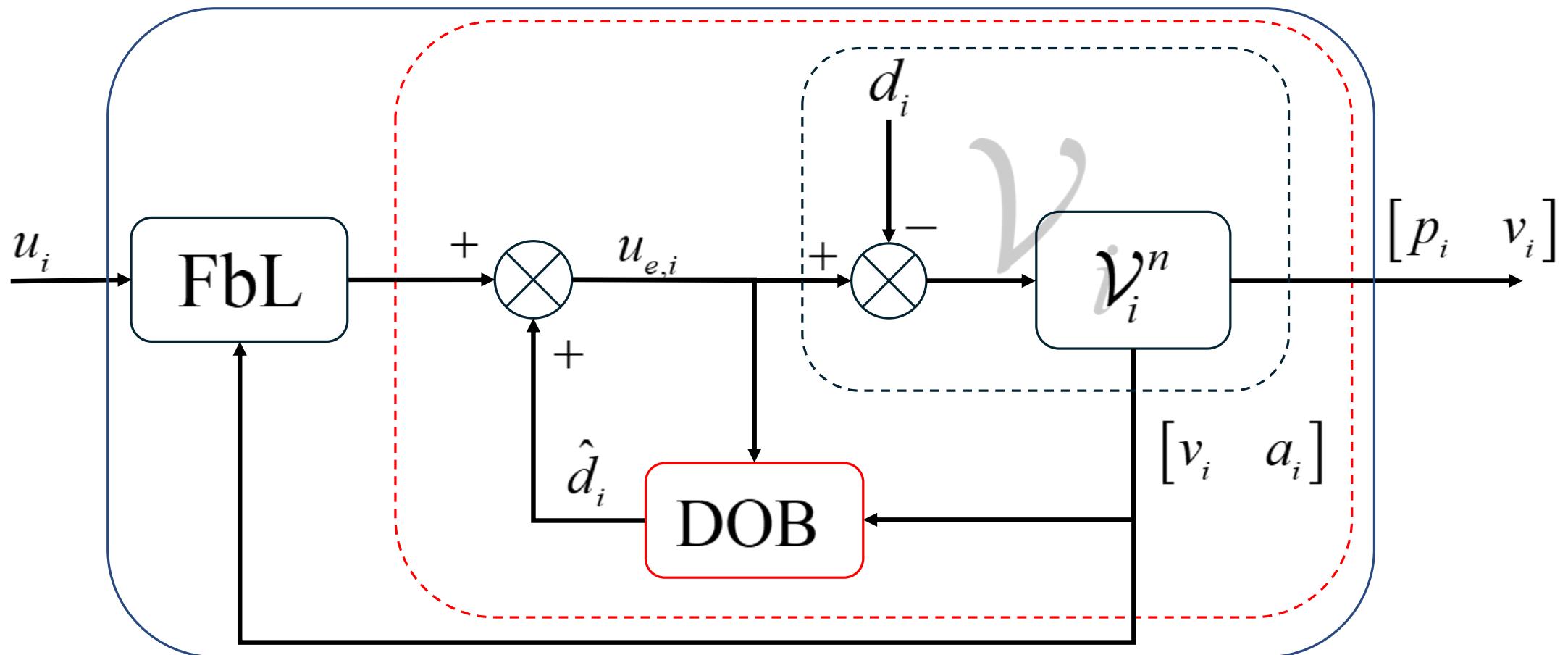
- All vehicles have the same  $\rho_d$

- Due to uncertainties the platoon is not fully homogenous

# Feedback linearization and DOB



# Feedback linearization and DOB



## □ Disturbance observer dynamics

$$\dot{\omega}_i = L_i(f_i^n(v_i, a_i) + b_i^n u_i - \hat{d}_i)$$

$$\hat{d}_i = \omega_i - L_i a_i$$

- Considering the estimation error:  $e_{d,i} = \hat{d}_i - d_i$
- Disturbance estimation error dynamics

$$\dot{e}_{d,i} = -L_i e_{d,i} + \dot{d}_i$$

$$\dot{d}_i \approx 0$$

Slow dynamics

$$e_{d,i} \rightarrow 0$$

# Literature Review

1

Some works in the literature assume that the system without disturbances or uncertainties (DOLK *et al.*, 2017, Ploeg *et al.*, 2011)

$$\dot{a}_i = -\frac{1}{\rho_d} a_i + \frac{1}{\rho_d} u_i + \hat{d}_i$$

2

Another work as (HUANG. KARIMI, 2021) assumes uncertainties or exogenous inputs but do not introduce any compensation

$$\dot{a}_i = -\frac{1}{\rho_d} a_i + \frac{1}{\rho_d} u_i + \hat{d}_i - d_i$$

# Literature Review

---

3

The works of (WANG *et al.*, 2022a, LUO *et al.*, 2021) propose compensation, but the conditions do not ensure **scalability**

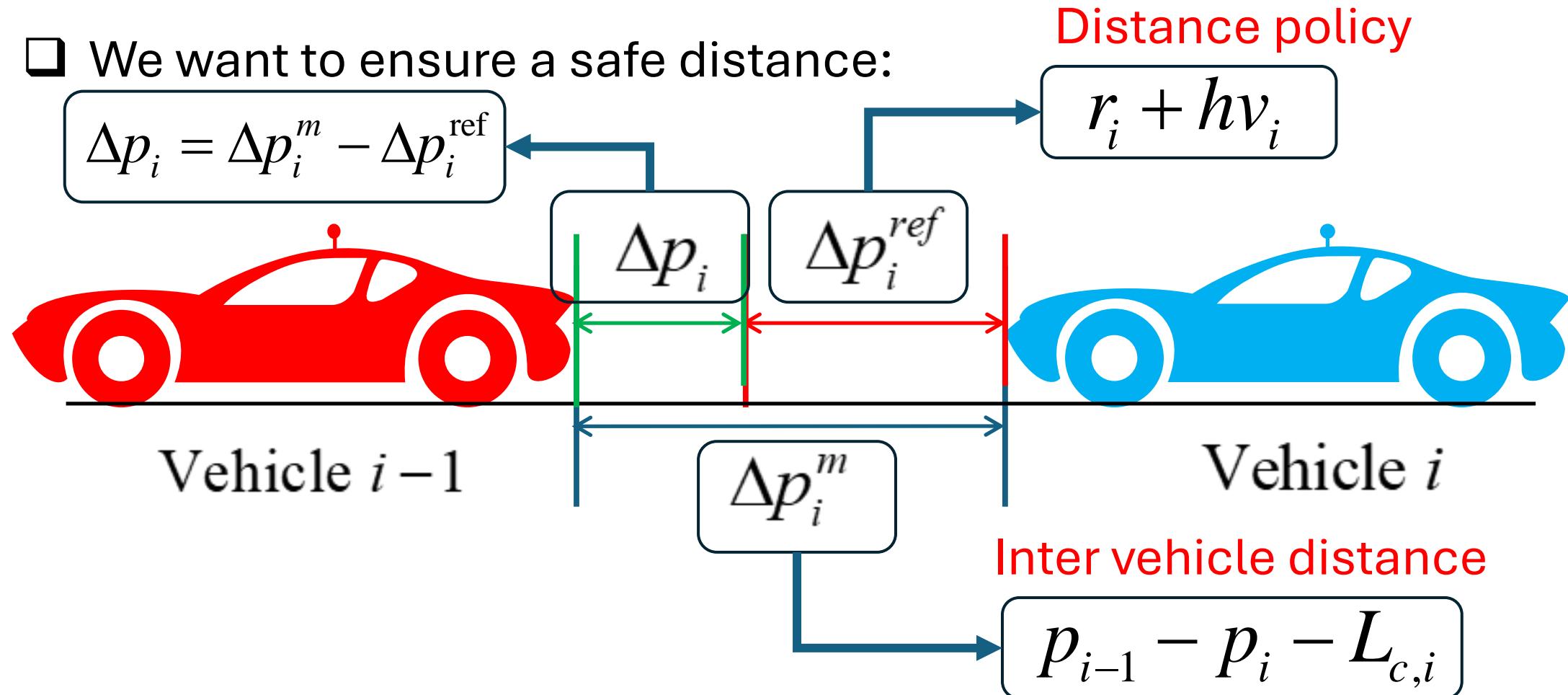
$$\dot{a}_i = -\frac{1}{\rho_d} a_i + \frac{1}{\rho_d} u_i + \hat{d}_i - d_i$$

Scalability

Stability conditions are independent of the number of vehicles

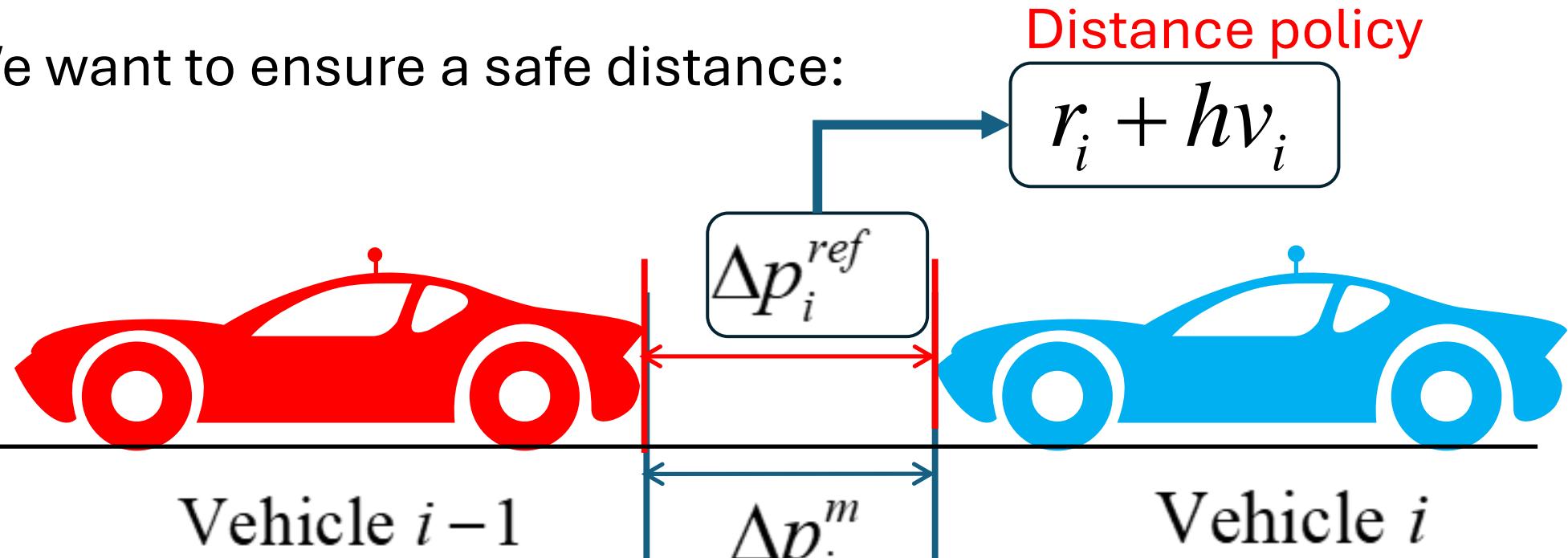
# Distance policy and Individual stability

- We want to ensure a safe distance:



# Distance policy and Individual stability

- We want to ensure a safe distance:



Ensure that  $\Delta p_i = \Delta p_i^m - \Delta p_i^{\text{ref}} \rightarrow 0$      $\Delta v_i = v_{i-1} - v_i \rightarrow 0$

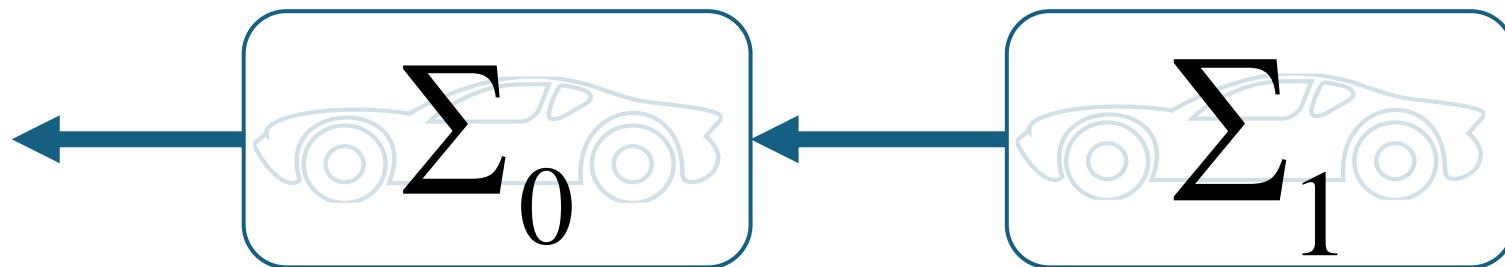
# DOB compensation simulations

**Main results assuming  
continuous communication**



## DOB COMPENSATION SIMULATION

- To evaluate the DOB performance, we consider a platoon of one leader and one follower:



- Distance policy

$$\Delta p_i^{\text{ref}} = 2.5 + 0.6v_i$$

- Vehicle parameters

Vehicle	$r$	$m$	$h_w$	$J_r$	$J_e$	$R_g$	$B$	$C$	$\rho$
$\Sigma_0$		1724	0.28	0.75	0.14	0.10	7.35	0.05	0.05
$\tilde{\Sigma}_0$		1724	0.25	1.05	0.14	0.13	8.09	0.06	0.08
$\Sigma_1$	2.5	2241	0.63	0.97	0.35	0.18	13.96	0.11	0.10
Simulation	$\tilde{\Sigma}_1$	2017	0.51	0.68	0.46	0.18	11.17	0.16	0.09

Compute the feedback linearization and DOB

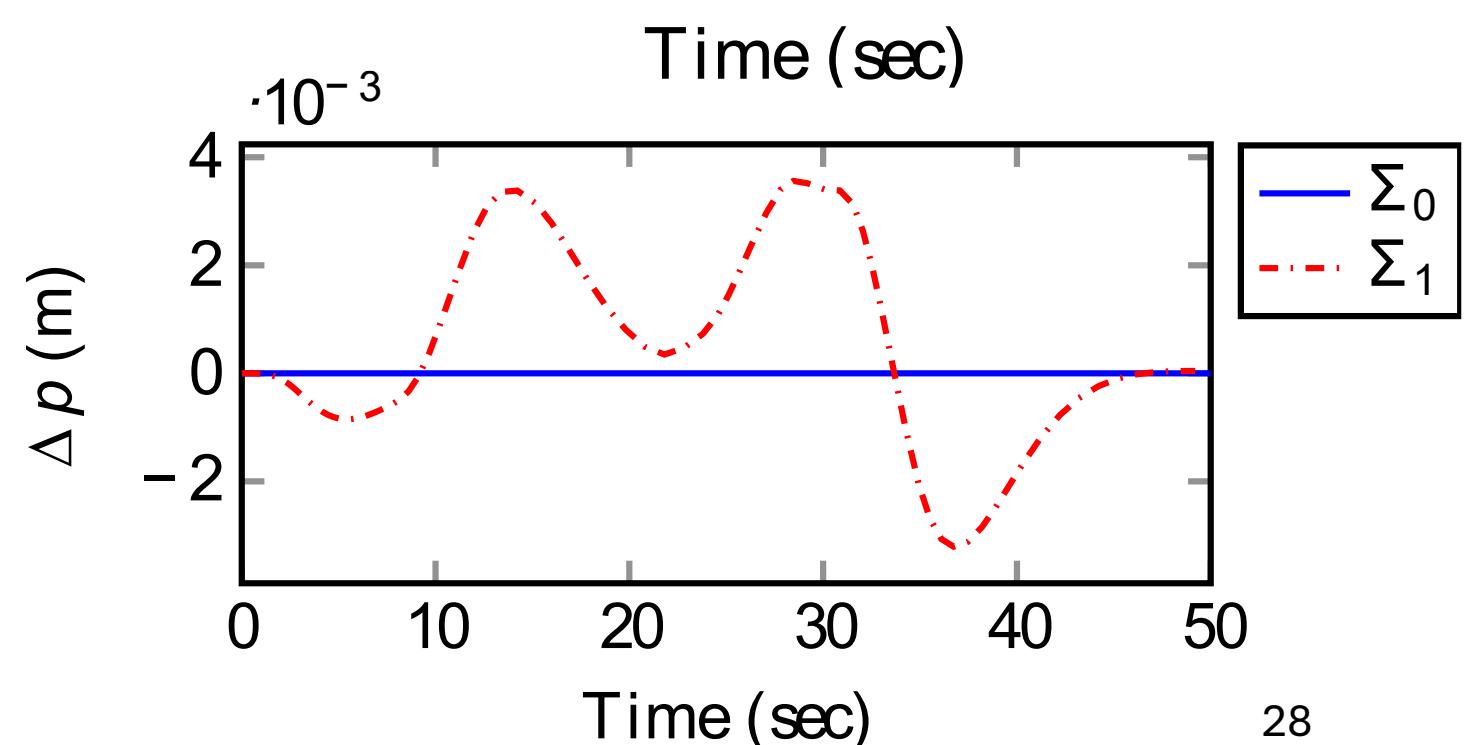
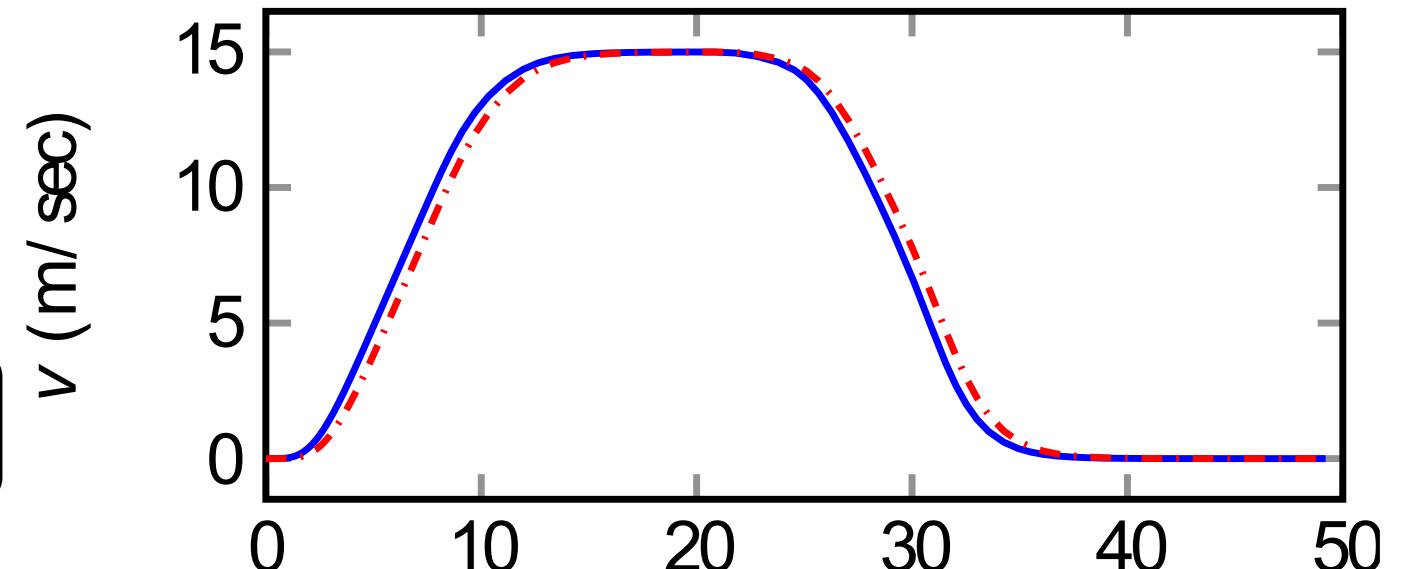
Simulation

Velocity profile and tracking

No mismatch  $d_1 = 0$

$$\Sigma_1 = \tilde{\Sigma}_1$$

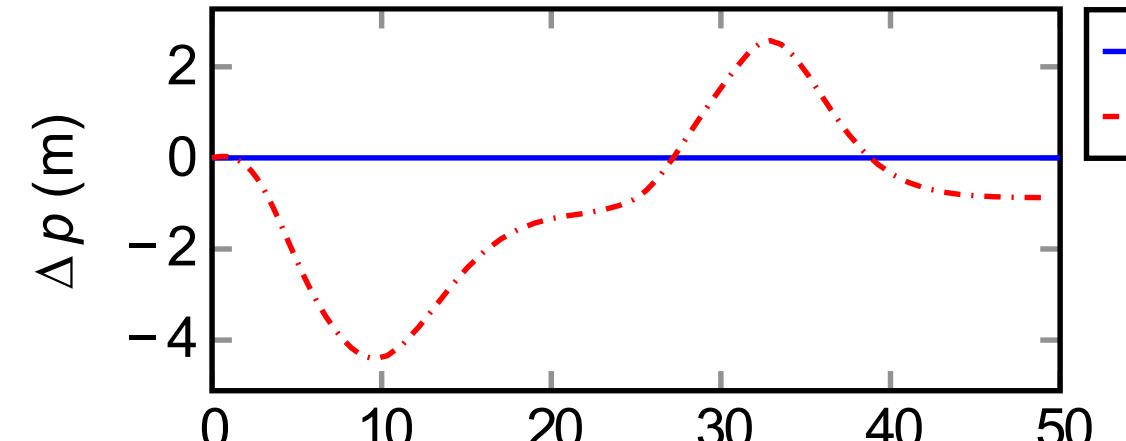
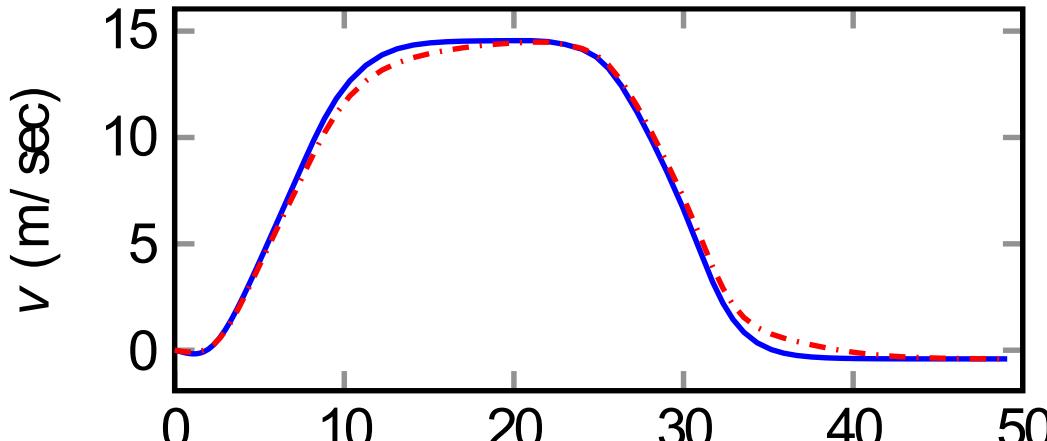
Distance policy error



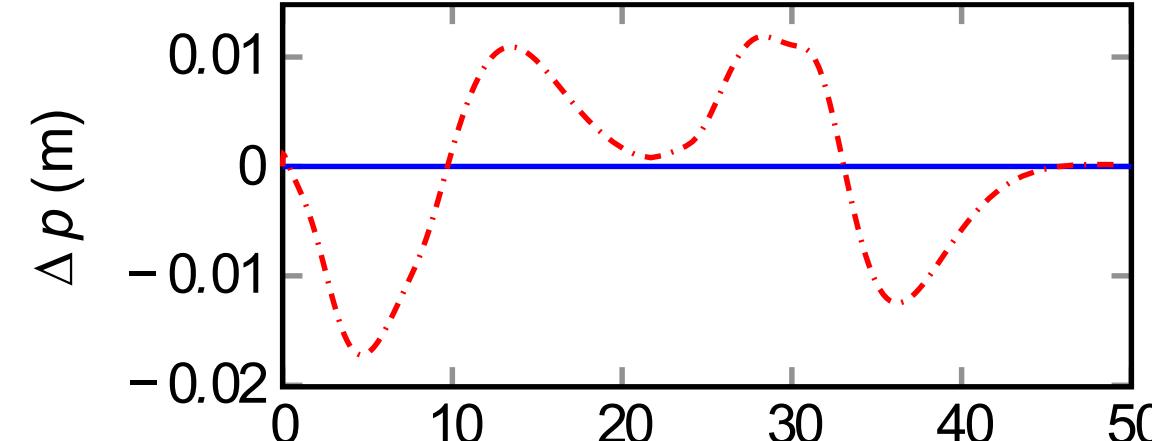
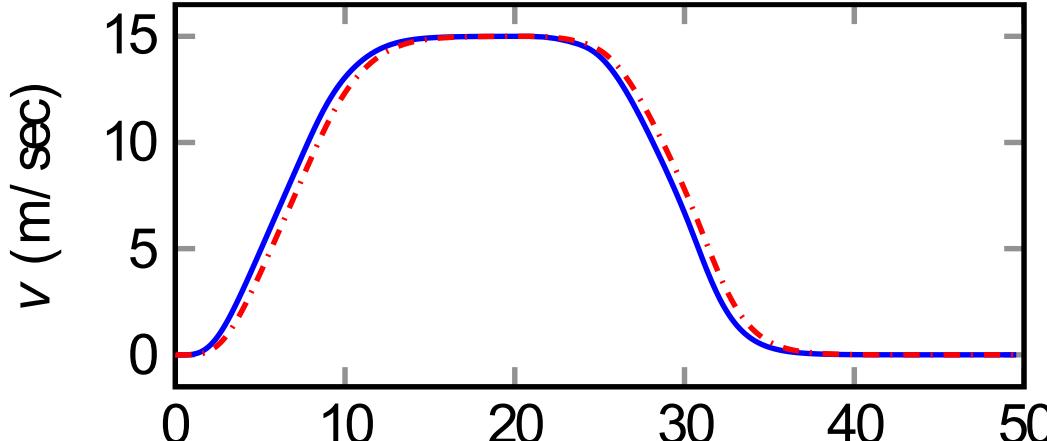
□ Uncertain dynamics

$$d_1 \neq 0 \quad \Sigma_1 \neq \tilde{\Sigma}_1$$

□ Without DOB compensation



□ DOB compensation

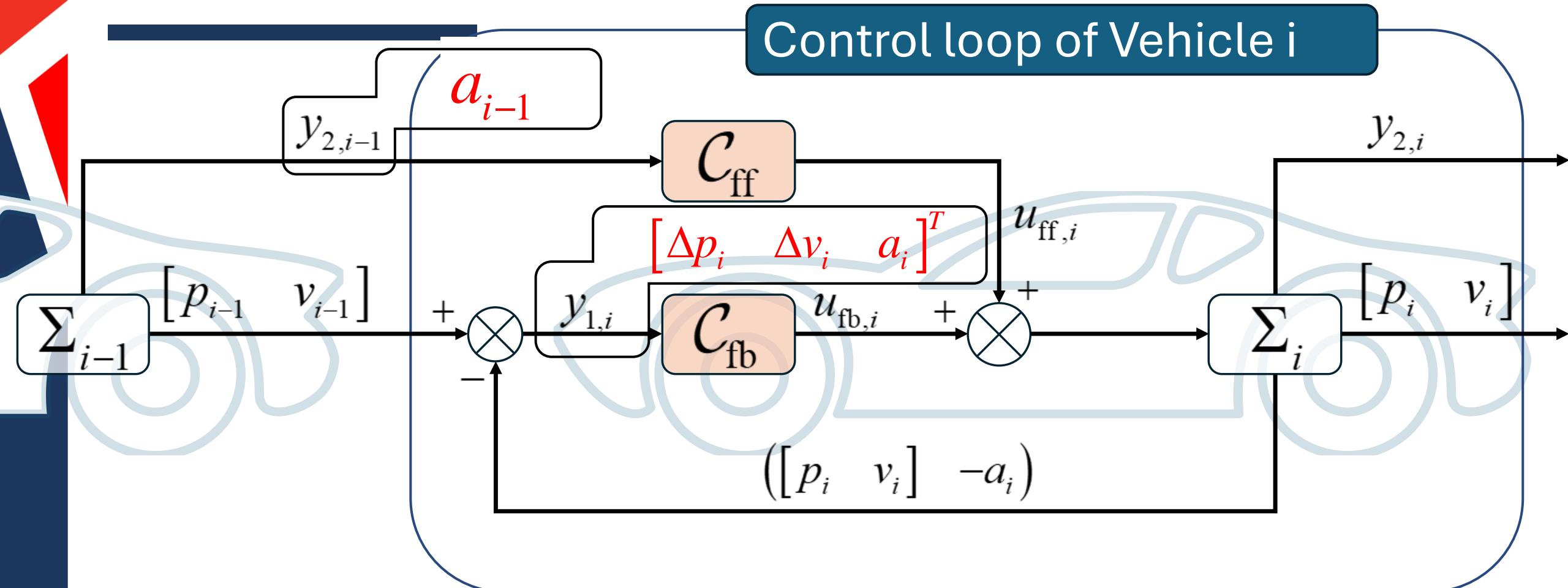


# Vehicles interactions

---

**Modeling vehicles interactions  
(communication)**

# Vehicles interactions



$$y_{2,i-1} = a_{i-1}$$

Transmitted

$$y_{1,i} = [\Delta p_i \quad \Delta v_i \quad a_i]^T$$

Measured

# Vehicle Linearized Dynamics

- ☐ Vehicle linearized dynamics with the distance policy error

$$\sum_i \begin{aligned} \Delta \dot{p}_i &= v_i \\ \Delta \dot{v}_i &= a_{i-1} - a_i \end{aligned}$$

$$\dot{a}_i = -\frac{1}{\rho_d} a_i + \frac{1}{\rho_d} u_i + e_{d,i}$$

$$y_{1,i} = [\Delta p_i \quad \Delta v_i \quad a_i]^T$$

$$y_{2,i-1} = a_{i-1}$$

- ☐ Vehicle control with feedback and feedforward components

$$u_i = u_{fb,i} + u_{ff,i}$$

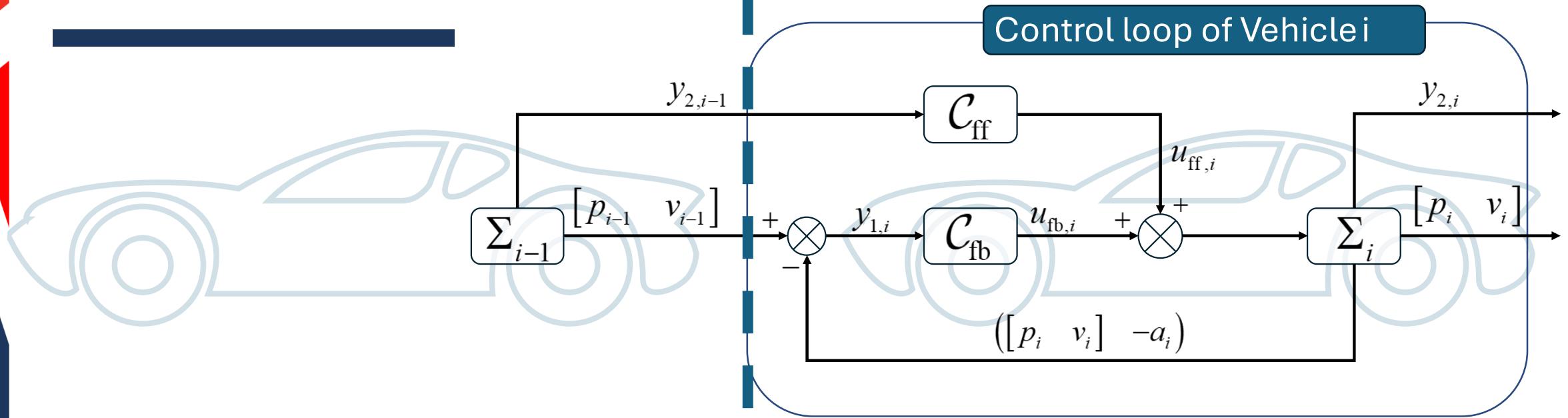
$$u_{fb,i} = K_1 y_{1,i}$$

Feedback

$$u_{ff,i} = K_2 y_{2,i-1}$$

Feedforward

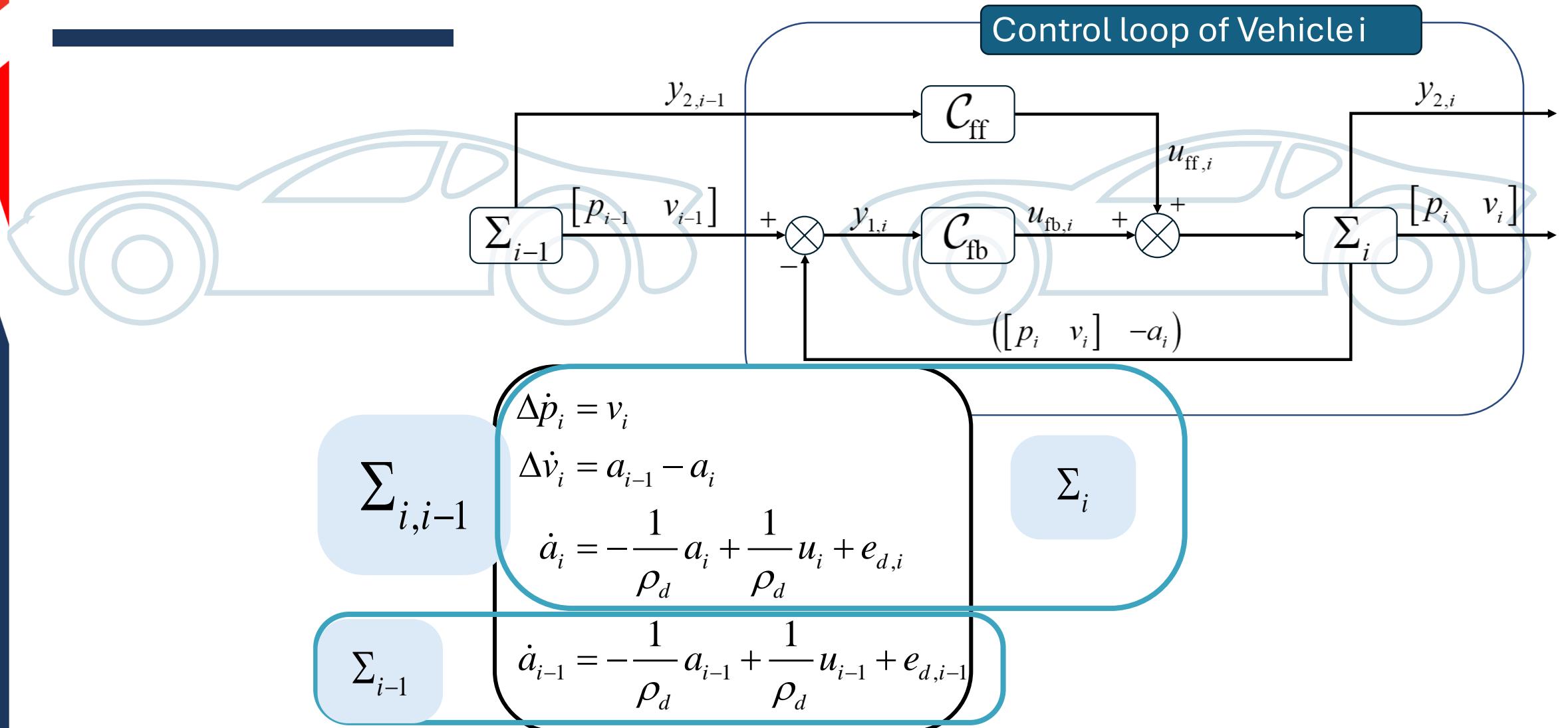
# Vehicles interactions



$$\begin{aligned}\Sigma_{i-1} \quad & \Delta \dot{p}_{i-1} = v_{i-1} \\ & \Delta \dot{v}_{i-1} = a_{i-2} - a_{i-1} \\ & \dot{a}_{i-1} = -\frac{1}{\rho_d} a_{i-1} + \frac{1}{\rho_d} u_{i-1} + e_{d,i-1}\end{aligned}$$

$$\begin{aligned}\Sigma_i \quad & \Delta \dot{p}_i = v_i \\ & \Delta \dot{v}_i = a_{i-1} - a_i \\ & \dot{a}_i = -\frac{1}{\rho_d} a_i + \frac{1}{\rho_d} u_i + e_{d,i}\end{aligned}$$

# Vehicles interactions



# Vehicle Linearized Dynamics

---

- Augmented dynamics model for vehicle interactions

$$\Delta \dot{p}_i = v_i$$

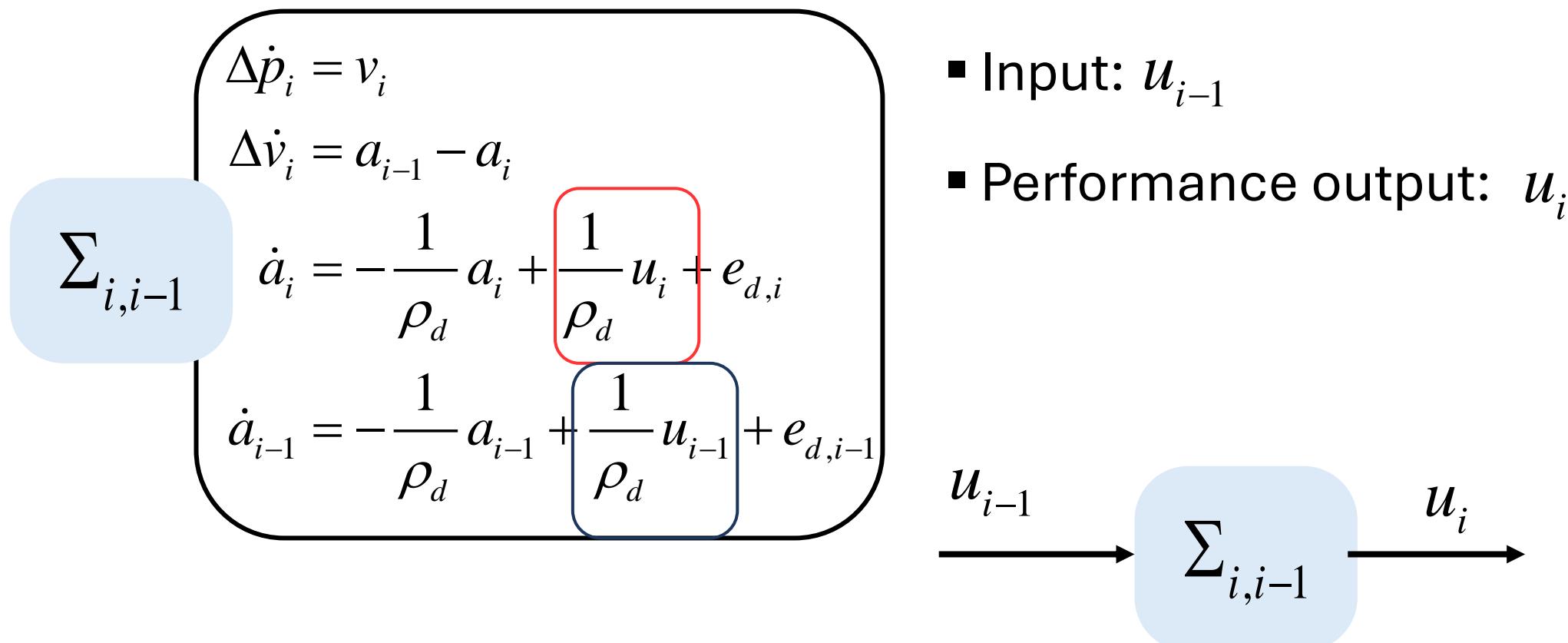
$$\Delta \dot{v}_i = a_{i-1} - a_i$$

$$\sum_{i,i-1} \dot{a}_i = -\frac{1}{\rho_d} a_i + \frac{1}{\rho_d} u_i + e_{d,i}$$

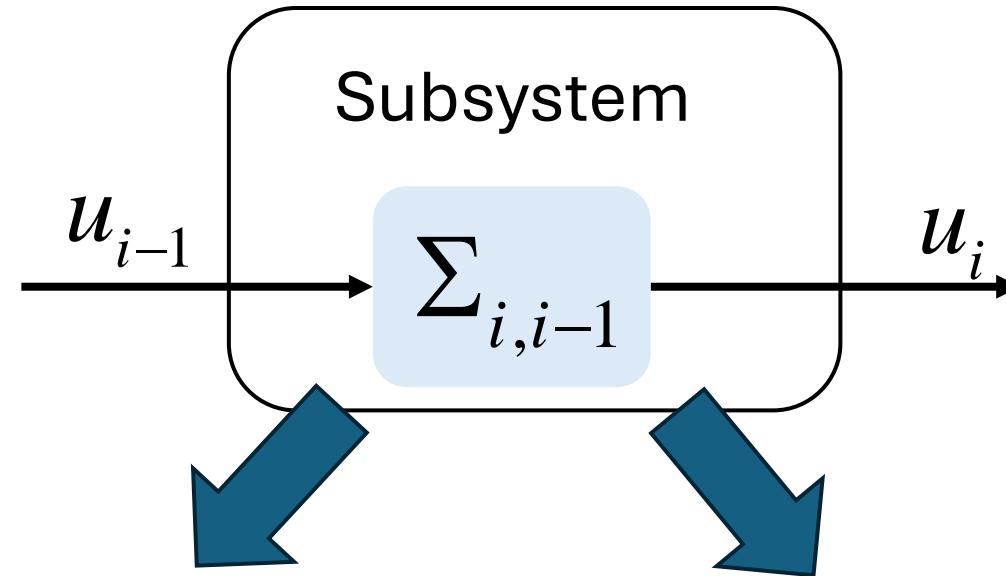
$$\dot{a}_{i-1} = -\frac{1}{\rho_d} a_{i-1} + \frac{1}{\rho_d} u_{i-1} + e_{d,i-1}$$

# Vehicles interactions

## ❑ Augmented dynamics model for vehicle interactions



# Vehicles interactions



Individual stability

String stability

Scalability

Stability conditions are independent of the number of vehicles

# Augmented dynamics

- From the augmented dynamics and the control law

$$\begin{aligned}\Delta \dot{p}_i &= v_i \\ \Delta \dot{v}_i &= a_{i-1} - a_i \\ \sum_{i,i-1} \dot{a}_i &= -\frac{1}{\rho_d} a_i + \frac{1}{\rho_d} u_i + e_{d,i} \\ \dot{a}_{i-1} &= -\frac{1}{\rho_d} a_{i-1} + \frac{1}{\rho_d} u_{i-1} + e_{d,i-1}\end{aligned}$$

- Control input

$$u_i = K_1 \begin{bmatrix} \Delta p_i \\ \Delta v_i \\ a_i \end{bmatrix} + K_2 a_{i-1}$$

# Augmented dynamics

□ Considering the closed-loop augmented system:

$$\sum_{i,i-1} \dot{x}_i = (A + B[K_1 \quad K_2])x_i + Du_{i-1} + E \begin{bmatrix} e_{d,i} \\ e_{d,i-1} \end{bmatrix}$$

$$z_i = [K_1 \quad K_2]x_i$$

$$y_{1,i} = C_1 x_i$$

$$y_{2,i-1} = C_2 x_i$$

■ System states:

$$x_i = \begin{bmatrix} \Delta p_i \\ \Delta v_i \\ a_i \\ a_{i-1} \end{bmatrix}$$

■ Input:  $u_{i-1}$

■ Performance output:  $z_i = u_i$

# Designing conditions (Interconnected)

- Considering the closed-loop augmented system:

$$\sum_{i,i-1} \dot{x}_i = (A + B[K_1 \quad K_2])x_i + Du_{i-1} + E \begin{bmatrix} e_{d,i} \\ e_{d,i-1} \end{bmatrix}$$
$$z_i = [K_1 \quad K_2]x_i$$

$$y_{1,i} = C_1 x_i$$
$$y_{2,i-1} = C_2 x_i$$

- Design a control law such that:

The platoon is string stable

# String stability analysis

- To account for uncertainties and ensure scalability, we propose

Considering the system  $\Sigma_{i,i-1}$ . Assuming that the uncertainties are bounded with  $\|w_{d,i}\| \leq w_{\max}$  and there exist positive scalars  $\gamma \leq 1$  and  $\beta$  such that :

$$\|z_i\| \leq \gamma \|z_{i-1}\| + \beta \|w_{d,i}\| \quad w_{d,i} = (e_{d,i}, e_{d,i-1})$$

then,

$$\|z_i\| \leq \|u_0\| + i\beta w_{\max}$$

# Designing conditions

- Lyapunov based stability conditions:

$$V(x_i) > 0, \quad \forall x_i \neq 0, \quad V(0) = 0,$$
$$\dot{V}(x_i) \leq -z_i^T z_i + \gamma u_{i-1}^T u_{i-1} + \beta w_{d,i}^T w_{d,i}$$

- LMI conditions: Determine  $\{P \quad L \quad \alpha \quad \beta\}$  such that:

$$\begin{bmatrix} \text{He}\{AP + BL\} + \alpha P & D & E & L \\ * & -\gamma^2 I & 0 & 0 \\ * & * & -\beta^2 I & 0 \\ * & * & * & -I \end{bmatrix} \prec 0 \quad \gamma \leq 1$$

$$K = LP^{-1}$$

Control gain

# PLATOON SIMULATION

**Main results assuming  
continuous communication**

☐ Vehicle base parameters values

Vehicle	$r$	$m$	$h_w$	$J_r$	$J_e$	$R_g$	$B$	$C$	$\rho$
$\Sigma_0^b$		1724	0.28	0.75	0.14	0.10	7.35	0.05	0.05
$\tilde{\Sigma}_0^b$		1724	0.25	1.05	0.14	0.13	8.09	0.06	0.08
$\Sigma_1^b$	2.5	2241	0.63	0.97	0.35	0.18	13.96	0.11	0.10
$\tilde{\Sigma}_1^b$		2017	0.51	0.68	0.46	0.18	11.17	0.16	0.09
$\Sigma_2^b$	2.5	2930	0.41	1.57	0.27	0.20	11.02	0.08	0.08
$\tilde{\Sigma}_2^b$		2637	0.33	1.26	0.40	0.30	8.82	0.05	0.06
$\Sigma_3^b$	2.5	3620	0.63	0.82	0.27	0.11	13.96	0.11	0.12
$\tilde{\Sigma}_3^b$		3258	0.89	1.15	0.40	0.08	15.36	0.16	0.12
$\Sigma_4^b$	2.5	3965	0.52	1.72	0.24	0.11	8.09	0.06	0.08
$\tilde{\Sigma}_4^b$		5947	0.63	2.41	0.29	0.14	11.32	0.08	0.11

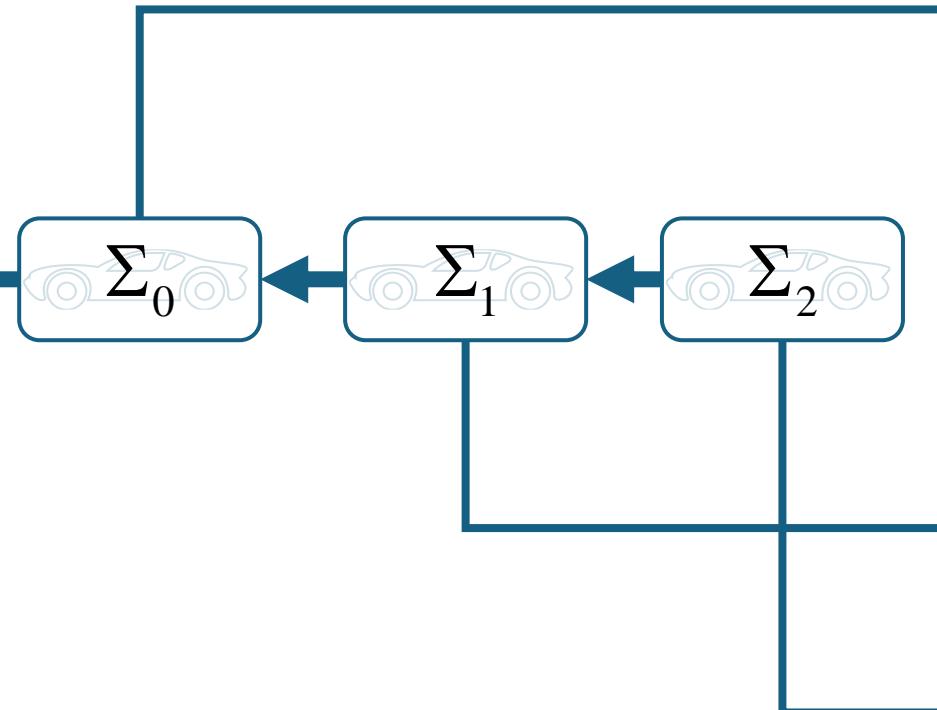
Vehicle base parameters values

Vehicle	$r$	$m$	$h_w$	$J_r$	$J_e$	$R_g$	$B$	$C$	$\rho$
$\Sigma_0^b$		1724	0.28	0.75	0.14	0.10	7.35	0.05	0.05
$\tilde{\Sigma}_0^b$		1724	0.25	1.05	0.14	0.13	8.09	0.06	0.08
$\Sigma_1^b$	2.5	2241	0.63	0.97	0.35	0.18	13.96	0.11	0.10
$\tilde{\Sigma}_1^b$		2017	0.51	0.68	0.46	0.18	11.17	0.16	0.09
$\Sigma_2^b$	2.5	2930	0.41	1.57	0.27	0.20	11.02	0.08	0.08
$\tilde{\Sigma}_2^b$		2637	0.33	1.26	0.40	0.30	8.82	0.05	0.06
$\Sigma_3^b$	2.5	3620	0.63	0.82	0.27	0.11	13.96	0.11	0.12
$\tilde{\Sigma}_3^b$		3258	0.89	1.15	0.40	0.08	15.36	0.16	0.12
$\Sigma_4^b$	2.5	3965	0.52	1.72	0.24	0.11	8.09	0.06	0.08
$\tilde{\Sigma}_4^b$		5947	0.63	2.41	0.29	0.14	11.32	0.08	0.11

# PLATOON SIMULATION

- Set of index:

$$\mathbb{J} = \{0, 3, 4\}$$



## ☐ Vehicle base parameters values

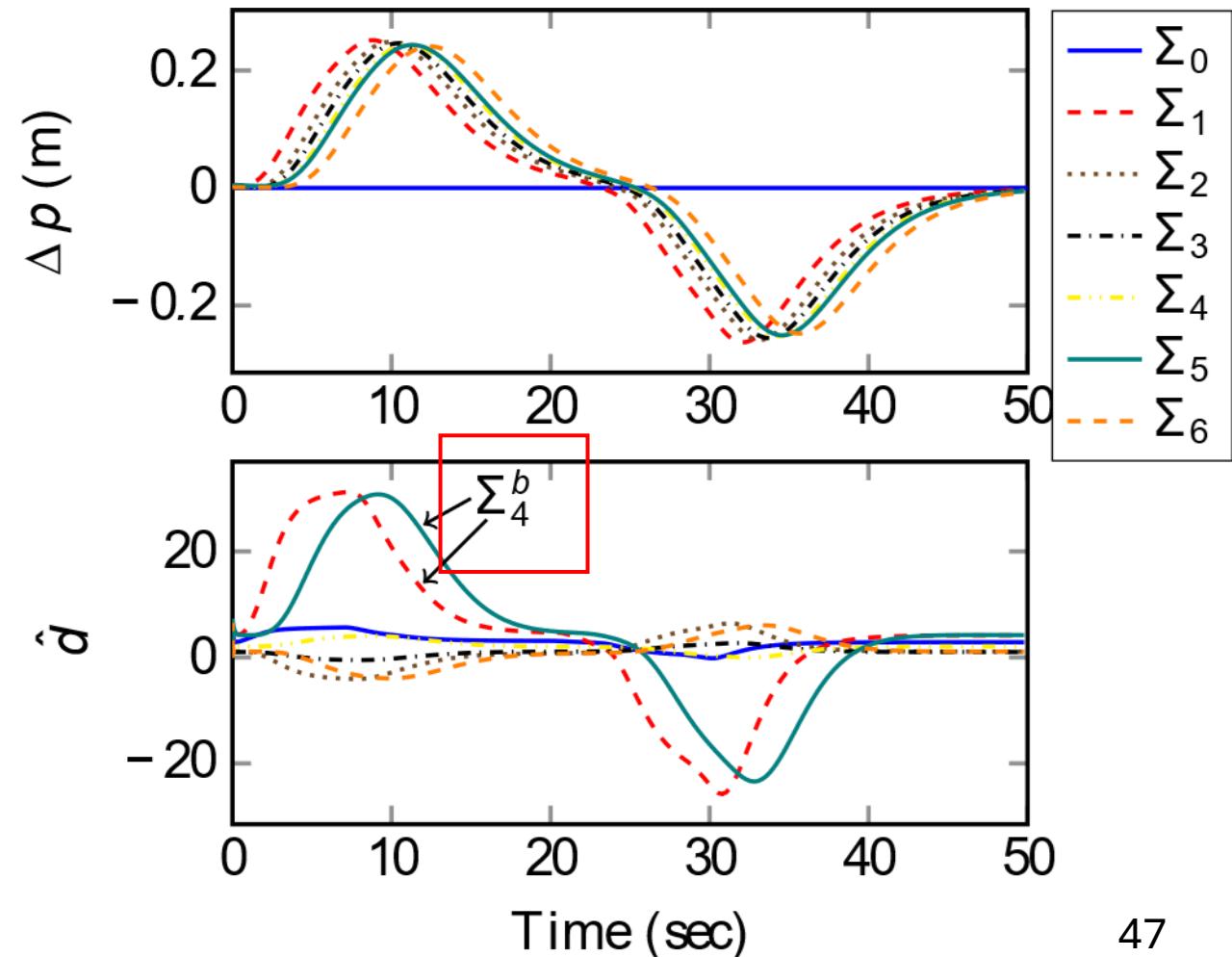
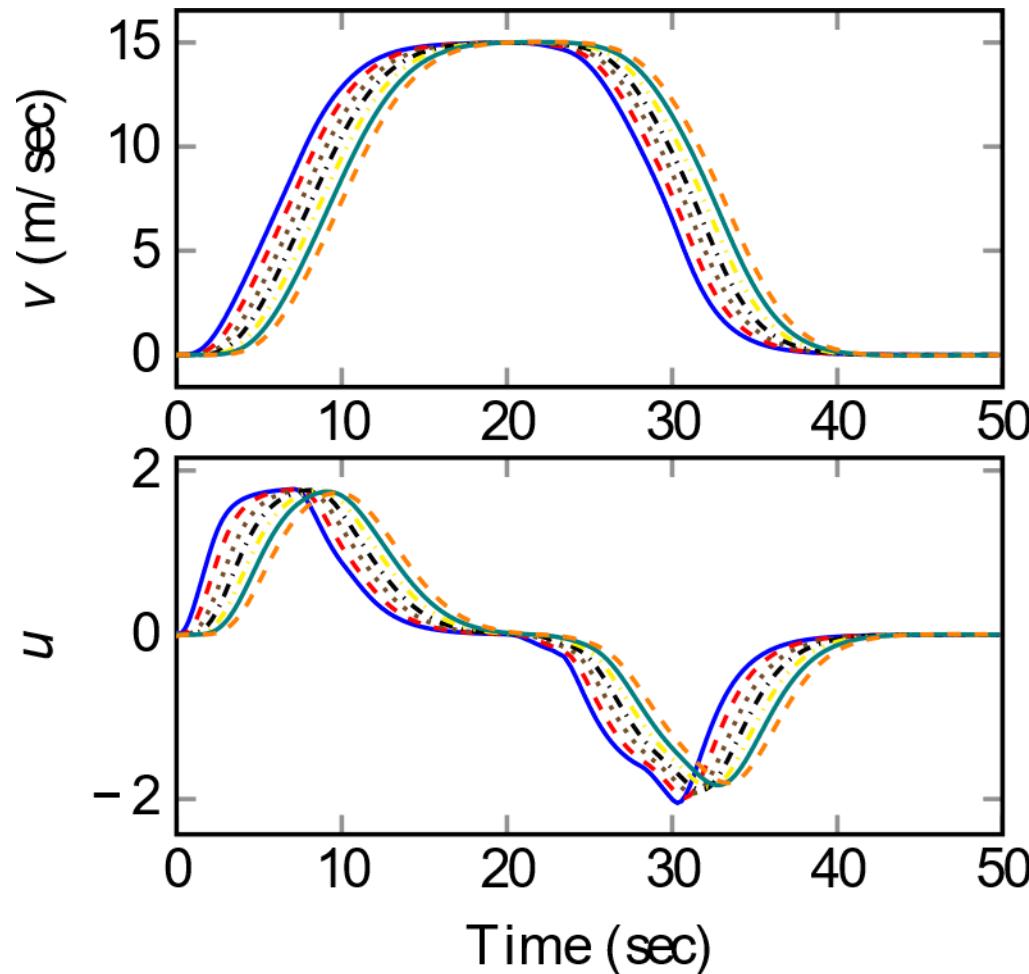
Vehicle	$r$	$m$	$h_w$	$J_r$	$J_e$	$R_g$	$B$	$C$	$\rho$
$\Sigma_0^b$		1724	0.28	0.75	0.14	0.10	7.35	0.05	0.05
$\tilde{\Sigma}_0^b$		1724	0.25	1.05	0.14	0.13	8.09	0.06	0.08
$\Sigma_1^b$	2.5	2241	0.63	0.97	0.35	0.18	13.96	0.11	0.10
$\tilde{\Sigma}_1^b$		2017	0.51	0.68	0.46	0.18	11.17	0.16	0.09
$\Sigma_2^b$	2.5	2930	0.41	1.57	0.27	0.20	11.02	0.08	0.08
$\tilde{\Sigma}_2^b$		2637	0.33	1.26	0.40	0.30	8.82	0.05	0.06
$\Sigma_3^b$	2.5	3620	0.63	0.82	0.27	0.11	13.96	0.11	0.12
$\tilde{\Sigma}_3^b$		3258	0.89	1.15	0.40	0.08	15.36	0.16	0.12
$\Sigma_4^b$	2.5	3965	0.52	1.72	0.24	0.11	8.09	0.06	0.08
$\tilde{\Sigma}_4^b$		5947	0.63	2.41	0.29	0.14	11.32	0.08	0.11

□ Interconnected system

$$\mathbb{J} = \{0, 4, 1, 3, 2, 4, 1\}$$

□ Control law designed via the optimization problem

$$K = [0.462 \quad 1.5488 \quad -1.760 \quad 0.1667]$$





# Main point

---

1

Input output dynamics (block) that represents the interactions between vehicles.

# Main point

---

2

Base on this augmented dynamics we can achieved scalable conditions accounting for:

- ETC:

SILVA, R.; NGUYEN, A.; GUERRA, T.-M.; SOUZA, F.; FREZZATTO, L. Switched dynamic event-triggered control for string stability of nonhomogeneous vehicle platoons with uncertainty compensation. *IEEE Transactions on Intelligent Vehicles*, p. 1–15, 2024.

- Time delays:

R. Nascimento Silva, “Event-triggered control for nonlinear systems : Application to vehicle platoons,” Theses, Université Polytechnique Hauts-de-France ; Universidade Federal de Minas Gerais, Jul. 2024.

# CONCLUSION

# Conclusion

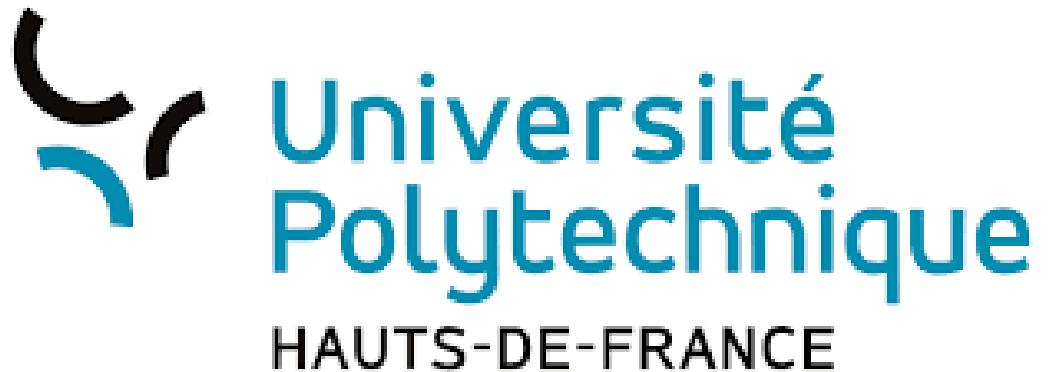
---

1

Conditions for longitudinal vehicle platoon were proposed such that:

- Individual and string stability can be ensured
- Uncertainties are compensated via DOB
- Design conditions are independent of the number of vehicles (scalability)

# Acknowledgments



Région  
Hauts-de-France





**THANK YOU ALL**