Projet Européen iGNC - Mission Mars Sample Return

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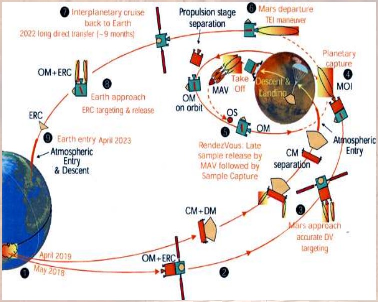
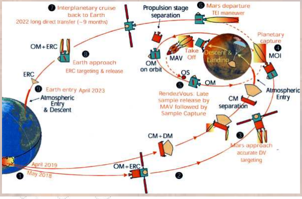
La mission MSR

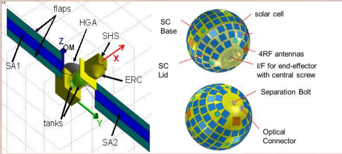
Mission MSR

1. un véhicule de transfert Terre-Mars
2. un orbiteur (chasseur)
3. un module de descente
4. un module de mise en orbite + canister (cible)
5. un véhicule de retour + rentrée atmosphérique



1. Orbiteur: 2 panneaux solaires (4 modes flexibles), 2 réservoirs (2 modes)
2. Canister: sphère de 23cm de diamètre





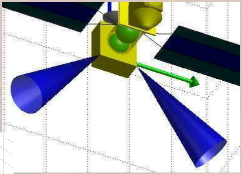
La mission MSR

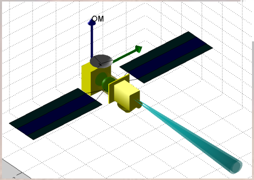
Instrumentation et Actionneurs

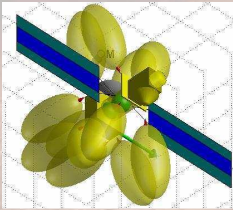
Absolute sensors	Nb.	Type/Supplier
Star-Tracker	2	Selex Galileo
IMU	2	MIMU Honeywell
Sun sensor	2	TNO Sun Acquisition Sensor (SAS)

Relative sensors	Nb.	Type/Supplier
RF Doppler	2	ELECTRA payload
NAC	2	IRIS-3 camera 5x5 deg 1024x1024 pxls

Actuator	Nb.	Type/Supplier
Reaction Wheels	4	RSI 25-220/45 Rockwell Collins
Thrusters	24	10N Astrium S10-26
Main Engine	1	424N Astrium S400-15







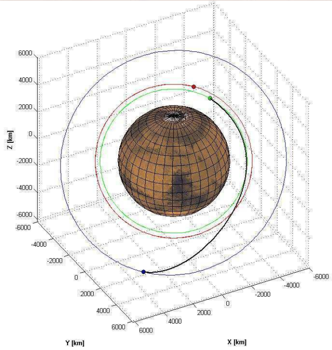
La phase de rendez vous

	Nominal OS	Nominal Orbiter (scenario 1)	Nominal Orbiter (scenario 2)	Contingency OS (MAV circularization failure)
Rp, Ra [km]	535x535 (minimum orbit altitude providing acceptable results in terms of OS orbit determination)	300x610 (12:1 resonant elliptic orbit with respect to MAV launch site)	455x455 (12:1 resonant circular orbit with respect to MAV launch site)	219x535
Inclination [deg]	40	40	40	40
RAAN [deg]	Ensuring initial illumination of the target	Ensuring initial illumination of the target	Ensuring initial illumination of the target	Ensuring initial illumination of the target
Arg. of pericenter [deg]	Ensuring initial illumination of the target	Ensuring initial illumination of the target	Ensuring initial illumination of the target	Ensuring initial illumination of the target
True Anomaly [deg]	-10	-10	-10	-10

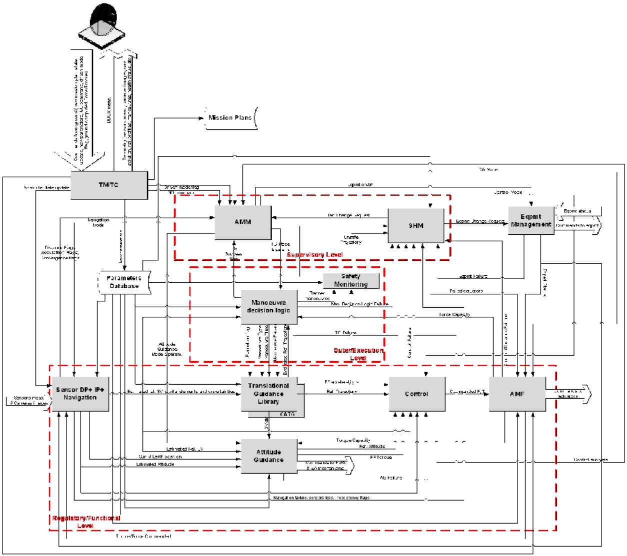
MAV injection accuracy
- Semi-major axis: 66 km
- Eccentricity: 0.013
- Inclination: 0.2 deg;
- RAAN: 0.1 deg;
- Argument of pericentre: 0.5 deg;
- True anomaly: 0.1 deg

Separation velocity: 1 m/s

OS initial knowledge accuracy: 20 km (3D)



Le software = GNC



- Compliant with Aurora Avionics Architecture
- Three-layers
  - AMM and SHM are part of a supervisory level
  - Manoeuvre decision logic and safety monitoring performs execution level
  - Control, navigation, attitude guidance, translational guidance library and control allocation belongs to the regulatory level
- Very high level of on-board autonomy is targeted

La fonction Autonomous Mission Management (AMM)

Agrégation	
Level 1 sensor checks	Monitoring of the outputs of all sensors. This level covers most of the sensor faults such as sudden sensor death and lock-in-place fault types
Level 2 IMU/IMU - IMU/STR	Interest is limited to the detection of failures not seen by level 1, e.g. slow drifts...
Level 3 thruster/IMU	Interest is faults in thrusters. The IMU hot redundancy enables to discard IMU failures, leading model-based techniques based on the IMUs to be viable candidates.
Level 3 wheel/tachometer	Covers wheels faults. The isolation is immediate since a tachometer is available on each wheel.
Level 4 approach corridors	Monitor the attitude/position/velocity of the chaser versus the approach corridors.
Level 4 collision risks	Detect if a collision may occur between the spacecraft
Level 4 mode success	Detect the divergence of the controllers outputs
Level 5 power alarm	Protection against ground operation errors and electrical subsystem failures

Solutions par approches à base de modèles:

1. Level 2 IMU/IMU (espace de parité statique + analyse de covariance) - IMU/STR (OBS NL optimal localement au sens H_∞)

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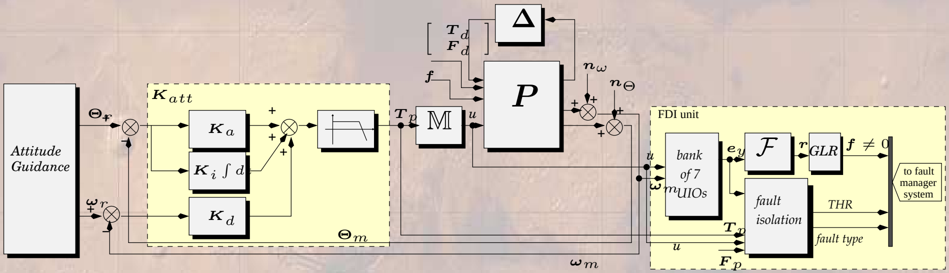
La fonction FDI des tuyères



1. une fonction detection robuste $\boldsymbol{r}(s) = \mathcal{F}(s)\boldsymbol{e}_y(s) \Rightarrow$ filtre $H_\infty/H_- +$ postanalyse μ_g (*"pire cas"*)
2. une fonction isolation $\Rightarrow$ circonscrit la faute aux tuyères engendrant le même couple / force (2 tuyères candidates) = stratégie à base de 7 UIOs t.q. $\left\{ \begin{array}{l} \dot{\boldsymbol{z}} = \boldsymbol{N}\boldsymbol{z} + \boldsymbol{G}\boldsymbol{u} + \boldsymbol{L}\boldsymbol{y} \\ \widehat{\boldsymbol{x}} = \boldsymbol{z} + \boldsymbol{H}\boldsymbol{y} \end{array} \right. , \quad \boldsymbol{e}_y = \boldsymbol{C}(\boldsymbol{x} - \widehat{\boldsymbol{x}})$
3. une fonction isolation finale qui isole la tuyère défaillante $\Rightarrow$ analyse de direction colinéaire (produit vectoriel)

Modélisation (dynamiques de rotation)

2nd loi d'Euler

$$\dot{\boldsymbol{\omega}} = \boldsymbol{J}^{-1}\sum_k \boldsymbol{T}_k - \boldsymbol{J}^{-1}\boldsymbol{\omega} \times \boldsymbol{J}\boldsymbol{\omega}, \quad \boldsymbol{\omega} = [p, q, r]^T$$

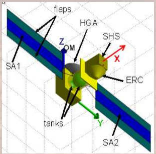
- couple de propulsion $\boldsymbol{T}_p$
- couples de perturbation $\boldsymbol{T}_d$ (pression solaire, gradient de gravité, vent de très haute atmosphère)
- couples "souples" liés aux panneaux solaires $\boldsymbol{T}_{sa}$
- couples de ballottement du carburant $\boldsymbol{T}_s$ (2 réservoirs à 50%)

Modes flexibles panneaux

$$\boldsymbol{T}_{sa} = -\underline{\boldsymbol{L}}\ddot{\boldsymbol{q}} - \sum_{i=1}^{n_p} \boldsymbol{J}_{SA_i}\dot{\boldsymbol{\omega}}, \quad J_{SA_i} = J_{0_i} + J_{transport}$$

$$\ddot{\boldsymbol{q}} + 2\boldsymbol{\xi}\boldsymbol{\omega_0}\dot{\boldsymbol{q}} + \boldsymbol{\omega_0^2}\boldsymbol{q} = \boldsymbol{L}^T\dot{\boldsymbol{\omega}}, \quad \boldsymbol{q} \in \mathbb{R}^{n_s.n_p}, \quad n_p = 2, n_s = 4$$

$$\underline{\boldsymbol{L}}_i = \boldsymbol{\mathcal{R}}_i(\alpha)\boldsymbol{B}_{R_i} + \boldsymbol{S}(\boldsymbol{d}_i)\boldsymbol{\mathcal{R}}_i(\alpha)\boldsymbol{B}_{T_i}$$



Modélisation (dynamiques de rotation)

Modes de ballottement

$\Rightarrow$ couples induits par une accélération
$\Rightarrow$ modèle masse-ressort-amortisseur (3D)

$$\ddot{\boldsymbol{x}}_{\boldsymbol{s}} + \frac{\boldsymbol{l}_s}{m_s}\dot{\boldsymbol{x}}_s + \frac{\boldsymbol{k}_s}{m_s}\boldsymbol{x}_s = \boldsymbol{\gamma} - \sum_k \gamma_k, \quad \boldsymbol{x}_s \in \mathbb{R}^3, \quad \boldsymbol{T}_{si} = \boldsymbol{r} \times (\boldsymbol{k}\boldsymbol{x}_s + \boldsymbol{l}\dot{\boldsymbol{x}}_s)$$

- m_s = masse du carburant, $\boldsymbol{r} \in \mathbb{R}^3$ distance CoM-centre de masse des réservoirs
- acceleration de coriolis: $\boldsymbol{\gamma}_1 = 2\boldsymbol{\omega} \times \dot{\boldsymbol{x}}_s$
- acceleration centrifuge: $\boldsymbol{\gamma}_2 = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times (\boldsymbol{r} + \boldsymbol{x}_s))$
- acceleration d'Euler: $\boldsymbol{\gamma}_3 = \dot{\boldsymbol{\omega}} \times (\boldsymbol{r} + \boldsymbol{x}_s)$
- $\boldsymbol{\gamma}$ = accélération au CoM (2nd Newton's law): $m\boldsymbol{\gamma} = \sum_k \boldsymbol{F}_k$
- m = masse du chasseur
- $\boldsymbol{F}_p$ = forces liées au système de propulsion
- $\boldsymbol{F}_d$ = forces de perturbations
- $\boldsymbol{F}_{sa}$ = panneaux solaires $\quad \boldsymbol{F}_{sa} = -\overline{\boldsymbol{L}}\ddot{\boldsymbol{q}} - \sum_{i=1}^{n_p} m_{sa_i}\boldsymbol{\gamma}$, $\quad \overline{\boldsymbol{L}} = [\ldots\overline{\boldsymbol{L}}_i\ldots], \ \overline{\boldsymbol{L}}_i = \boldsymbol{\mathcal{R}}_i(\alpha)\boldsymbol{B}_{\boldsymbol{T}_i}$

Modélisation (dynamiques de rotation)

Modèle du système de propulsion

$$\boldsymbol{T}_p = [M_{T_1} ... M_{T_{12}}]\boldsymbol{u} = \boldsymbol{M}_T\boldsymbol{u}$$

$$\boldsymbol{F}_p = [M_{F_1} ... M_{F_{12}}]\boldsymbol{u} = \boldsymbol{M}_F\boldsymbol{u} \quad \boldsymbol{M} = \left[\boldsymbol{M}_T^T\ \boldsymbol{M}_F^T\right]^T$$

$$\left[\begin{array}{cc} \boldsymbol{T}_p^T & \boldsymbol{F}_p^T \end{array}\right]^T = \boldsymbol{M}\boldsymbol{u}(t-\tau)$$

Modèle complet

1. linéarisation autour de la trajectoire de rendez vous ($\omega_0 = 0, \Theta_0 = 0$)
2. approximation de Padé pour les retards

$$\Rightarrow \left\{\begin{array}{l} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u} + \boldsymbol{E}\left[\begin{array}{c} \boldsymbol{T}_d \\ \boldsymbol{F}_d \end{array}\right] \\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{n}_\omega = \boldsymbol{\omega} + \boldsymbol{n}_\omega \\ \boldsymbol{u}(s) = \boldsymbol{K}(s)\left[\begin{array}{c} \boldsymbol{\omega}_m(s) \\ \boldsymbol{\Theta}_m(s) \end{array}\right] \end{array}\right.$$

$\mathbf{dim(x) = 32}$
$\mathbf{dim(x_K) = 13}$
$\mathbf{dim(u) = 12}$
$\mathbf{dim(y) = 3}$

Modélisation (dynamiques de rotation)

List of considered uncertainties:

dBR sa x mode 2: 2 occurrences
dBR sa y mode 1: 2 occurrences
dBR sa y mode 3: 2 occurrences
dBR sa z mode 4: 2 occurrences
dBT sa x mode 1: 4 occurrences
dBT sa x mode 3: 4 occurrences
dBT sa x mode 4: 4 occurrences
dBT sa y mode 2: 4 occurrences
dCOM x: 4 occurrences
dCOM y: 4 occurrences
dCOM z: 4 occurrences
dIxx: 2 occurrences
dIyy: 2 occurrences
dIzz: 2 occurrences
d sa angle: 8 occurrences
dangle gyr x: 2 occurrences
dangle gyr y: 2 occurrences
dangle gyr z: 2 occurrences
dangle tthrust x: 2 occurrences
dangle tthrust y: 2 occurrences
dangle tthrust z: 2 occurrences
ddamp sa mode 1: 2 occurrences
ddamp sa mode 2: 2 occurrences
ddamp sa mode 3: 2 occurrences
ddamp sa mode 4: 2 occurrences
dfreq sa mode 1: 4 occurrences
dfreq sa mode 2: 4 occurrences
dfreq sa mode 3: 4 occurrences
dfreq sa mode 4: 4 occurrences
dslosh damping sm1: 3 occurrences
dslosh damping sm2: 3 occurrences
dslosh mass sm1: 3 occurrences
dslosh mass sm2: 3 occurrences
dslosh stiffness sm1: 3 occurrences
dslosh stiffness sm2: 3 occurrences
dtao thr: 3 occurrences

1. 36 incertitudes (dépendance nonlinéaire)
2. contrainte dure ESA:
   preuve formelle de robustesse ⇒ stratégie de reconfiguration

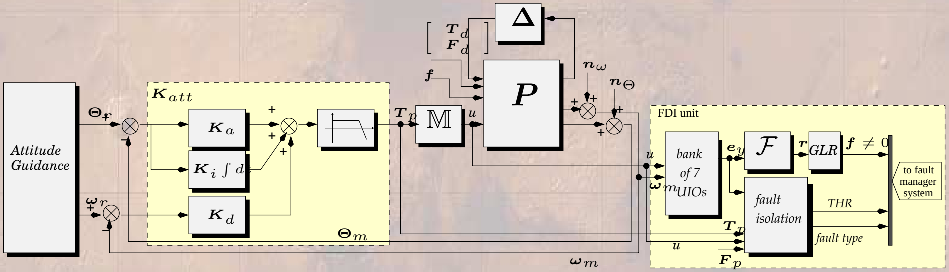
$$\Rightarrow \boldsymbol{y}(s) = F_u(\boldsymbol{P}(s), \boldsymbol{\Delta}) \begin{bmatrix} \boldsymbol{T}_d(s) \\ \boldsymbol{F}_d(s) \\ \boldsymbol{u}(s) \end{bmatrix} + \boldsymbol{n}_\omega(s)$$

$$\boldsymbol{u}(s) = \boldsymbol{K}(s) \begin{bmatrix} \boldsymbol{\omega}_m(s) \\ \boldsymbol{\Theta}_m(s) \end{bmatrix}$$

$$\begin{aligned}\boldsymbol{\Delta} = \quad & blocdiag(\delta_1 \boldsymbol{I}_2, \delta_2 \boldsymbol{I}_2, \delta_3 \boldsymbol{I}_2, \delta_4 \boldsymbol{I}_2, \delta_5 \boldsymbol{I}_4, \delta_6 \boldsymbol{I}_4, \\ & \delta_7 \boldsymbol{I}_4, \delta_8 \boldsymbol{I}_4, \delta_9 \boldsymbol{I}_4, \delta_{10} \boldsymbol{I}_4, \delta_{11} \boldsymbol{I}_4, \delta_{12} \boldsymbol{I}_2, \delta_{13} \boldsymbol{I}_2, \\ & \delta_{14} \boldsymbol{I}_2, \delta_{15} \boldsymbol{I}_8, \delta_{16} \boldsymbol{I}_2, \delta_{17} \boldsymbol{I}_2, \delta_{18} \boldsymbol{I}_2, \delta_{19} \boldsymbol{I}_2, \delta_{20} \boldsymbol{I}_2, \delta_{21} \boldsymbol{I} \\ & \delta_{25} \boldsymbol{I}_2, \delta_{26} \boldsymbol{I}_4, \delta_{27} \boldsymbol{I}_4, \delta_{28} \boldsymbol{I}_4, \delta_{29} \boldsymbol{I}_4, \delta_{30} \boldsymbol{I}_3, \\ & \delta_{31} \boldsymbol{I}_3, \delta_{32} \boldsymbol{I}_3, \delta_{33} \boldsymbol{I}_3, \delta_{34} \boldsymbol{I}_3, \delta_{35} \boldsymbol{I}_3, \delta_{36} \boldsymbol{I}_3), \\ & ||\Delta||_\infty \leq 1\end{aligned}$$

$$\boldsymbol{\Delta} \in \mathbb{R}^{107 \times 107}$$

Solution proposée



1. une stratégie uniquement basée sur mesure IMS (ω) car diagnostiqués par une approche signal au niveau antérieur (Level 2)
2. 7 UIOs $\Rightarrow$ circonscrit la faute aux tuyères engendrant le même couple / force (2 tuyères candidates)
3. une fonction détection robuste avec preuve formelle de robustesse $\forall \Delta, T_d, F_d, n_\omega$.

Les UIOs

1. Une théorie LMI $\Rightarrow$ plus robuste numériquement $(dim(x) = 32)$
2. Tous les degrés de liberté vont être utilisés pour la fonction d'isolation $(r \perp$ sous ensemble de fautes)
3. $\Delta = 0 \Rightarrow$ aucune garantie de robustesse $\Delta, T_d, F_d, n_\omega$.

Théorie générale (dans notre cas $\boldsymbol{\Phi}(\boldsymbol{x}) = 0$)

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{\Phi}(\boldsymbol{x}(t)) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t) + \boldsymbol{E}\boldsymbol{d}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \end{cases}$$

$\boldsymbol{E}$ va être choisi égal à $\mathbb{M}_i, i = \{1,2,3,4\}, i = \{5,12\}, i = \{6,11\}, i = \{7,10\}, i = \{8,9\}, i = \{1,4\}, i = \{2,3\}$ = les colonnes de la matrice de configuration des tuyères.

Hypothèses:

1. $\boldsymbol{\Phi}(\boldsymbol{x})$ est localement Lipschitz, i.e.$\|\boldsymbol{\Phi}(\boldsymbol{x}_1) - \boldsymbol{\Phi}(\boldsymbol{x}_2)\| \leq \gamma\|\boldsymbol{x}_1 - \boldsymbol{x}_2\|, \forall(\boldsymbol{x}_1, \boldsymbol{x}_2) \in \mathcal{S}$.
2. $\boldsymbol{E}$ est de plein rang colonne $\text{rank}(\boldsymbol{C}\boldsymbol{E}) = \text{rank}(\boldsymbol{E})$.

Les UIOs

Définition N.UIOs

$$\begin{aligned}\dot{\boldsymbol{z}}(t) &= \boldsymbol{N}\boldsymbol{z}(t) + \boldsymbol{G}\boldsymbol{u}(t) + \boldsymbol{L}\boldsymbol{y}(t) + \boldsymbol{M}\boldsymbol{\Phi}\left(\widehat{\boldsymbol{x}}(t)\right)\\ \widehat{\boldsymbol{x}}(t) &= \boldsymbol{z}(t) + \boldsymbol{H}\boldsymbol{y}(t)\\ \boldsymbol{r}(t) &= C(\boldsymbol{x}(t) - \widehat{\boldsymbol{x}}(t))\end{aligned} \quad \Rightarrow \boldsymbol{r(t)} \perp \boldsymbol{d(t)}$$

La solution générale s'écrit

$$\begin{aligned}\boldsymbol{N} &= \boldsymbol{M}\boldsymbol{A} - \boldsymbol{K}\boldsymbol{C},\\ \boldsymbol{L} &= \boldsymbol{K}(\boldsymbol{I} - \boldsymbol{C}\boldsymbol{H}) + \boldsymbol{M}\boldsymbol{A}\boldsymbol{H},\\ \boldsymbol{M} &= \boldsymbol{I} - \boldsymbol{H}\boldsymbol{C},\\ \boldsymbol{G} &= \boldsymbol{M}\boldsymbol{B}\\ (\boldsymbol{I} - \boldsymbol{H}\boldsymbol{C})\boldsymbol{E} &= \boldsymbol{0}\end{aligned}$$

où $\boldsymbol{H} = \boldsymbol{U} + \boldsymbol{Y}\boldsymbol{V}$, $\boldsymbol{Y}$ doit être choisi t.q. $\boldsymbol{H}$ est de plein rang et

$$\boldsymbol{U} = \boldsymbol{E}(\boldsymbol{C}\boldsymbol{E})^{\dagger}, \quad \boldsymbol{V} = \boldsymbol{I} - (\boldsymbol{C}\boldsymbol{E})(\boldsymbol{C}\boldsymbol{E})^{\dagger}$$

Les UIOs

Théorème (Fonod et Henry, 2014)

L'UIO non linéaire est asymptotiquement stable et admet la constante max de Lipschitz γ^* avec une atténuation $\mathcal{L}_2$ de $\Delta\boldsymbol{B}.\boldsymbol{u}$ sur $\boldsymbol{e}$ bornée par $\kappa > 0$, s.si $\exists \boldsymbol{P} = \boldsymbol{P}^T > 0$ et $\bar{\boldsymbol{K}}$, $\bar{\boldsymbol{Y}}$ solution de

$$\max_{P,\bar{K},\bar{Y}} \xi \quad s.c. \begin{bmatrix} \boldsymbol{\Psi}_{11} + \boldsymbol{\Gamma}_{11} & \boldsymbol{\Omega}_{12} & \boldsymbol{\Omega}_{13} & \mathbf{0} & \mathbf{0} \\ * & -\boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -\boldsymbol{I} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\kappa^2\boldsymbol{I} & \boldsymbol{S}_2\boldsymbol{B_T} \\ * & * & * & * & -\boldsymbol{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \xi & \gamma \\ * & 1 \end{bmatrix} \geq 0 \tag{1}$$

$$\boldsymbol{\Psi}_{11} = ((\boldsymbol{I} - \boldsymbol{UC})\boldsymbol{A})^T\boldsymbol{P} + \boldsymbol{P}(\boldsymbol{I} - \boldsymbol{UC})\boldsymbol{A} + (1+\xi)\boldsymbol{I} \tag{2}$$

$$\boldsymbol{\Gamma}_{11} = -(\boldsymbol{VCA})^T\bar{\boldsymbol{Y}}^T - \bar{\boldsymbol{Y}}\boldsymbol{VCA} - \boldsymbol{C}^T\bar{\boldsymbol{K}}^T - \bar{\boldsymbol{K}}\boldsymbol{C} \tag{3}$$

$$\boldsymbol{\Omega}_{12} = \boldsymbol{P}(\boldsymbol{I} - \boldsymbol{UC}) - \bar{\boldsymbol{Y}}\boldsymbol{VC} \tag{4}$$

$$\boldsymbol{\Omega}_{13} = \boldsymbol{P}(\boldsymbol{I} - \boldsymbol{UC})\boldsymbol{R}_2 - \bar{\boldsymbol{Y}}\boldsymbol{VCR}_2 \tag{5}$$

$$\tag{6}$$

Alors $\boldsymbol{K} = \boldsymbol{P}^{-1}\bar{\boldsymbol{K}}, \quad \boldsymbol{Y} = \boldsymbol{P}^{-1}\bar{\boldsymbol{Y}}, \quad \gamma^* = \sqrt{\xi}$

Les UIOs

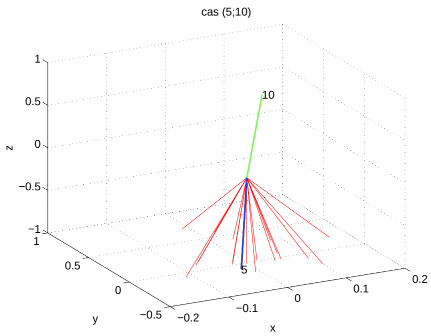
1. L'UIO non linéaire est robuste vis-à-vis de toute incertitude additive $\boldsymbol{\Delta\Phi}(\boldsymbol{x})$ t.q. $\boldsymbol{\Phi}_{\triangle}(\boldsymbol{x}) = \boldsymbol{\Phi}(\boldsymbol{x}) + \boldsymbol{\Delta\Phi}(\boldsymbol{x})$ admettant une constante de Lipschitz inférieure où égale à $\gamma^* - \gamma$.
2. La maximisation de la constante de Lipschitz γ^* peut entraîner une dynamique très élevée de l'UIO non linéaire. La solution consiste alors à utiliser les régions LMIs ($\mathcal{D}$-stabilité), ϵMI et $\epsilon\epsilon MI$ ($\mathcal{D}_{\mathcal{U}}$-stabilité) pour contourner ce problème. En effet, on montre (linéarisation LMI):

$$\boldsymbol{N}^T = \boldsymbol{A}^T - (\boldsymbol{UCA})^T - (\bar{\boldsymbol{Y}}\boldsymbol{VCA})^T\boldsymbol{P}^{-1} - (\bar{\boldsymbol{K}}\boldsymbol{C})^T\boldsymbol{P}^{-1}$$

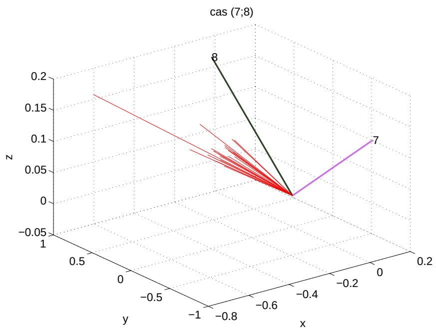
On peut donc appliquer une contrainte sur les valeurs propres de $\boldsymbol{N}$ en utilisant directement les résultats sur les régions LMIs, ϵMI et $\epsilon\epsilon MI$.

Isolation finale

1. 7 UIOs $\Rightarrow$ circonscrit la faute aux tuyères engendrant le même couple / force (2 tuyères candidates)
2. Discrimination par mesure de colinéarité entre $\vec{e}_{yi} = \vec{\omega} - \widehat{\vec{\omega}}$ et $\vec{\mathbb{M}}_j$.



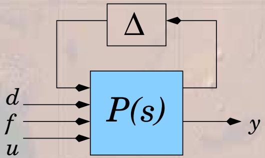
$\vec{r}_4 \bullet \vec{\mathbb{M}}_j, j = 5, 10$



$\vec{r}_6 \bullet \vec{\mathbb{M}}_j, j = 7, 8$

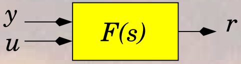
La fonction détection robuste

- Given the (controled) uncertain model in the LFR $y = F_u(P,\Delta)\begin{pmatrix} d \\ f \\ u \end{pmatrix}$, where $u(s) = K(s)y(s)$, $\Delta \in \underline{\Delta} : ||\Delta||_\infty \leq 1, \underline{\Delta} = \{\text{block diag}(\delta_1^r I_{k_1}, ..., \delta_{m_r}^r I_{k_{m_r}}, \delta_1^c I_{k_{m_r+1}}, ..., \delta_{m_c}^c I_{k_{m_r+m_c}}, \Delta_1^C, ..., \Delta_{m_C}^C), \delta_i^r \in \mathbb{R}, \delta_i^c \in \mathbb{C}, \Delta_i^C \in \mathbb{C}\}$.



- The goal is to find A_F, B_F, C_F, D_F:

$$r(s) = (C_F(sI - A_F)^{-1}B_F + D_F)\begin{pmatrix} y(s) \\ u(s) \end{pmatrix}$$

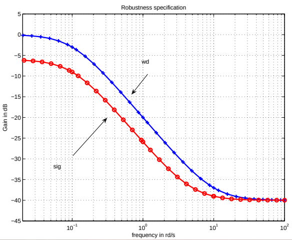


1. $\min_F \gamma_1$ $\quad \forall \Delta \in \underline{\Delta} : ||\Delta||_\infty \leq 1$ s.t. $||T_{rd}||_\infty < \gamma_1$ (Robustness)
2. $\max_F \gamma_2$ $\quad \forall \Delta \in \underline{\Delta} : ||\Delta||_\infty \leq 1$ s.t. $||T_{rf}||_- > \gamma_2 \quad \forall \omega \in \Omega$ (Fault sensitivity)

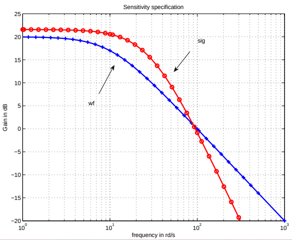
where $||P||_- = \inf_{\omega \in \Omega} \underline{\sigma}(P(j\omega)), \Omega = [\omega_1 ; \omega_2]$

The general solution

- The method parallels the well known H_∞ design / μ analysis cycle
- Thought better than the "μ-synthesis" technique (smaller order filter)
- Specify the robustness and sensitivity objectives through shaping filters $W_d(s)/W_f(s)$



H.frq noise rejection: attenuation of $40dB$ (**at least**) for $\omega \in [10rd/s; +\infty[$



L. frq fault amplification of $20dB$ (**at least**) for $\omega \in]0; 100]rd/s$

LMI formulation (Henry, 2005, 2005b)

Bounded real lemma (Boyd, 1994) and projection lemma (Gahinet & Apkarian, 1994) using an appropriate basis

The general solution

Unfortunately:

- The procedure involves sufficient conditions ($H_- \to H_\infty$, small gain theorem)
- The nature (i.e. real and/or complex) and the structure (block diagonal) of Δ is not taken into account.

$\Rightarrow$ $\gamma < 1 \to$ what about the conservativeness ?

$\Rightarrow$ $\gamma \geq 1 \to F(s)$ may be an admissible solution !

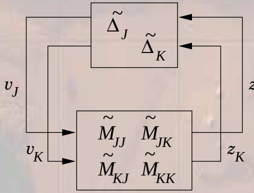
Post-analysis of robust fault detection performance

$\Rightarrow$ the generalized structured singular value μ_g (Henry 2002;Henry,2005; Henry,2006)

Definition: μ_g

Let $\underline{\tilde{\Delta}}_J = \Big\{\text{bloc diag}(\delta_1^r I_{k_1}, \dots, \delta^r_{m_{rJ}} I_{k_{m_{rJ}}}, \delta_1^c I_{k_{m_{rJ}+1}}, \dots,$
$\delta^c_{m_{cJ}} I_{k_{m_{rJ}+m_{cJ}}}, \Delta^C_{J1}, \dots, \Delta^C_{Jm_{CJ}})\Big\}$ and
$\underline{\tilde{\Delta}}_K = \Big\{\text{bloc diag}(\Delta^C_{K1}, \dots, \Delta^C_{Km_{CK}})\Big\}$.

Let $\widetilde{M} = \begin{pmatrix} \widetilde{M}_{JJ} & \widetilde{M}_{JK} \\ \widetilde{M}_{KJ} & \widetilde{M}_{KK} \end{pmatrix}$ and $\tilde{\Delta} = \begin{pmatrix} \tilde{\Delta}_J & 0 \\ 0 & \tilde{\Delta}_K \end{pmatrix} \in$
$\underline{\tilde{\Delta}} = \begin{pmatrix} \underline{\tilde{\Delta}}_J & 0 \\ 0 & \underline{\tilde{\Delta}}_K \end{pmatrix}$



closed-loop system $M - \Delta$

Definition (Henry,2005;2005b)

$$\mu_{g\underline{\tilde{\Delta}}}(\widetilde{M}) \triangleq \max_{||v||=1} \left\{ \gamma : \begin{array}{l} ||v_j||\gamma \le ||z_j||, j = 1, \dots, m_{\tilde{\Delta}J}, m_{CJ} \neq 0 \\ ||z_k||\gamma \le ||v_k||, k = 1, \dots, m_{CK} \end{array} \right\} \qquad (7)$$

with

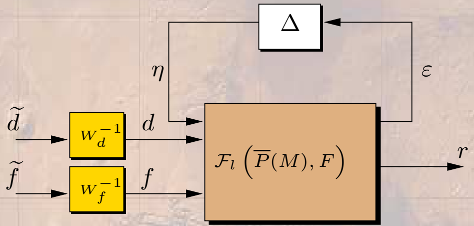
$$\widetilde{M} \in \text{dom}(\mu_g) \quad \text{iff} \quad \widetilde{M}_{KK} v_K = 0 \Rightarrow v_K = 0 \qquad (8)$$

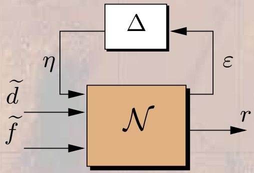
Interpretation: Like a μ problem = a robust stability problem:

The smallest (structured) uncertainty $\tilde{\Delta}_J$ that destabilizes the closed-loop system $M - \Delta$ is $||\tilde{\Delta}_J|| = 1/\mu_g$ and, simultaneously, the biggest (structured) uncertainty $\tilde{\Delta}_K$ that destabilizes the closed-loop system is $||\tilde{\Delta}_K|| = \mu_g$.

The μ_g analysis procedure

Consider W_d and W_f. With some LFR manipulations ....





Problem formulation

With the computed solution $F(s)$:

$$||T_{r\tilde{d}}||_\infty < 1 \text{ and } ||T_{r\tilde{f}}||_- > 1 \quad \forall \Delta \in \underline{\Delta} : ||\Delta||_\infty \leq 1$$

Theorem (Henry,2005): A necessary and sufficient condition for $F(s)$ to satisfy the requirements W_d/W_f is:

$$\boxed{\sup_{\omega\in\mathbf{\Omega}} \mu_{g\hat{\underline{\Delta}}}(\mathcal{N}(j\omega)) < 1}$$

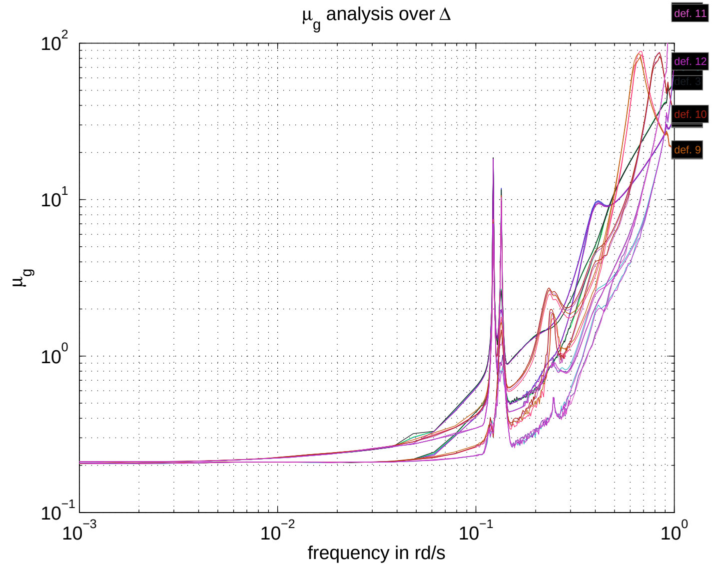
En résumé

The H_∞/H_- FD filter is derived according to the following procedure

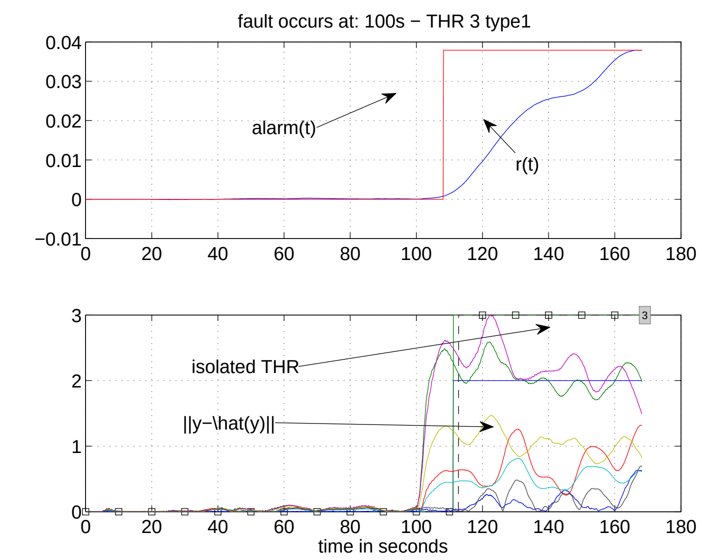
1. Specify the robustness and sensitivity objectives → ”shaping filters” .
2. Solve the LMI problem (SDPT 3) to derive A_F, B_F, C_F, D_F:
$r(s) = (C_F(sI - A_F)^{-1}B_F + D_F)\begin{pmatrix} y(s) \\ u(s) \end{pmatrix}$
3. Use the μ_g post-analysis procedure to analyze $F(s)$. Go to step 1 (i.e. refine the objectives) until:

$$\boxed{\sup_{\omega\in\Omega} \mu_{g\underline{\hat{\Delta}}}(\mathcal{N}(j\omega)) < 1}$$

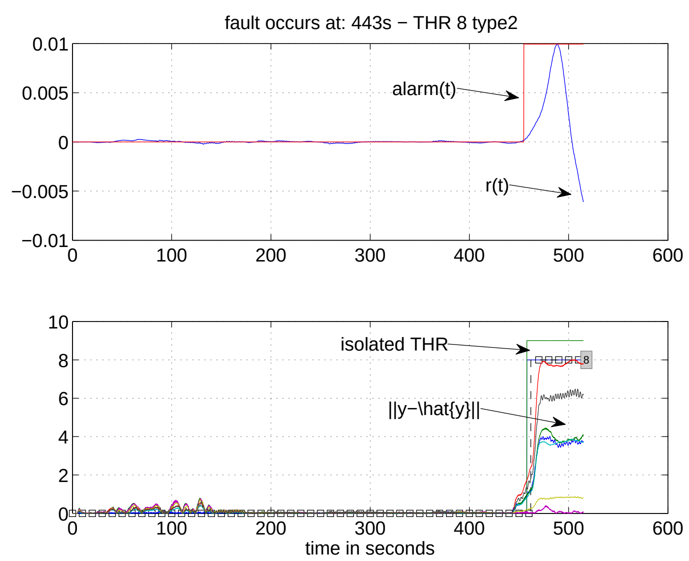
Résultats de synthèse



Résultats de simulation non linéaire



Résultats de simulation non linéaire



Tests industriels

Résultats des tests industriels............

.... TRL 5