Subspace-based fault detection and isolation for structural health monitoring

Michael Döhler and Laurent Mevel

Inria
Centre Rennes - Bretagne Atlantique, Rennes

I4S Research Team

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Outline

Outline

Motivation: FDI for structural health monitoring

Context

- Fault detection and isolation in structural engineering
- Vibration monitoring of civil, aeronautical or mechanical structures

FDI problem

- **Fault detection:** detect structural damage
- **Fault isolation:** locate damage in the structure

How?

Hypothesis tests on parameterized Gaussian residual vector

- zero mean in reference state
- non-zero mean in faulty state

Models

- Stationary linear dynamical system

$$\mathcal{M}\ddot{z}(t) + \mathcal{C}\dot{z}(t) + \mathcal{K}z(t) = v(t)$$

  ... observed at some sensor coordinates
- Discrete-time state space model for identification

$$\begin{cases} x_{k+1} & = & A\ x_k + v_k \\ y_k & = & C\ x_k + w_k \end{cases}$$

- “Input” v_k is unmeasured non-stationary noise
- Model order is large (in the 100’s)

Parameters

- **Damage** produces change in structural properties and modal parameters
- **Structural parameters:**
  - Finite element model of a structure
  - E.g. element mass, element stiffness
  - Model-based parameters, not from measurements

$$\mathcal{M}(\theta)\ddot{z}(t) + \mathcal{C}\dot{z}(t) + \mathcal{K}(\theta)z(t) = v(t)$$

- **Modal parameters:**
  - Natural frequencies, damping ratios, mode shapes
  - Found in eigenstructure of (C, A)
  - Can be obtained from system identification (e.g. Stochastic Subspace Identification)

$$\begin{cases} x_{k+1} & = & A(\theta)\ x_k + v_k \\ y_k & = & C(\theta)\ x_k + w_k \end{cases}$$

Faults

Damage characterization

- **Damage detection:** is there a change in θ?
  - θ = modal or structural parameter
  - Fault detection
- **Damage localization:** which components of θ changed?
  - θ = local structural parameter
  - Fault isolation
- **Damage quantification:** estimate $\Delta\theta$

Outline

Local approach

Ingredients

1. Gaussian residual function (computed on data, parameterized by θ)
2. Close hypothesis
3. Generalized likelihood ratio test

Benveniste, Basseville & Moustakides, *The asymptotic local approach to change detection and model validation*, IEEE Transactions on Automatic Control, AC-32(7):583-592, 1987.

Local approach

Step 1: residual definition

- Block Hankel matrix of correlations $R_i = \mathbf{E}\ \left(y_k\ y_{k-i}^T\right)$

$$\mathcal{H} = \begin{bmatrix} R_1 & R_2 & \dots & R_q \\ R_2 & R_3 & \dots & R_{q+1} \\ \vdots & \ddots & \dots & \vdots \\ R_{p+1} & \ddots & \dots & R_{p+q} \end{bmatrix} = \mathcal{O}\mathcal{C}, \quad \mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^p \end{bmatrix}$$

- Left null space S in reference state: $S^T\mathcal{O} = S^T\mathcal{H} = 0$

$$\Rightarrow \quad \mathbf{E}_\theta(S^T\widehat{\mathcal{H}}) = 0 \quad \text{iff} \quad \theta = \theta_0$$

- Define residual

$$\zeta_N = \sqrt{N}\,\mathrm{vec}(S^T\widehat{\mathcal{H}})$$

with estimate $\widehat{\mathcal{H}}$ from $\widehat{R}_i = \frac{1}{N}\sum_{k=1}^{N} y_k\ y_{k-i}^T$

Basseville, Abdelghani & Benveniste, *Subspace-based fault detection algorithms for vibration monitoring*, Automatica, 36(1):101-109, 2000.

Local approach

Step 2: close hypotheses

$$\begin{array}{lll} \mathbf{H}_0 : & \theta = \theta_0 & \text{(reference system)} \\ \mathbf{H}_1 : & \theta = \theta_0 + \delta/\sqrt{N} & \text{(faulty system)} \end{array}$$

δ: unknown but fixed

- Central Limit Theorem

$$\zeta_N \xrightarrow{d} \begin{cases} \mathcal{N}(0, \Sigma(\theta_0)) & \text{under } \mathbf{H}_0 \\ \mathcal{N}(\mathcal{J}(\theta_0)\,\delta, \Sigma(\theta_0)) & \text{under } \mathbf{H}_1 \end{cases}$$

- "Local interpretation": $\delta = \sqrt{N}(\theta - \theta_0)$
  - $(\theta - \theta_0)$ small, N large
  - $(\theta - \theta_0)$ not so small, N not so large

Local approach

Step 3: generalized likelihood ratio test

... adapted for detection / isolation

Outline

Fault detection

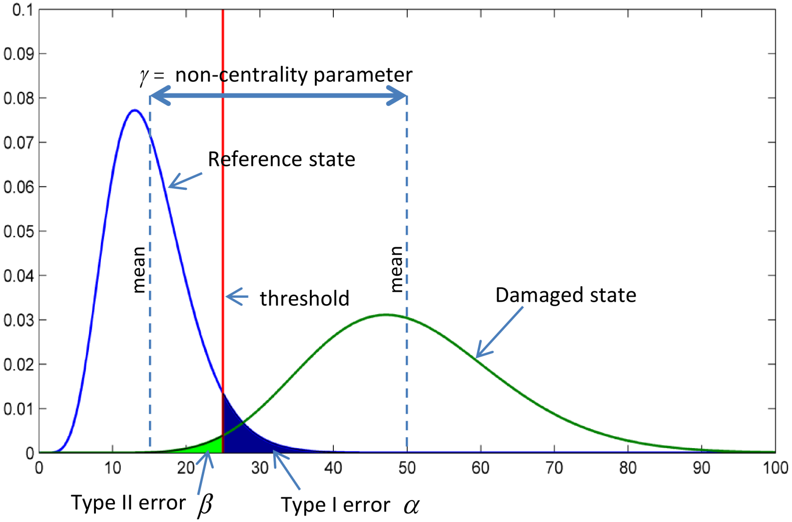
$$\zeta_N \xrightarrow{d} \begin{cases} \mathcal{N}(0, \Sigma(\theta_0)) & \text{under } \mathbf{H}_0 \\ \mathcal{N}(\mathcal{J}(\theta_0)\,\delta, \Sigma(\theta_0)) & \text{under } \mathbf{H}_1 \end{cases}$$

Generalized likelihood ratio test

$$\chi^2_N = \zeta_N^T \widehat{\Sigma}^{-1} \widehat{\mathcal{J}} (\widehat{\mathcal{J}}^T \widehat{\Sigma}^{-1} \widehat{\mathcal{J}})^{-1} \widehat{\mathcal{J}}^T \widehat{\Sigma}^{-1} \zeta_N$$

- χ^2 distributed, $\dim(\theta_0)$ degrees of freedom
- Non-centrality parameter: $\delta^T F \delta$, $F = \mathcal{J}^T \Sigma^{-1} \mathcal{J}$
- Compare χ^2_N to a threshold for decision

Fault detection



Fault detection

Numerical problems for test computation

$$\chi_N^2 = \zeta_N^T \widehat{\Sigma}^{-1} \widehat{\mathcal{J}} (\widehat{\mathcal{J}}^T \widehat{\Sigma}^{-1} \widehat{\mathcal{J}})^{-1} \widehat{\mathcal{J}}^T \widehat{\Sigma}^{-1} \zeta_N$$

- $\widehat{\Sigma}$ rank deficient or badly conditioned
- χ_N^2 too unstable

Numerically robust computation

1. $\Sigma = \Sigma^{1/2}(\Sigma^{1/2})^T$, get $\widehat{\Sigma}^{1/2}$ directly from data
2. $(\widehat{\Sigma}^{1/2})^\dagger \widehat{\mathcal{J}} = QR$
3. $\chi_N^2 = \alpha^T \alpha$ where $\alpha = Q^T (\widehat{\Sigma}^{1/2})^\dagger \zeta_N$

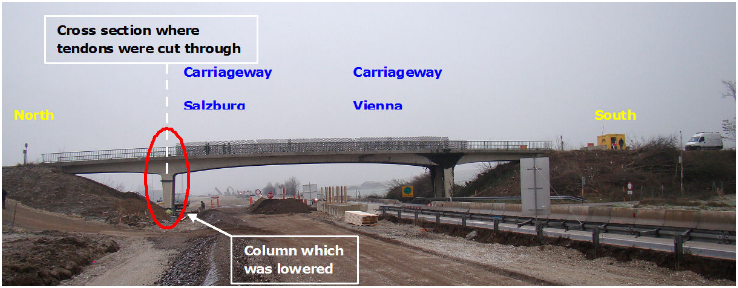
Zhang & Basseville, *Advanced numerical computation of chi2-tests for fault detection and isolation*, SAFEPROCESS, 2003.

Döhler & Mevel, *Robust subspace based fault detection*, IFAC World Congress, 2011.

Application: S101 Bridge

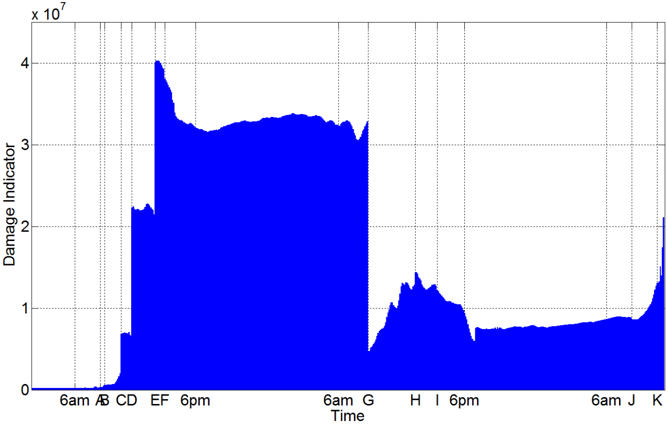
Damage detection on S101 Bridge

- In FP7 IRIS: Large scale progressive damage test as benchmark for damage identification
- 4 days of measurements with different damage actions
  - Lowering a column in 3 steps
  - Cutting the prestressing cables

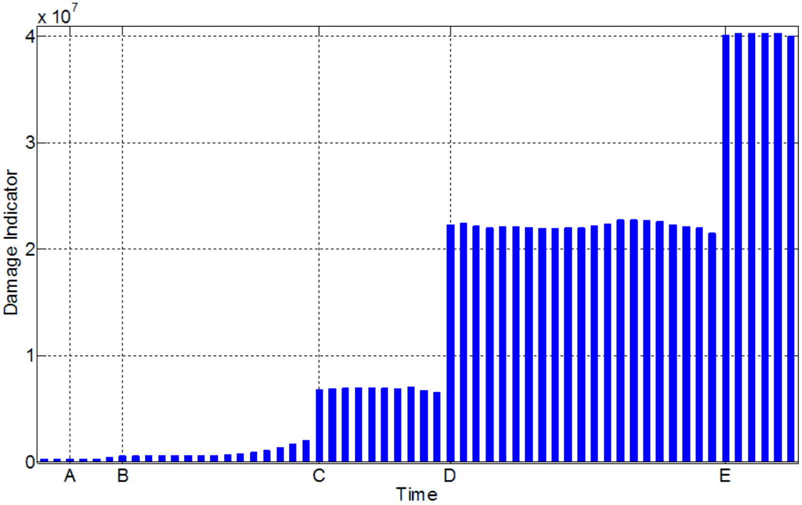


Döhler, Hille, Mevel & Rücker, *Structural health monitoring with statistical methods during progressive damage test of S101 Bridge*, Engineering Structures 69:183-193, 2014.

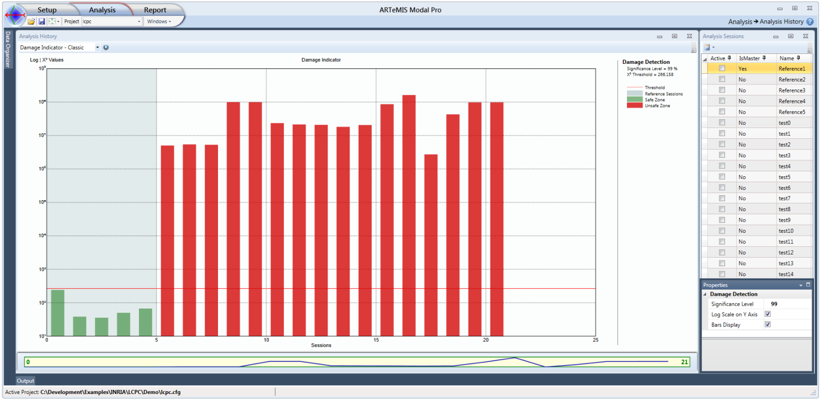
Application: S101 Bridge



Application: S101 Bridge



Commercial software



ARTeMIS Damage Detection Plugin, Structural Vibration Solutions A/S

Fault detection

What if ambient excitation properties change?

$$\begin{cases} x_{k+1} &= A\, x_k + v_k \\ y_k &= C\, x_k + w_k \end{cases} \qquad \zeta_N = \sqrt{N}\,\mathrm{vec}(S^T \widehat{\mathcal{H}})$$

$Q = \mathbf{E}(v_k v_k^T)$ changes $\Rightarrow \widehat{\mathcal{H}}$ changes $\Rightarrow \chi^2_N$ changes
without any change in the structural parameters

Solutions

- Recompute $\widehat{\Sigma}$ on tested dataset
  - Computationally expensive
  - Often more data available in reference state than in possibly damaged state
- NEW: define residual vector robust to excitation changes

Döhler & Mevel, *Subspace-based fault detection robust to changes in the noise covariances*, Automatica 49(9): 2734-2743, 2013.

Döhler, Mevel & Hille, *Subspace-based damage detection under changes in the ambient excitation statistics*, Mechanical Systems and Signal Processing 45(1):207-224, 2014.

Fault detection

Definition of residual robust to excitation change

Instead of using $\widehat{\mathcal{H}}$, take orthogonal basis of its image

- SVD

$$\widehat{\mathcal{H}} = \begin{bmatrix} \widehat{U}_1 & \widehat{U}_0 \end{bmatrix} \begin{bmatrix} \widehat{\Delta}_1 & 0 \\ 0 & \widehat{\Delta}_0 \end{bmatrix} \begin{bmatrix} \widehat{V}_1^T \\ \widehat{V}_0^T \end{bmatrix}, \quad \widehat{\Delta}_0 \approx 0$$

- Properties in reference state do not change

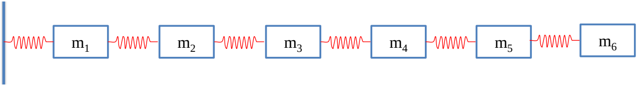
$$S^T \widehat{\mathcal{H}}^{(0)} \approx S^T \widehat{U}_1^{(0)} \approx 0$$

- Define residual vector

$$\xi_N = \sqrt{N}\,\mathrm{vec}(S^T \widehat{U}_1)$$

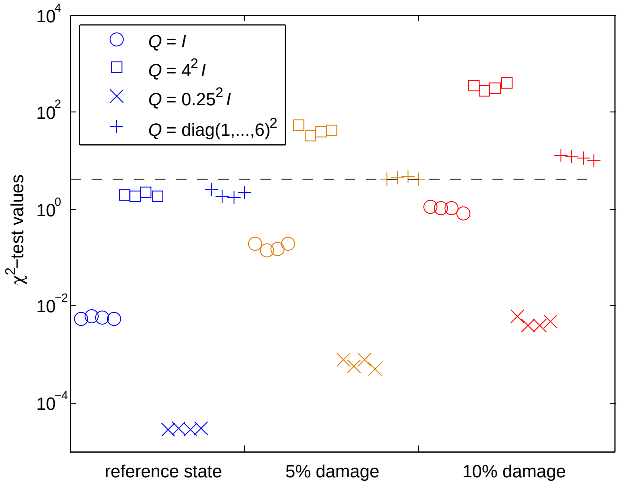
Application: simulated mass-spring chain

- 6 DOF simulated mass-spring chain



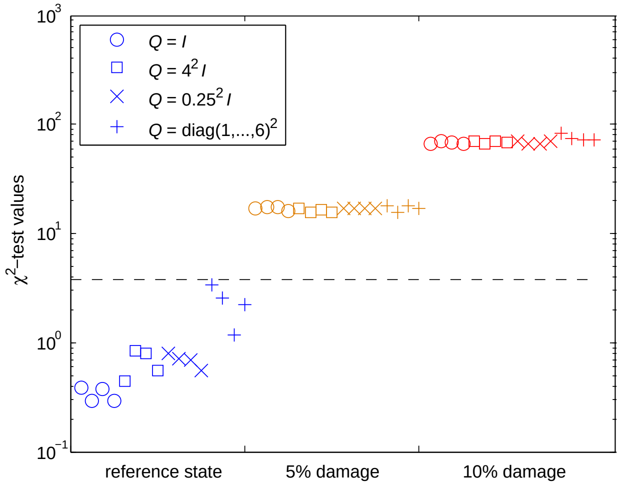
- 3 different structural states
  - reference state
  - 5% stiffness reduction in spring 2
  - 10% stiffness reduction in spring 2
- Ambient excitation with different covariances $Q = \mathbf{E}(v_k v_k^T)$
  - $Q = I_6$
  - $Q = 4^2 I_6$
  - $Q = 0.25^2 I_6$
  - $Q = \mathrm{diag}(1, 2, 3, 4, 5, 6)^2$

Application: simulated mass-spring chain



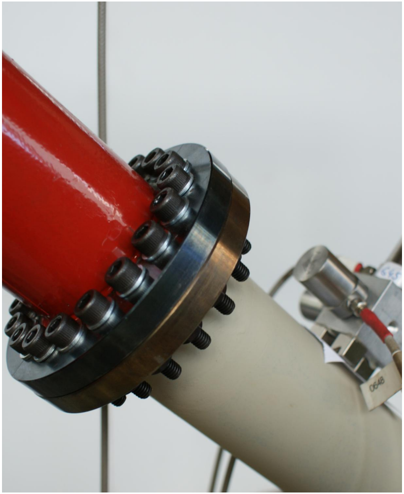
Residual $\zeta_N = \sqrt{N}\,\mathrm{vec}(S^T\widehat{\mathcal{H}})$, $\widehat{\Sigma}_\zeta$ from reference state

Application: simulated mass-spring chain



Residual $\xi_N = \sqrt{N}\,\mathrm{vec}(S^T \widehat{U}_1)$, $\widehat{\Sigma}_\xi$ from reference state

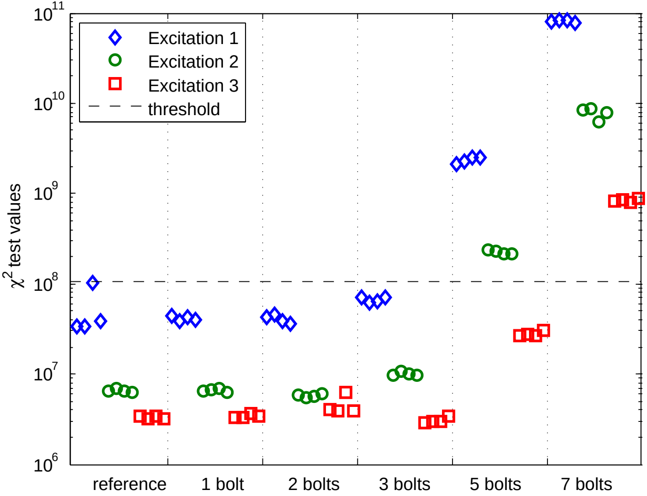
Application: steel-frame structure



Application: steel-frame structure

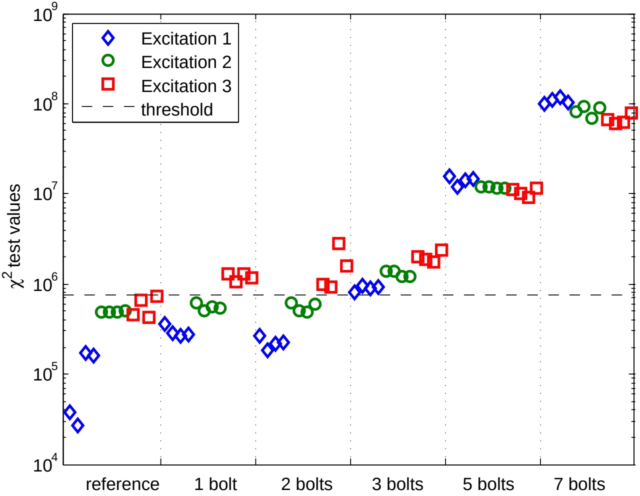
- Jacket-like steel frame structure
- Damage: 1, 2, 3, 5 or 7 adjacent bolts were unscrewed
  - 3 loose bolts $\approx$ reduction of bending stiffness by 3%
  - 7 loose bolts $\approx$ reduction of bending stiffness by 30%
- Excitation: white noise at three different levels
  - Full scale level
  - 5 dB reduction ($\approx$0.56% of amplitude)
  - 10 dB reduction ($\approx$0.31% of amplitude)
- Signals were measured for 16.4 s at 2500 Hz

Application: steel-frame structure



Residual $\zeta = \sqrt{N}\,\mathrm{vec}(S^T \widehat{\mathcal{H}})$, $\widehat{\Sigma}_\zeta$ from reference state

Application: steel-frame structure



Residual $\xi = \sqrt{N}\,\mathrm{vec}(S^T \widehat{U}_1)$, $\widehat{\Sigma}_\xi$ from reference state

Outline

Problem setting

Given: structural parameter vector θ, Gaussian residual vector ζ

Hypotheses

$$\begin{array}{lll} \mathbf{H}_0: & \theta = \theta_0 & \text{(reference system)} \\ \mathbf{H}_1: & \theta = \theta_0 + \delta/\sqrt{N} & \text{(faulty system)} \end{array}$$

Residual distribution

$$\zeta(\theta) \sim \begin{cases} \mathcal{N}(0, \Sigma) & \text{under } \mathbf{H}_0 \\ \mathcal{N}(\mathcal{J}\,\delta, \Sigma) & \text{under } \mathbf{H}_1 \end{cases}$$

- δ: parameter change
- $\mathcal{J}$, Σ: sensitivity and covariance of residual vector

Which parts of θ changed, i.e. which parts of δ are $\neq 0$?

Problem setting

Isolation tests

- Consider different partitions of the vector δ into two subvectors
- For each partition: decide if the first subvector is zero or not

$$\delta = \begin{bmatrix} \delta_a \\ \delta_b \end{bmatrix}$$

Corresponding partitions

$$\mathcal{J} = \begin{bmatrix} \mathcal{J}_a & \mathcal{J}_b \end{bmatrix}$$

$$F \stackrel{\text{def}}{=} \mathcal{J}^T \Sigma^{-1} \mathcal{J} = \begin{bmatrix} \mathcal{J}_a^T \Sigma^{-1} \mathcal{J}_a & \mathcal{J}_a^T \Sigma^{-1} \mathcal{J}_b \\ \mathcal{J}_b^T \Sigma^{-1} \mathcal{J}_a & \mathcal{J}_b^T \Sigma^{-1} \mathcal{J}_b \end{bmatrix} = \begin{bmatrix} F_{aa} & F_{ab} \\ F_{ba} & F_{bb} \end{bmatrix}$$

F ... Fisher information of parameter θ in residual $\zeta(\theta)$

Sensitivity test

The test

- Assume $\delta_b = 0$, thus $\zeta \sim \mathcal{N}(\mathcal{J}_a \delta_a, \Sigma)$
- GLR test

$$t_{\text{sens}} = \zeta^T \Sigma^{-1} \mathcal{J}_a \left(\mathcal{J}_a^T \Sigma^{-1} \mathcal{J}_a \right)^{-1} \mathcal{J}_a^T \Sigma^{-1} \zeta$$

Properties

- χ^2 distributed with $\dim(\theta_a)$ degrees of freedom
- Non-centrality parameter $\delta_a^T F_{aa} \delta_a$, if $\delta_b = 0$ is true
- Compare t_{sens} to a threshold for decision

Minmax test

The test

- Replace δ_b by least favorable value for decision about δ_a
- Define partial residuals $\zeta_a \stackrel{\text{def}}{=} \mathcal{J}_a^T \Sigma^{-1} \zeta$, $\zeta_b \stackrel{\text{def}}{=} \mathcal{J}_b^T \Sigma^{-1} \zeta$
- Robust residual

$$\zeta_a^* \stackrel{\text{def}}{=} \zeta_a - F_{ab} F_{bb}^{-1} \zeta_b \quad \sim \quad \mathcal{N}\left(F_a^*\, \delta_a,\ F_a^*\right)$$

- GLR test

$$t_{\mathrm{mm}} = \zeta_a^{*T} F_a^{*-1} \zeta_a^*$$

Properties

- χ^2 distributed with $\dim(\theta_a)$ degrees of freedom
- Non-centrality parameter $\delta_a^T F_a^*\, \delta_a$, independently of δ_b
- Compare t_{mm} to a threshold for decision

Döhler, Mevel & Hille, *Efficient computation of minmax tests for fault isolation and their application to structural damage localization*, IFAC World Congress, 2014.

Damage localization

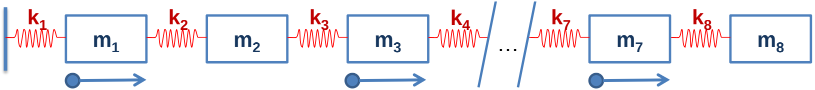
Damage localization

- Each structural parameter θ_i in $\theta = \begin{bmatrix} \theta_1 & \theta_2 & \dots & \theta_{n_\theta} \end{bmatrix}^T$ corresponds to an element of a structure
- For each parameter θ_i, perform FDI test
- Damage is located at elements i for which test reacts

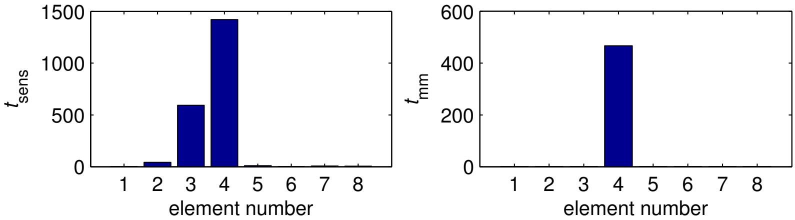
Application: simulated mass-spring chain

Test case

- 8 elements, 4 sensors
- Damage: reduction of spring stiffness in ...
  - element 4 (by 10%)
  - elements 2 (by 5%) and 4 (by 10%)
  - elements 3 (by 5%) and 4 (by 10%)
- Generation of $N = 100\,000$ acceleration samples from random excitation, 5% measurement noise

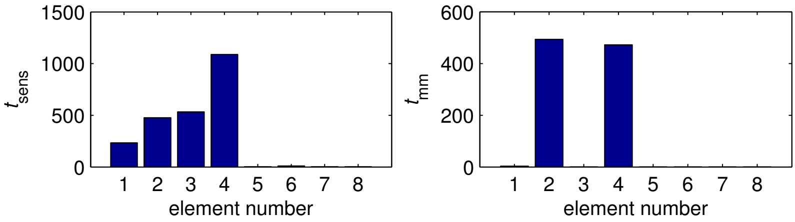


Application: simulated mass-spring chain



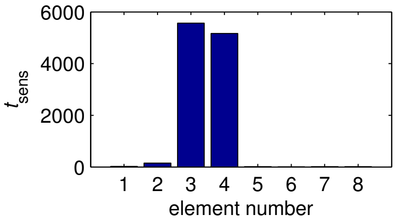
Sensitivity tests (left) and minmax tests (right) for 10% damage in element 4.

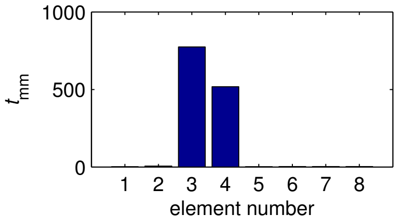
Application: simulated mass-spring chain



Sensitivity (left) and minmax tests (right) for 5% damage in element 2 and 10% damage in element 4.

Application: simulated mass-spring chain





Sensitivity (left) and minmax tests (right) for 5% damage in element 3 and 10% damage in element 4.

Outline

Damage quantification

Estimate δ_a for parameters θ_a whose “localization tests” reacted

Based on sensitivity test...

- Given (assuming $\delta_b = 0$):

$$\zeta \sim \mathcal{N}\left(\mathcal{J}_a \delta_a, \Sigma\right)$$

- $\widehat{\delta}_a^{\text{sens}} \stackrel{\text{def}}{=} (\mathcal{J}_a^T \Sigma^{-1} \mathcal{J}_a)^{-1} \mathcal{J}_a^T \Sigma^{-1} \zeta$, then:

$$\widehat{\delta}_a^{\text{sens}} \sim \mathcal{N}\left(\delta_a, F_{aa}^{-1}\right)$$

Damage quantification

Based on minmax test...

- Given:

$$\zeta_a^* \sim \mathcal{N}\left(F_a^*\,\delta_a,\ F_a^*\right)$$

- $\widehat{\delta}_a^{\mathrm{mm}} \stackrel{\mathrm{def}}{=} (F_a^*)^{-1}\zeta_a^*$, then:

$$\widehat{\delta}_a^{\mathrm{mm}} \ \sim\ \mathcal{N}(\delta_a, (F_a^*)^{-1})$$

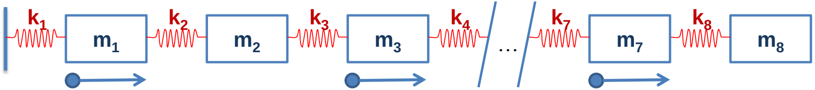
Döhler & Mevel, *Fault isolation and quantification from Gaussian residuals with application to structural damage quantification*, submitted to SAFEPROCESS, 2015.

Application: simulated mass-spring chain, cont'd

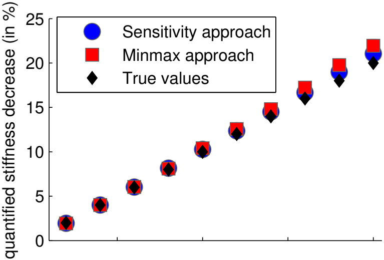
Test case

- 8 elements, 4 sensors
- Damage: reduction of spring stiffness in ...
  - element 4
  - elements 2 and 4
  - elements 3 and 4

  ... for different damage extents
- Generation of $N = 100\,000$ acceleration samples from random excitation, 5% measurement noise

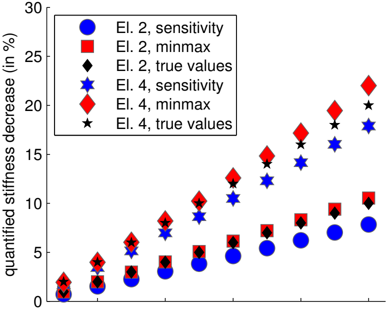


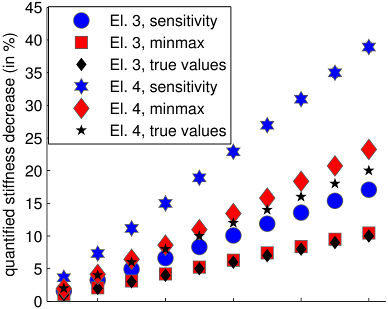
Application: simulated mass-spring chain



Quantification of different damage extents in element 4.

Application: simulated mass-spring chain





Quantification of different damage extents in elements 2 and 4 (left), 3 and 4 (right).

Outline

Conclusions

- Common statistical framework for *damage detection*, *localization* and *quantification*
  - Hypotheses testing on asymptotically Gaussian residuals
  - Residual based on subspace properties
- Required parameterizations:
  - *Detection:* modal parameters (from data)
  - *Localization, quantification:* structural parameters (from FE model)
- Suitable framework for structural health monitoring
  - Output-only measurements, no inputs
  - Large systems
  - Numerical robustness of tests and estimators
- Different maturity of approaches
  - *Detection:* successful on real structures in the field
  - *Localization, quantification:* still on simulation level