Control of heterogeneity in actuator degradation for over-actuated systems

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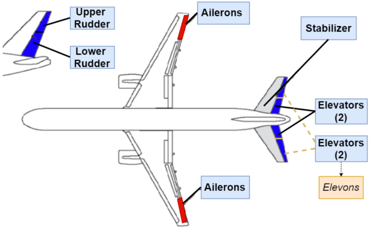
Over-actuated system

- **Definition:** More actuators than necessary to control its degrees of freedom.
- **Typical applications:**
  - Over-actuated aircraft / UAVs
  - Electric vehicles (independent motors)
- **Key characteristics:**
  - Redundancy
  - Fault tolerance
  - Performance optimization



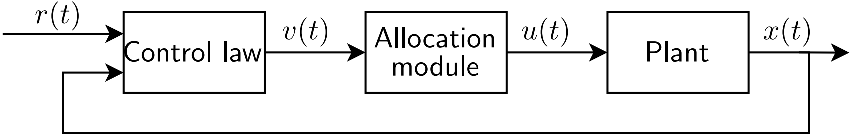
Example of actuator redundancy in aircraft[1]

Key Challenges

- How to distribute control effort among redundant actuators?
- **How to adapt the control strategy as the system degrades?**

[1] Reconfigurable Dynamic Control Allocation with SDRE As a FTFC for NASA GTM Design.

Control allocation (CA)



$$\dot{x}(t) = Ax(t) + B_u u(t) \quad \Rightarrow \quad \begin{array}{ll} ① & \dot{x}(t) = Ax(t) + B_v v(t), \\ ② & \mathbf{v}(\mathbf{t}) = \mathbf{B}\mathbf{u}(\mathbf{t}), \quad B = \left(B_v^T B_v\right)^{-1} B_v^T B_u \end{array}$$

Example Given $\dot{x} = u_1 + u_2$ with desired $v = 1$:

- $u_1 = 1, \ u_2 = 0$
- $u_1 = -10, \ u_2 = 11$
- $u_1 = u_2 = 0.5$
- $\cdots$

Redundancy yields infinitely many solutions for CA problem.

Identical control performance, but **different** long-term consequences.

CA strategy determines actuator degradation

While all solutions satisfy $v = Bu$, they lead to distinct actuator usage patterns and degradation behaviors.

Control inputs $u(t)$ are not "free" — high usage drives degradation:

$$\|u(t)\| \uparrow \implies \text{Degradation} \uparrow \implies \text{Actuator effectiveness} \downarrow$$

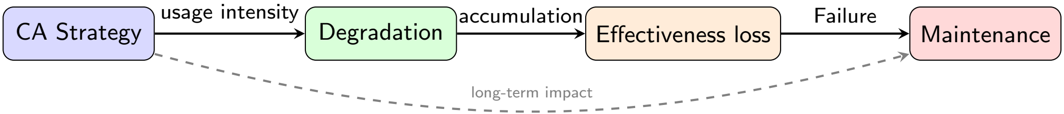
Example: $\dot{x} = u_1 + u_2$, desired $v = 1$

Strategy →	**Usage pattern** →	**Impact**
$u_1{=}1,\ u_2{=}0$	Actuator 1 fully stressed	Unbalanced
$u_1{=}u_2{=}0.5$	Load shared equally	Balanced
$u_1{=}-10,\ u_2{=}11$	Both degrade rapidly	Wasteful

Open question

How to design a CA strategy that optimize long-term degradation?

Joint control & maintenance optimization



Key idea: *CA is not only a control problem, but also a maintenance action with long-term impact.*

Research objective

Reduce long-run maintenance cost rate.

Level 1 ($\rightarrow u$): Control (degradation-aware allocation)

- Design CA strategies to balance degradation across actuators.
- Avoid premature failure of individual actuators to preserve system reliability.

Level 2 ($\rightarrow T$): Maintenance (block replacement policy)

- All actuators are periodically replaced every T.

KL-Based CA: Concept

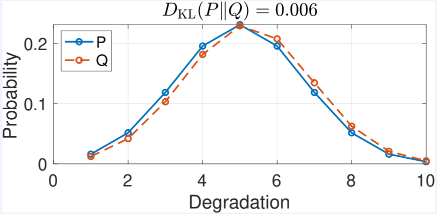
Goal: Achieve **balanced degradation** in a statistical way.
Motivation: Actuator degradation is stochastic.
Tool: Kullback–Leibler (KL) divergence.

What is KL divergence?

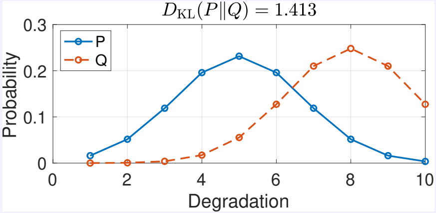
Statistical *distance* between two distributions P and Q: $D_{\mathrm{KL}}(P\|Q) = \sum_i P_i \ln \frac{P_i}{Q_i}$

Distributions are close $\Rightarrow$ small KL value

Distributions are far $\Rightarrow$ large KL value



Actuators degrade similarly



Actuators degrade differently

KL-Based CA: Formulation

Goal: Achieve **balanced degradation** in a statistical way.
Motivation: Actuator degradation is stochastic.
Tool: Kullback–Leibler (KL) divergence.

How to use it for CA?

Treat actuator degradation levels with a stochastic distribution P_i.

Minimizing $D_{\mathrm{KL}} \Rightarrow$ pushing all actuators toward **uniform degradation**.
Optimization formulation:

$$u(t_n) = \arg\min_u K(P_1, \ldots, P_m) \quad \text{s.t. } Bu(t_n) = v(t_n), \quad \underline{u} \leq u(t_n) \leq \bar{u}$$

with average KL divergence $K(P_1, \ldots, P_m) = \frac{1}{m(m-1)} \sum_{i \neq j} D_{KL}(P_i \| P_j)$.

- Provides a **mathematical guarantee** on balancing degradation.

Plant Modeling

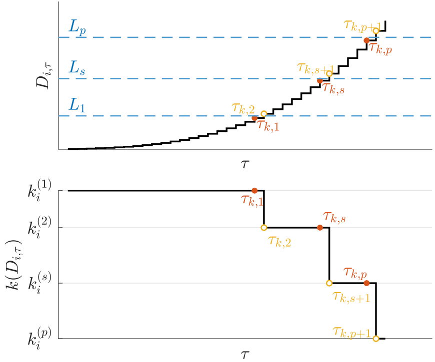
System dynamics $\dot{x}(t) = Ax(t)+B_u K(D_t)u(t)$

Effectiveness model

$$k_i(D_{i,t}) = \begin{cases} k_i^{(1)}, & D_{i,t} < L_1, \\ k_i^{(2)}, & L_1 \leq D_{i,t} < L_2, \\ \vdots & \\ k_i^{(p)}, & D_{i,t} \geq L_{p-1} \end{cases}$$

- $k_i^{(s)}$: effectiveness levels
- L_s: degradation thresholds
- $D_{i,t} = D_{i,\tau_k}$: degradation level

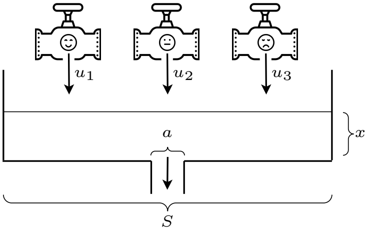
Degradation and effectiveness model explanation

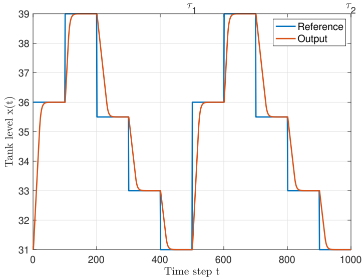


Degradation model $D_i(\tau_{k+1}) = D_i(\tau_k) + \alpha_i \int_{\tau_k}^{\tau_{k+1}} |\dot{u}_i(t)|\, dt + \beta_i \int_{\tau_k}^{\tau_{k+1}} u_i^2(t)\, dt + \Delta B_i$

- $\tau_k = k\Delta\tau$: degradation update instant
- $\Delta\tau = N_D \Delta t$: degradation sampling period
- Δt: control sampling period

Case study





System dynamics $\rho S\dot{x}(t) = -\rho a\sqrt{2gx(t)} + B_u K(D_t)u(t)$

Virtual controller $v(t) = K_p \cdot \left(e(t) + \frac{1}{T_i} \cdot \int_0^t e(z)dz + T_d \cdot \frac{de(t)}{dt}\right)$

Effectiveness model

$$k_i(D_{i,t}) = \begin{cases} 1, & D_{i,t} < 0.25\,D_{\max}, \\ 0.75, & 0.25\,D_{\max} \le D_{i,t} < 0.5\,D_{\max}, \\ 0.5, & 0.5\,D_{\max} \le D_{i,t} < 0.75\,D_{\max}, \\ 0.25, & 0.75\,D_{\max} \le D_{i,t} < D_{\max}, \\ 0, & D_{i,t} \ge D_{\max}, \end{cases}$$

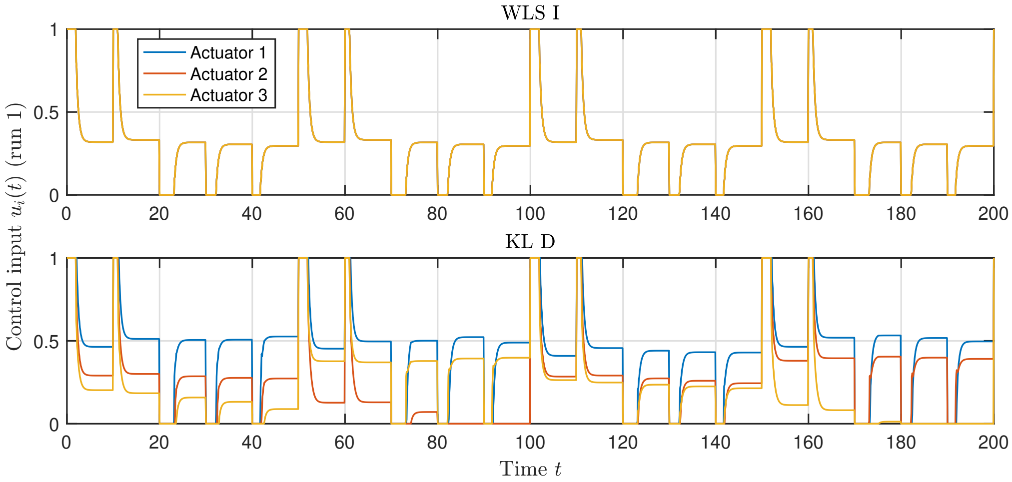
Degradation model

$$D_i(\tau_{k+1}) = D_i(\tau_k) + \alpha_i \int_{\tau_k}^{\tau_{k+1}} |\dot{u}_i(t)|\,dt + \beta_i \int_{\tau_k}^{\tau_{k+1}} u_i^2(t)\,dt + \Delta B_i$$

Compared CA strategies

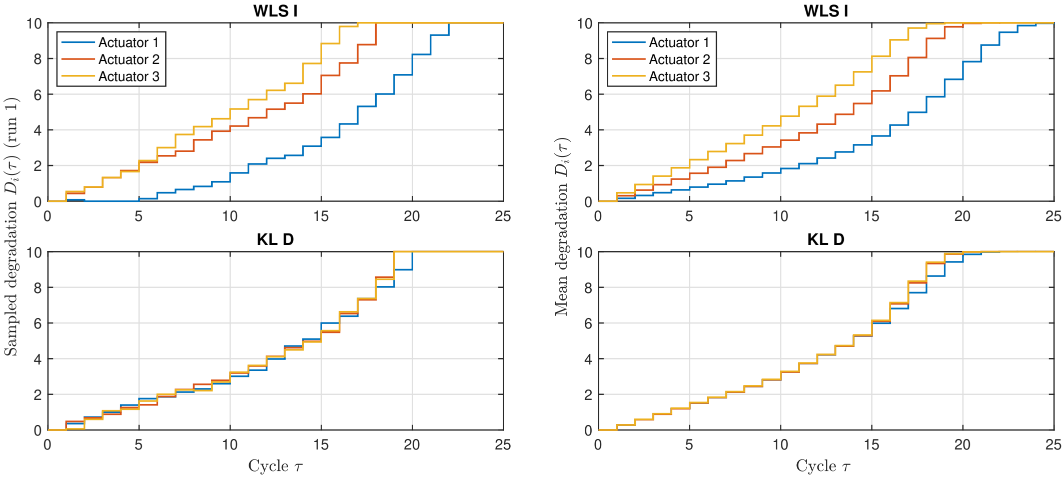
- WLS I: minimize energy consumption
- KL D: minimize KL divergence

Control input



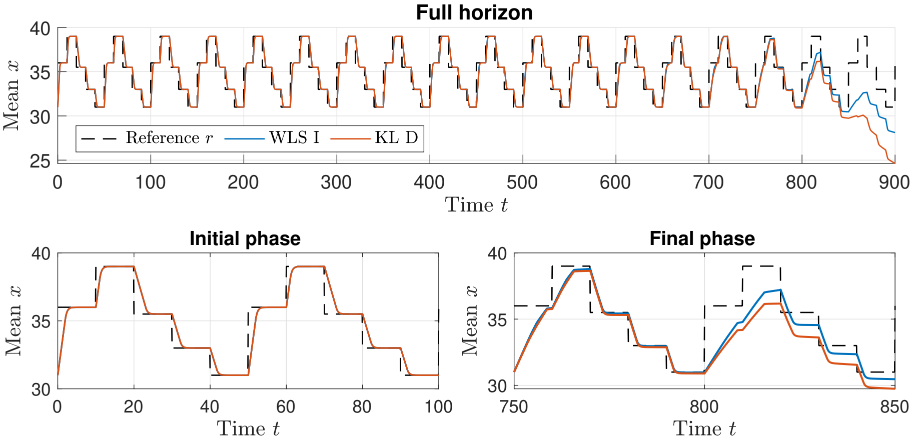
Each strategy exhibits a distinct control usage pattern.

Degradation



The performance in controlling degradation heterogeneity is: KL D > WLS I.

System behaviour



Redundancy preserves tracking performance until severe actuator degradation occurs.
KL D improves degradation balance, but accelerates performance loss and leads earlier failure.

Maintenance cost evaluation

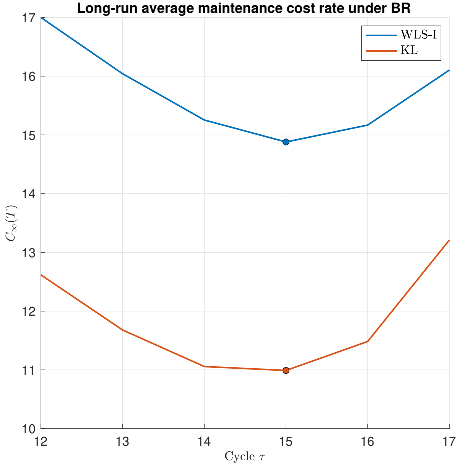
$$C_\infty(T) = \frac{1}{E[\tau]} E\left[\sum_{i=1}^{m} (C_p 1_{T<T_{f,i}} + C_c 1_{T \geq T_{f,i}}) + C_l \int_0^{\min(T,T_f)} P_{\text{loss}}(s)\, ds + C_d (T - T_f) 1_{T \geq T_f} \right] + E\left[C_v \mathrm{Var}^{(r)}(T) 1_{T<T_{f,i}} \right]$$

Main terms

- T: maintenance interval (decision variable)
- $T_{f,i}$: failure time of actuator i
- T_f: failure time of system

- C_p: preventive replacement cost
- C_c: corrective replacement cost
- C_l: performance loss cost
- C_d: downtime cost
- C_v: unbalanced degradation cost

Comparative performance: KL D < WLS I



Conclusion and perspective

Conclusion

✓ **Joint optimization of control and maintenance:**
Co-design of CA strategy and replacement interval to reduce long-run maintenance cost rate — treating CA as a maintenance action, not only a control problem.

✓ **Degradation-aware CA frameworks:**
Propose a CA strategy that distribute actuator usage based on health state to balance degradation across actuators.

Perspective

➔ Control the level of heterogeneity of degradation.

Thank you for your attention!