

Traitement statistique de signaux pour la Surveillance d'Intégrité de Structures

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Traitement **statistique**
de l'**information** multi-capteurs
sur la base de **modèles physiques**
pour la **surveillance** et le **diagnostic**
de **systèmes dynamiques** complexes

Problématique privilégiée :
surveillance de systèmes soumis à des **vibrations**

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Problème : surveillance **en fonctionnement**

- The **excitation** is typically:
 - natural, **not controlled**.
 - **not measured**:
 - * buildings, bridges, offshore structures,
 - * rotating machinery (e.g. steam flowing),
 - * cars, trains, aircrafts.
 - **nonstationary** (e.g., turbulent).
- The data contain slow and fast non-stationarities
- How to **merge** multiple measurements setups
e.g. in case of **moving** sensors?
- How to **detect** and **localize small** damages ?

Model-based statistical approach - Key issues

- **9.** Modelling
- **11.** Output-only covariance-driven subspace identification
- **13.** Merging multiple measurements setups
- **19.** Robustness to nonstationary excitation
- **21.** Structural monitoring: damage detection and diagnostics
- 30.** Examples: Z24 bridge & flutter monitoring
- 45.** Free toolbox

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Vibrations : **problème de structure propre**

FE model:
$$\begin{cases} M \ddot{Z}(s) + C \dot{Z}(s) + K Z(s) = \nu(s) \\ Y(s) = L Z(s) \end{cases}$$

$(M\mu^2 + C\mu + K) \Psi_\mu = 0, \quad \psi_\mu = L \Psi_\mu$

State space:
$$\begin{cases} X_{k+1} = F X_k + V_k \\ Y_k = H X_k \end{cases}$$

$F \varphi_\lambda = \lambda \varphi_\lambda, \quad \phi_\lambda \triangleq H \varphi_\lambda$

Parameter:
$$\underbrace{e^{\delta\mu}}_{\text{modes}} = \lambda, \quad \underbrace{\psi_\mu = \phi_\lambda}_{\text{mode shapes}}; \quad \theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec } \Phi \end{pmatrix}$$

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Vibrations : **problème ARMA non-stationnaire**

$$\begin{cases} X_{k+1} = F X_k + V_k \\ Y_k = H X_k \end{cases}$$

$$Y_k = \sum_{i=1}^p A_i Y_{k-i} + \sum_{j=0}^q B_j(k) V_{k-j}, \quad H F^p = \sum_{i=1}^p A_i H F^{p-i}$$

- Monitor AR part, with nonstationary MA part.
- Modal changes not visible on spectra (1%).
- Likelihood : no hope !

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Output-only **covariance-driven subspace** identification

$$\begin{cases} X_{k+1} = F X_k + V_k & G \triangleq E(X_k Y_k^T) \\ Y_k = H X_k & R_i \triangleq E(Y_k Y_{k-i}^T) \\ & \text{ok if stationary !} \end{cases}$$

$$\mathcal{H} \triangleq \begin{pmatrix} R_0 & R_1 & R_2 & \dots \\ R_1 & R_2 & R_3 & \dots \\ R_2 & R_3 & R_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}, \quad \mathcal{O} \triangleq \begin{pmatrix} H \\ HF \\ HF^2 \\ \vdots \end{pmatrix}, \quad \mathcal{C} \triangleq (G \quad FG \quad F^2G \quad \dots)$$

$$R_i = H F^i G \implies \mathcal{H} = \mathcal{O} \mathcal{C}$$

$$\mathcal{H} \longrightarrow \mathcal{O} \longrightarrow (H, F) \longrightarrow (\lambda, \varphi_\lambda)$$

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Implementation

$$\hat{R}_i \triangleq \frac{1}{N} \sum_{k=1}^N Y_k Y_{k-i}^T, \quad \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

ok when nonstationary !

$$\hat{\mathcal{H}} \approx \hat{\mathcal{O}} \hat{\mathcal{C}}$$

$$\text{SVD}(\hat{\mathcal{H}}) + \text{truncation} \longrightarrow \hat{\mathcal{O}} \longrightarrow (\hat{H}, \hat{F}) \longrightarrow (\hat{\lambda}, \hat{\varphi}_\lambda)$$

$$\hat{\mathcal{H}} = U \Delta W^T = U \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_0 \end{pmatrix} W^T; \quad \hat{\mathcal{O}} = U \Delta_1^{1/2}$$

$$\mathcal{O}_p^\dagger(H, F) = \mathcal{O}_p(H, F) F$$

$$\det(F - \lambda I) = 0, \quad F \Phi_\lambda = \lambda \Phi_\lambda, \quad \varphi_\lambda = H \Phi_\lambda$$

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Model-based statistical approach - Key issues

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- Output-only covariance-driven subspace identification
- **Merging multiple measurements setups**
- Robustness to nonstationary excitation
- Structural monitoring: damage detection and diagnostics

Examples: Z24 bridge & flutter monitoring

Free toolbox

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Merging multiple measurements setups (1)

$$\underbrace{\begin{bmatrix} Y_k^{(0,1)} \\ Y_k^{(1)} \end{bmatrix}}_{\text{Record 1}} \quad \underbrace{\begin{bmatrix} Y_k^{(0,2)} \\ Y_k^{(2)} \end{bmatrix}}_{\text{Record 2}} \quad \dots \quad \underbrace{\begin{bmatrix} Y_k^{(0,J)} \\ Y_k^{(J)} \end{bmatrix}}_{\text{Record J}}$$

$$\begin{cases} X_{k+1}^{(j)} = F X_k^{(j)} + V_k^{(j)} \\ Y_k^{(0,j)} = H_0 X_k^{(j)} & \text{(the reference)} \\ Y_k^{(j)} = H_j X_k^{(j)} & \text{(sensor pool } n^o j) \end{cases}$$

$$R_i^{0,j} \triangleq E Y_k^{(0,j)} Y_{k-i}^{(0,j)T}, \quad R_i^j \triangleq E Y_k^{(j)} Y_{k-i}^{(j)T}$$

$$E Y_k^{(j)} Y_{k-i}^{(j)T} \text{ not used, } E Y_k^{(j')} Y_{k-i}^{(j)T} (j \neq j') \text{ not available}$$

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Merging multiple measurements setups (2)

Stationary excitation

$$\text{cov} V_k^{(j)} = Q, \quad G \triangleq E X_k^{(j)} Y_k^{(0,j)T}$$

$$R_i^{0,j} = H_0 F^i G \triangleq R_i^0, \quad R_i^j = H_j F^i G$$

$$R_i^\pi \triangleq \begin{bmatrix} R_i^0 \\ R_i^1 \\ \vdots \\ R_i^J \end{bmatrix} = H F^i G, \quad H \triangleq \begin{bmatrix} H_0 \\ H_1 \\ \vdots \\ H_J \end{bmatrix}$$

Nonstationary excitation

$$\text{cov} V_k^{(j)} = Q_j, \quad G_j \triangleq E X_k^{(j)} Y_k^{(0,j)T}$$

$$R_i^{0,j} = H_0 F^i G_j, \quad R_i^j = H_j F^i G_j$$

Hint: right **renormalization** of the covariances.

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Merging multiple measurements setups (3)

Reference sensors: using the **redundancy**

$$R_i^{0,j} = H_0 F^i G_j, \quad G \triangleq (G_1 \ G_2 \ \dots \ G_J)$$

$$R_i^0 \triangleq (R_i^{0,1} \ R_i^{0,2} \ \dots \ R_i^{0,J}) = H_0 F^i G$$

$$\mathcal{H}_0 \triangleq \begin{pmatrix} R_0^0 & R_1^0 & R_2^0 & \dots \\ R_1^0 & R_2^0 & R_3^0 & \dots \\ R_2^0 & R_3^0 & R_4^0 & \dots \\ \vdots & \vdots & \dots & \vdots \end{pmatrix} = \mathcal{O}(H_0, F) \mathcal{C}(F, G) \longrightarrow \mathcal{C}(F, G_j)$$

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Merging multiple measurements setups (4)

Moving sensors: normalizing the data

$$R_i^j = H_j F^i G_j, \quad \mathcal{H}_j \triangleq \begin{pmatrix} R_0^j & R_1^j & R_2^j & \dots \\ R_1^j & R_2^j & R_3^j & \dots \\ R_2^j & R_3^j & R_4^j & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \mathcal{O}(H_j, F) \mathcal{C}(F, G_j)$$

$$\bar{\mathcal{H}}_j \triangleq \mathcal{H}_j \left(\mathcal{C}^T(F, G_j) \left(\mathcal{C}(F, G_j) \mathcal{C}^T(F, G_j) \right)^{-1} \mathcal{C}(F, G_1) \right)$$

$$\bar{\mathcal{H}}_j \leftrightarrow \bar{R}_i^j \triangleq H_j F^i G_1 \quad (1 \leq j \leq J)$$

$$\bar{R}_i \triangleq \begin{pmatrix} \bar{R}_i^0 \\ \bar{R}_i^1 \\ \vdots \\ \bar{R}_i^J \end{pmatrix}, \quad \bar{R}_i^0 \triangleq R_i^{0,1} = H_0 F^i G_1$$

$$\bar{\mathcal{H}} \triangleq \begin{pmatrix} \bar{R}_0 & \bar{R}_1 & \bar{R}_2 & \dots \\ \bar{R}_1 & \bar{R}_2 & \bar{R}_3 & \dots \\ \bar{R}_2 & \bar{R}_3 & \bar{R}_4 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} = \mathcal{O}(H, F) \mathcal{C}(F, G_1)$$

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Model-based statistical approach - Key issues

- Modelling
- Output-only covariance-driven subspace identification
- Merging multiple measurements setups
- **Robustness to nonstationary excitation**
- Structural monitoring: damage detection and diagnostics

Examples: Z24 bridge & flutter monitoring

Free toolbox

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Merging multiple measurements setups (5)

- Build the J Hankel matrices $\mathcal{H}_{0,j} = \text{Hank}(R_i^{(0,j)})$
- $\mathcal{H}_0 \triangleq$ **interleaving** the block-columns of the $\mathcal{H}_{0,j}$'s
- **SVD** (\mathcal{H}_0) + truncation $\rightarrow \mathcal{C}(F, G) \triangleq \mathcal{C}$
- Partition $\mathcal{C} = (\mathcal{C}_1 \mathcal{C}_2 \dots \mathcal{C}_J)$
- Compute $(\mathcal{C}_j^T (\mathcal{C}_j \mathcal{C}_j^T)^{-1} \mathcal{C}_1)$
- Build the J Hankel matrices $\mathcal{H}_j = \text{Hank}(R_i^j)$
- **Renormalize:** $\bar{\mathcal{H}}_j \triangleq \mathcal{H}_j (\mathcal{C}_j^T (\mathcal{C}_j \mathcal{C}_j^T)^{-1} \mathcal{C}_1)$; $\bar{\mathcal{H}}_0 \triangleq \mathcal{H}_{0,1}$
- $\bar{\mathcal{H}} \triangleq$ **interleaving** the block-rows of the $(J+1)$ $\bar{\mathcal{H}}_j$'s
- Apply the subspace algorithm to $\bar{\mathcal{H}}$.

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Robustness to nonstationary excitation

Approximate factorization of covariances : $\hat{R}_i \approx H F^i \hat{G}$

Consistency : $T^{-1} \hat{F} T \rightarrow F, \hat{H} \rightarrow H; (\hat{\lambda}, \hat{\varphi}_\lambda) \rightarrow (\lambda, \varphi_\lambda)$

Holds true when merging multiple setups as well.

Theory and experience show that the **combination of:**

- the key **factorization** property of the covariances,
- the **averaging** operation in their computation,

allows to **cancel out nonstationarities in the excitation**

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Model-based statistical approach - Key issues

- Modelling
- Output-only covariance-driven subspace identification
- Merging multiple measurements setups
- Robustness to nonstationary excitation
- **Structural monitoring: damage detection and diagnostics**

Examples: Z24 bridge & flutter monitoring

Free toolbox

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Structural monitoring : Eigenstructure monitoring

$$\begin{cases} X_{k+1} = F X_k + V_k & F \varphi_\lambda = \lambda \varphi_\lambda \\ Y_k = H X_k & \phi_\lambda \triangleq H \varphi_\lambda \end{cases}$$

Canonical parameter : $\theta \triangleq \begin{pmatrix} \Lambda \\ \text{vec } \Phi \end{pmatrix}$ **modes**
mode shapes

Observability in modal basis : $\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi \Delta \\ \vdots \\ \Phi \Delta^p \end{pmatrix}$

System parameter characterization:

$\mathcal{H}_{p+1,q}$ and $\mathcal{O}_{p+1}(\theta)$ have the **same left kernel**.

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Eigenstructure monitoring - Contd

θ_0 : reference parameter, known (or identified)

Y_k : N -size sample of new measurements

Test statistics $\zeta_N(\theta_0)$: Which **estimating function** ?

Local approach (**small** deviations)

Test $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_0 + \delta\theta/\sqrt{N}$

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Eigenstructure monitoring - Contd

Fresh data $\rightarrow \hat{R}_i \rightarrow \hat{\mathcal{H}} = \begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \hat{R}_2 & \dots \\ \hat{R}_1 & \hat{R}_2 & \hat{R}_3 & \dots \\ \hat{R}_2 & \hat{R}_3 & \hat{R}_4 & \dots \\ \vdots & \vdots & \dots & \vdots \end{pmatrix}$

Nominal model : $\mathcal{O}(\theta_0) = \begin{pmatrix} \Phi \\ \Phi \Delta \\ \Phi \Delta^2 \\ \vdots \end{pmatrix}$ Observability
in modal basis

! $\mathcal{H} = \mathcal{O} \mathcal{C}$! $\ker \hat{\mathcal{H}}^T \stackrel{?}{=} \ker \mathcal{O}^T(\theta_0)$

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Eigenstructure monitoring - Contd

System parameter characterization:

$\mathcal{H}_{p+1,q}$ and $\mathcal{O}_{p+1}(\theta)$ have the **same left kernel**.

$\exists S, S^T S = I_s, S^T \mathcal{O}_{p+1}(\theta_0) = 0; \text{ say } S(\theta_0)$

$\theta_0 \leftrightarrow (R_i^0)_i$ characterized by: $S^T(\theta_0) \hat{\mathcal{H}}_{p+1,q}^0 = 0$

The **subspace-based residual** $\zeta_N(\theta_0)$:

$$\zeta_N(\theta_0) \triangleq \sqrt{N} \text{vec} \left(S^T(\theta_0) \hat{\mathcal{H}}_{p+1,q} \right)$$

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Model-based monitoring - **Generalization**

Any **estimating function** can play the role of a **residual**

Warning:

The **prediction error** is OK for sensor faults,
NOT for structural damages !

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The **test statistics** is asymptotically **Gaussian**

Mean **sensitivity** (Jacobian) $\mathcal{J}(\theta_0)$ and **covariance** $\Sigma(\theta_0)$

$$\zeta_N(\theta_0) \rightarrow \begin{cases} \mathcal{N}(0, \Sigma(\theta_0)) & \text{under } P_{\theta_0} \\ \mathcal{N}(\mathcal{J}(\theta_0) \delta\theta, \Sigma(\theta_0)) & \text{under } P_{\theta_0 + \frac{\delta\theta}{\sqrt{N}}} \end{cases}$$

(GLR) χ^2 -test for modal monitoring

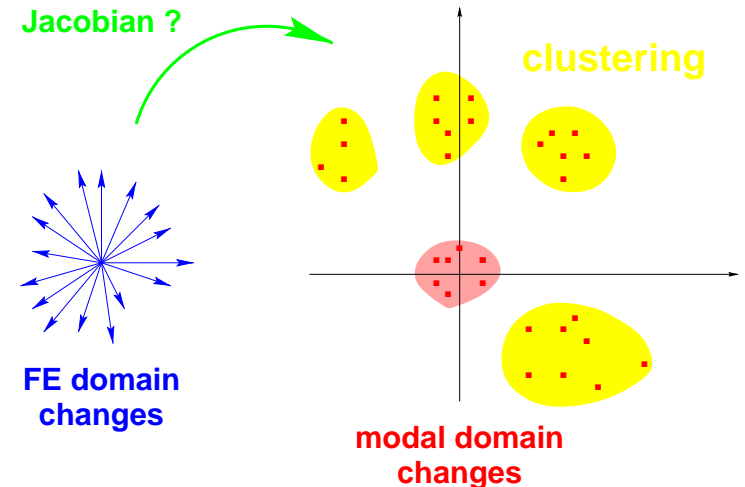
$$\zeta_N^T \Sigma^{-1} \mathcal{J} (\mathcal{J}^T \Sigma^{-1} \mathcal{J})^{-1} \mathcal{J}^T \Sigma^{-1} \zeta_N \geq h$$

(GLR) **Directional** χ^2 -test for modal **diagnosis**

$$\zeta_N^T \Sigma^{-1} \mathcal{J}_i (\mathcal{J}_i^T \Sigma^{-1} \mathcal{J}_i)^{-1} \mathcal{J}_i^T \Sigma^{-1} \zeta_N \geq h$$

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On-board damage diagnostics: projecting changes



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$$\zeta \sim \mathcal{N}(\mathcal{I} \delta\theta, \Sigma), \quad \delta\theta = \mathcal{I} \mathcal{J}_{(M_0^*, K_0^*)} \begin{pmatrix} \delta M \\ \delta K \end{pmatrix}$$

(M_0^*, K_0^*) : design model

Jacobian : $(\delta M, \delta K) \xrightarrow{\mathcal{J}_{(M_0^*, K_0^*)}} (\delta\mu, \delta\psi_\mu)$

Reduction: \mathcal{I} matching computed/identified modes

Problem : $\dim \begin{pmatrix} M \\ K \end{pmatrix} \gg \dim \theta$

Hint: Cluster the vectors $(\delta\mu, \delta\psi_\mu)$ using the χ^2 -metric

- Example 1 - Z24 bridge
- Example 2 - Beam within thermal chamber
- Example 3 - Aircraft flutter monitoring
- Test cases processed so far

<http://www.irisa.fr/sisthem/sisthem-testcases.pdf>

Processed test cases (1) - Simulators and laboratory test-beds

Task	Structure	Our algos tested at	Partner
I & D	18 masses & springs	Sisthem	CNEXO
I & D	Rotating shaft	Sisthem	EDF
I	Triangular structure	AS&I Dataid	
I & D	In-flight rocket	Sisthem	Ecole Centrale P.
I & D	Steelquake simulator	Sisthem	
I & D	Flutter simulator	Sisthem	V.U.Brussels
I & D	In-flight data	Sisthem	Airbus France
I	Launcher	EADS L.V.	CNES
I & D	Bridge deck	Sisthem	Ecole Centrale P.
I & D	Cantilever beam	Sisthem	EDF
I & D	Clamped & free aluminum beam	LMS	
I & D	Steel frame breadboard model	LMS	
I & D	Sport car scale model	LMS	
I & D	Steelquake model	Sisthem	COST F3
I & D	Composite plate	U. Siegen	
I & D	Reticular structure	LMS	
I & D	Pre-stressed beam	Sisthem	LCPC
I	Large scale pre-flexed beam	LCPC, Sisthem	
I & D	Aircraft model	Sisthem	U. Liège
I & D	Composite plastic engine oil pan	LMS, Sisthem	

Processed test cases (2) - Real structures

Task	Structure	Our algos tested at	Partner
I & D	Offshore platforms	CNEXO, Sisthem	Elf Norway
I & D	Turbo-alternator	Sisthem	EDF
I	Subway carrying bogies	AS&I Dataid	Ratp
I	Washing machine	AS&I Dataid	
I & D	Paris MS760 aircraft	Sopemea, Sisthem	
I & D	Sport car	LMS	
I	Helicopter	PZL-Swidnik	
I & D	Z24 bridge	LMS, Sisthem	COST F3
I & D	Slat track	LMS	
I & D	Ariane5 launcher, ground tests	Sisthem	EADS L.V.
I & D	Ariane5, flights 501 & 502	CNES, Sisthem	EADS L.V.
I	Launcher	EADS Launch Vehicles	CNES
I	Airbus A320 - In-flight	Airbus France, Sisthem	Airbus Fr.
I	Airbus A340 - In-flight	Sisthem	Airbus Fr.
I	Rafale - Ground tests	Sopemea, Sisthem	Dassault
I	Rafale - In-flight	Sopemea, Dassault, Sisthem	Dassault
I	Falcon - In-flight	Sopemea, Dassault, Sisthem	Dassault
I	Bradford stadium	Sisthem	U.Sheffield
I	Manchester stadium	Sisthem	U.Sheffield

Example 1 - Z24 bridge

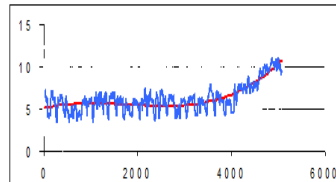
- A benchmark of the BRITE/EURAM project SIMCES and of the European COST action F3
- Response to traffic excitation under the bridge measured over one year in 139 points
- Two damage scenarios (DS1 and DS2): pier settlements of 20mm and 80mm.

Identified first four natural frequencies / Test values
(Results with four sensors)

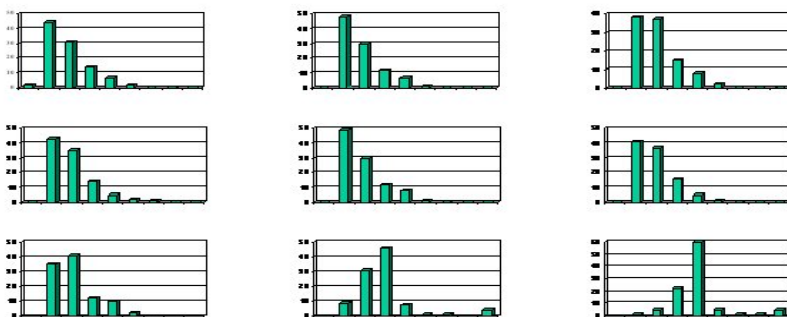
	Mode	1	2	3	4	χ^2
Safe	Freq.(Hz)	3.88	5.01	9.80	10.30	$8.80 \cdot 10e2$
Dam. DS1	Freq.(Hz)	3.87	5.06	9.79	10.32	$8.00 \cdot 10e5$
Dam. DS2	Freq.(Hz)	3.76	4.93	9.74	10.25	$3.96 \cdot 10e6$

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Test values over nine months (log-scale).

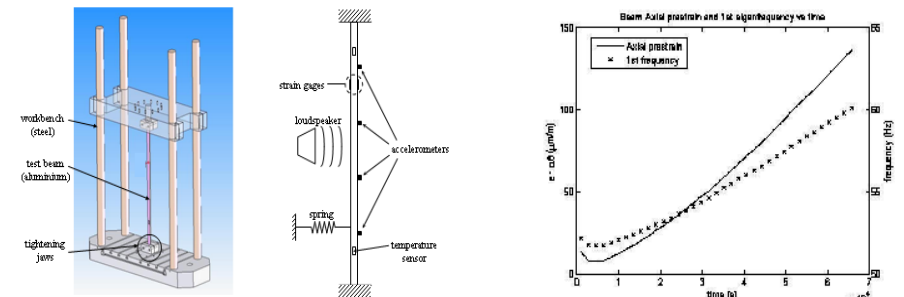


Histograms of the test values for each of the nine months.

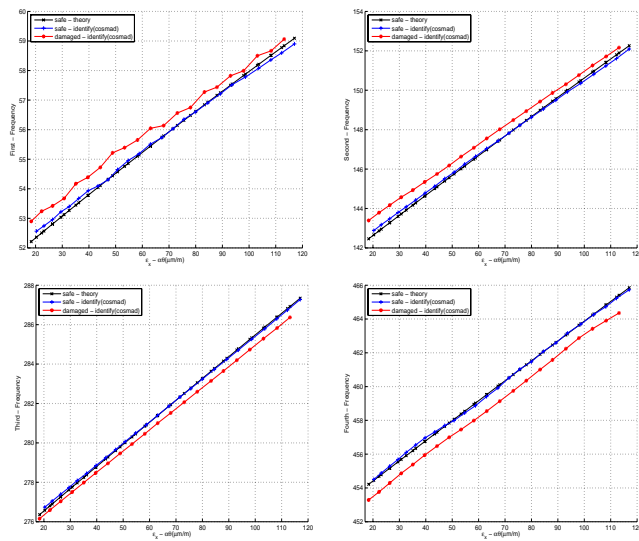
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Example 2 - Beam within thermal chamber

- A laboratory test-case provided by LCPC
- Vertical clamped beam, subject to increasing temperatures
- Local damage: horizontal clamped spring attached to the beam, with tunable stiffness and height
- Thermal effect modelling, analytical computations for temperature updated modal parameters



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First 4 frequencies vs. thermal constraint.
Computed (black) and identified (blue: safe, red: damaged)

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Example 3 - Aircraft flutter monitoring

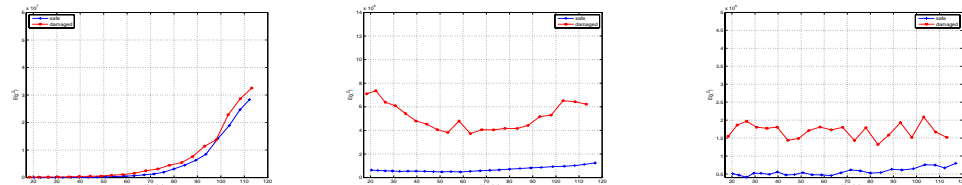
- Aero-elastic flutter: critical instability phenomenon
- Flight flutter testing procedure
- Objective: **on-line** in-flight exploitation of test data
- On-line flight **flutter monitoring** problem:
monitoring some specific **damping coefficient**
- Decide whether $\rho < \rho_c$, critical value

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Robustness to **temperature** effects

New results - 3 solutions

- **Empirical** kernel, **merging** many temperature conditions
- Simplified model of the temperature effect
Analytical **kernel updating**
- Simplified model of the temperature effect
Statistical **nuisance rejection** techniques



Original χ^2 -test After **empirical merging** After **kernel updating**
Safe (blue) and damaged (red)

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Monitoring **on-line** a **damping** coefficient

- Write the **subspace-based test-statistics** ζ
as a **cumulative sum**
- Test $\rho \geq \rho_c$ against $\rho < \rho_c$
Non local ! Use a **different asymptotics** for ζ
- Introduce a **minimum change magnitude**
(actual change magnitude unknown)
- Run two **CUSUM tests** in parallel
(actual change direction unknown)

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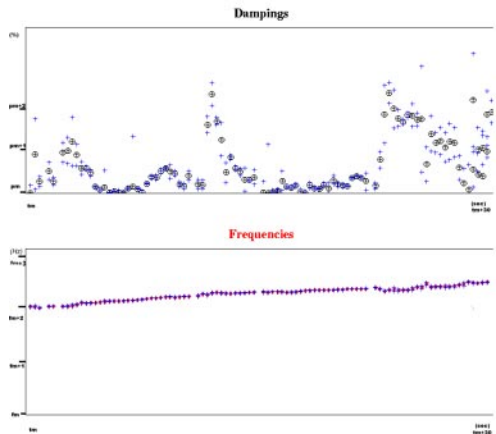
Monitoring a **damping** coefficient - Details

$$\bar{\zeta}_n(\rho_0) \triangleq \mathcal{J}(\rho_0)^T \Sigma(\theta_0)^{-1} \zeta_n(\theta_0) = \frac{n-p}{\sum_{k=q}^{n-p}} Z_k(\rho_0) / \sqrt{n}$$

$$Z_k(\rho_0) \triangleq \mathcal{J}(\rho_0)^T \Sigma(\theta_0)^{-1} \text{vec} \left(U(\theta_0)^T \mathcal{Y}_{k,p+1}^+ \mathcal{Y}_{k,q}^-^T \right)$$

$Z_k \sim \mathcal{N}(\nu, \cdot)$, Z_k 's independent

$$\bar{H}_0 : \nu \geq + \nu_m/2 \quad \text{and} \quad \bar{H}_1 : \nu < - \nu_m/2$$



Ariane booster launcher during a launch scenario on the ground Automatic identification. Each symbol: processing 5 sec. data.

Monitoring a **damping** coefficient - Details - Contd

Decreasing mean

$$S_n^{(-)} \triangleq \Sigma^{-1/2} \sum_{k=q}^{n-p} (Z_k + \nu_m)$$

$$T_n^{(-)} \triangleq \max_{q \leq k \leq n-p} S_k^{(-)}$$

$$g_n^- \triangleq T_n^{(-)} - S_n^{(-)}$$

$$g_n^- \geq \gamma^-$$

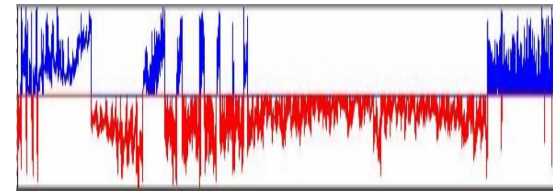
Increasing mean

$$S_n^{(+)} \triangleq \Sigma^{-1/2} \sum_{k=q}^{n-p} (Z_k - \nu_m)$$

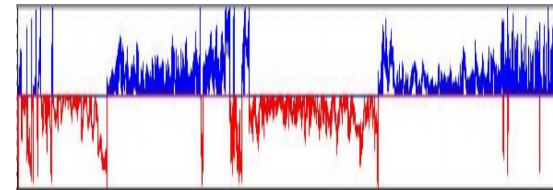
$$T_n^{(+)} \triangleq \min_{q \leq k \leq n-p} S_k^{(+)}$$

$$g_n^+ \triangleq S_n^{(+)} - T_n^{(+)}$$

$$g_n^+ \geq \gamma^+$$



Test for $\rho_c = \rho_c^{(1)}$.



Test for $\rho_c = \rho_c^{(2)} < \rho_c^{(1)}$.

Bottom: $-g_n^-$ reflects $\rho < \rho_c$. **Top:** g_n^+ reflects $\rho > \rho_c$.

Toolbox COSMAD

COvariance Subspace Modal Analysis and Diagnosis

<http://www.irisa.fr/sisthem/cosmad/>

A **toolbox** to be used within **Scilab**
a **free** matrix computing environment

with support of **Eurêka** projects **SINOPSYS** and **FLiTE**

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Experimented for and/or within

- LMS International
- EADS Space Transportation and CNES
- Dassault Aviation
- Eurocopter
- SOPEMEA (Test Lab.)
- LCPC/SMI (Civil Eng. Inst. / Metrology dept.)

Current research topics and developments

- **Flutter** monitoring (aircrafts)
- Rejecting **temperature** effects (civil eng.)

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Functions

- **Output-only** subspace-based **identification**
- **Input-output** subspace-based identification
- **Automated** subspace **modal analysis** and **tracking**
- **Automated recursive** subspace modal analysis
- Subspace-based **moving sensors data fusion**
- **Damage detection**
- **Damage monitoring**
- **Modal diagnosis**
- **Damage localization**
- **Optimal sensors positioning** for monitoring

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References

- M. Basseville, A. Benveniste, M. Goursat, L. Hermans, L. Mevel, H. Van der Auweraer, Output-only subspace-based structural identification, from theory to industrial testing practice. *ASME Jnl Dynamic Systems Measurement and Control, Special Issue on Identification of Mechanical Systems*, V.123, N.4, Dec. 2001, pp.668-676.
- L. Mevel, M. Basseville, A. Benveniste, M. Goursat, Merging sensor data from multiple measurement setups for nonstationary subspace-based modal analysis. *Jnl Sound and Vibration*, V.249, N.4, Jan. 2002, pp.719-741.
- L. Mevel, A. Benveniste, M. Basseville, M. Goursat, Blind subspace-based eigenstructure identification under nonstationary excitation using moving sensors. *IEEE Trans. Signal Processing*, V.50, N.1, Jan. 2002, pp.41-48.
- M. Basseville, M. Abdelghani, A. Benveniste, Subspace-based fault detection algorithms for vibration monitoring. *Automatica*, V.36, N.1, Jan. 2000, pp.101-109.
- M. Basseville, L. Mevel, M. Goursat, Statistical model-based damage detection and localization, subspace-based residuals and damage-to-noise sensitivity ratios. *Jnl Sound and Vibration*, V.275, N.3-5, Aug. 2004, pp.769-794.
- L. Mevel, M. Basseville, A. Benveniste, Fast in-flight detection of flutter onset - A statistical approach. *AIAA Jnl Guidance, Control, and Dynamics*, V.28, N.3, May 2005, pp.431-438.
- M. Basseville, A. Benveniste, M. Goursat, L. Mevel, Subspace-based algorithms for structural identification, damage detection, and sensor data fusion. *Jnl Applied Signal Processing, Special Issue on Advances in Subspace-Based Techniques for Signal Processing and Communications*, under minor revision, Feb. 2006.

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