Synergy of canonical control and unfalsified control concept to achieve fault tolerance

Tushar Jain, Joseph J. Yamé, Dominique Sauter

tushar.jain@cran.uhp-nancy.fr

Centre de Recherche en Automatique de Nancy
CRAN-CNRS UMR 7039
Group: SURFDIAG

Outline

Motivations and Objective

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- Model based FTC requires precise knowledge of plant parameters during FDI operation
  - Information about plant is not known
  - Generates false alarm on mismatching

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Objective

To develop generic methods of FTC based upon real trajectories from the system subjected to fault rather than through models resulting from a priori assumptions

Fault tolerant control problem

Standard control problem

Solve the problem $<\mathfrak{O}, \mathfrak{C}(\theta), \mathfrak{U}>$.

- $\mathfrak{O}$: objectives which the system is expected to achieve
- $\mathfrak{C}(\theta)$: functional relations (parameter θ) that controlled system satisfy over time
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Impact of fault on control problem

Occurrence of fault transform the constraints $\mathfrak{C}(\theta)$ from $\mathfrak{C}_n(\theta_n)$ into $\mathfrak{C}_f(\theta_f)$, $f \in \mathcal{F}$, where $\mathcal{F}$ indexes the set of all considered faults.

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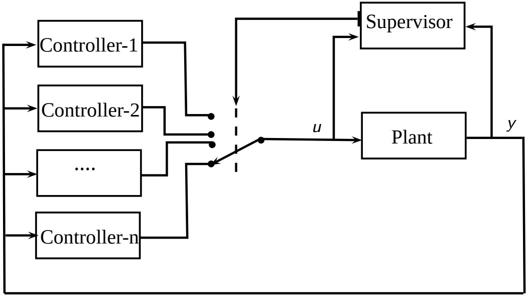
Model based FTC

Constraints $\mathfrak{C}_f(\theta)_f$ are estimated first in "fault detection and isolation" module and then new control law is applied in fault accommodation phase.

Logic based switching control

Controller reconfiguration

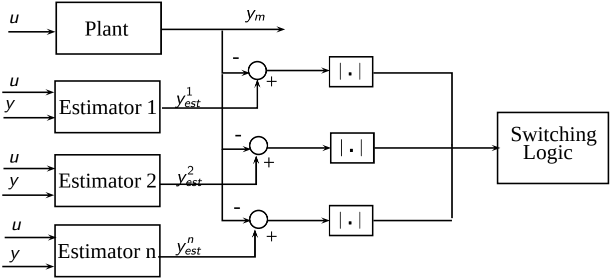
- Construct bank of controllers, each one being associated to a healthy or a faulty plant working mode.
- Selection of controller for present working mode assumes to be achieved with some delay.



Logic based switching control

Features

- theory of switching control relies on a bank of controllers
- supervisor is made of a set of estimators that gives the information about the fault
- each estimator reconstructs the plant output in either one of the healthy or faulty working modes



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Shortcomings

- partial or complete knowledge of plant model must be known
- stability issues concerned with multiple switching
- the presence of good controller in the controller bank is assumed

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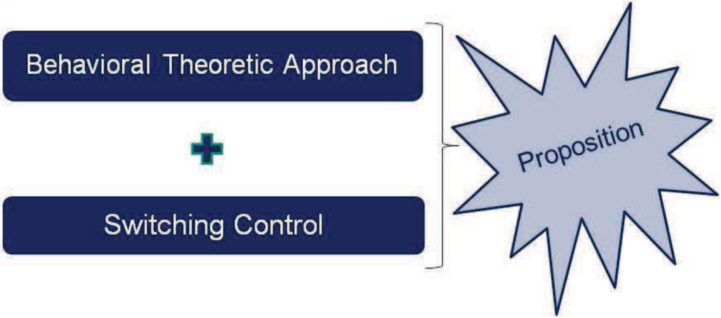
Direction

Behavioral System theoretical approach

Proposition



Features of behavioral theoretical approach

- Does not take input-output structure as the starting point
- Mathematical model viewed as any dynamical relation among variables
  - manifest variables or to-be controlled variables
  - latent variables or control variables
- Dynamical relation constraints the time evolution
- Collection of time trajectories defines the $behavior$ of dynamical system

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Definition

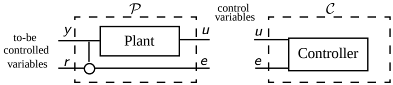
Dynamical system Σ is represented by a triple $\Sigma = (\mathbb{T}, \mathbb{W}, \mathfrak{B})$ where $\mathbb{T} \subseteq \mathbb{R}$, called the time axis, $\mathbb{W} \subseteq \mathbb{R}^{\mathtt{w}}$ called the signal space and $\mathfrak{B} \subseteq \mathbb{W}^{\mathbb{T}}$ called the behavior. A trajectory is a function

$$\mathbf{w} : \mathbb{T} \to \mathbb{W}, \; t \mapsto \mathbf{w}(t)$$

Control in Behavioral Context

- Control problem is viewed as an interconnection of two dynamical subsystems with behavior (Plant: $\mathcal{P}$, Controller: $\mathcal{C}$) such that it gives the controlled behavior, $\mathcal{K}$

$$\mathcal{K} = (\mathcal{P} \wedge_{\mathtt{c}} \mathcal{C})_{\mathtt{w}} \subseteq (\mathcal{P})_{\mathtt{w}}$$



w : to-be controlled variables;

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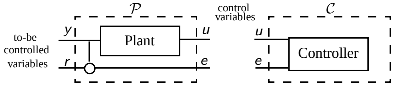

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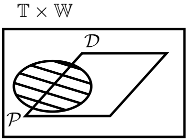

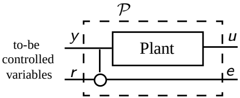

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γ is a real bound and Ψ is the performance index

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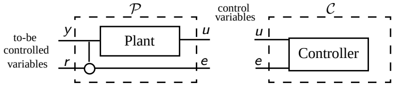

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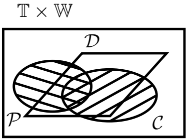

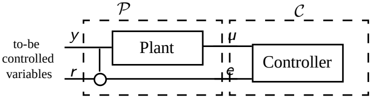

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$(\mathcal{P} \wedge_c \mathcal{C})_w = \{w = (u, y, r) \in \mathbb{W}^{\mathbb{T}} : u(t) = C.(r(t) - y(t))\} \subseteq \mathcal{D}$ consistent

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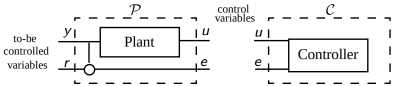

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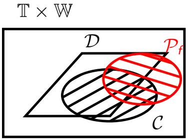

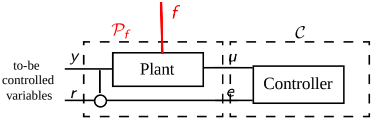

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Unfalsified control concept

Definition

A controller $K \in \mathfrak{U}$ is said to be *falsified* by measurement information if this information is sufficient to deduce that the performance specification $(\mathrm{r},\mathrm{y},\mathrm{u}) \in \mathfrak{D}\ \forall\ \mathrm{r} \in \mathcal{R}$ would be violated if that controller were in the feedback loop. Otherwise the control law K is said to be *unfalsified*

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Problem

Given

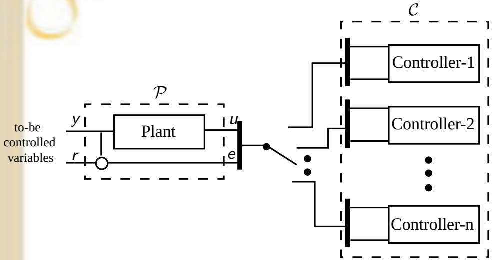
- (a) a measurement information set $P_{data} \subset \mathcal{R} \times \mathcal{Y} \times \mathcal{U}$;
- (b) a performance specification set $\mathfrak{O} \subset \mathcal{R} \times \mathcal{Y} \times \mathcal{U}$;
- (c) a class $\mathfrak{U}$ of admissible control laws;

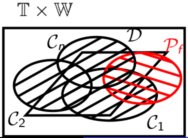
determine the subset $\mathfrak{U}_{OK}$ of control laws $K \in \mathfrak{U}$ whose ability to meet the specification $(\mathrm{r,y,u}) \in \mathfrak{O}\ \forall\ \mathrm{r} \in \mathcal{R}$ is not falsified by $P_{\mathtt{data}}$

FTC in behavioral context

Solve the problem $<\mathfrak{D}, \mathfrak{C}(\theta), \mathfrak{U}>$.

For any fault, $f_i \in \mathcal{F}, i = 1, ..., n$, construct a set $K \in \mathfrak{U}$ corresponding to each faulty mode instead of determining $\mathfrak{C}(\theta)$, each time fault occur.





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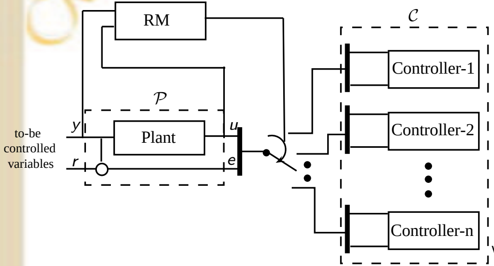
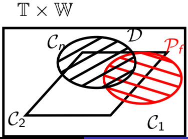


$$r_{C_i}^{fict} = C_i^{-1} u(t) + y(t)$$
$$w_i := (r_{C_i}^{fict}, u, y)$$
$$\mathcal{D} = \{w_i \mid J(w_i) \le \gamma\}$$

fictitious reference signal is the reference signal that would have generated the data (u, y), if controller C_i would have been in the loop



FTC in Behavioral Context

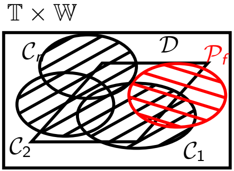
Features

- Not all the controllers need to be put into the closed loop to see which controller satisfies the performance specifications.
- Ofcourse, the scheme is completely model free. Only the presence of right controller in the controller bank that can achieve the desired performance is assumed.
- Shortcomings seen in model based FTC is lowered down.
- In this model free FTC, a set of pre-determined control laws for each (healthy or faulty) mode is equipped instead of estimating the constraints as seen in model based FTC.

Further issues

To be resolved

What will happen if the right controller is not present in the controller bank ?



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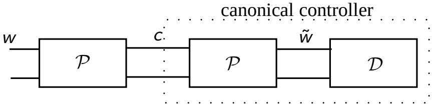
Canonical Control in Behavioral framework: is a controller which is generated by the interconnection of plant and desired behavior. The only condition is that the desired behavior must be achievable.

- $\mathfrak{O}$: the *desired behavior* , $\mathcal{D}$
- $\mathfrak{C}(\theta)$: behavior of the controlled system satisfies over time
- $\mathfrak{U}$: the *controller behavior*, $\mathcal{C}$

Here we imposed an assumption on desired trajectory that the canonical controller designed achieves the desired behavior

Canonical Control

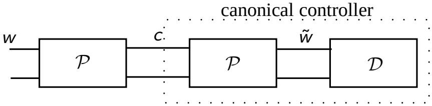


$$\mathcal{P} = \{(w, c) \in \mathbb{R}^{q+p} \mid R(\xi)w = M(\xi)c\}$$

to-be-controlled variables, $w := (r, y)$; control variables , $c := (u, e)$

$$R = \begin{pmatrix} 1 & -1 \\ 0 & R_y \end{pmatrix}, M = \begin{pmatrix} 1 & 0 \\ 0 & R_u \end{pmatrix}, G = R_y^{-1} R_u$$

Canonical Control



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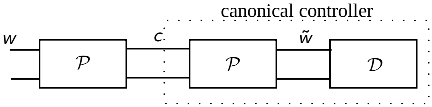
$$\mathcal{C} = \{c \in \mathbb{R}^p \mid H(\xi)c = 0\}, C = C_e^{-1} C_u$$

C_e, C_u, R_y, R_u are the coprime polynomial factorization

Canonical Control



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$$\mathcal{D} = \{w \in \mathbb{R}^q \mid \exists c \in \mathcal{C} \text{ such that } (w, c) \in \mathcal{P} \mid D(\xi)w = 0\}$$

Canonical Control

Theorem

Let $\mathcal{P} \subset (\mathbb{R}^{q+p})^{\mathbb{T}}$ be a given plant system, and let $\mathcal{C} \subset (\mathbb{R}^{p})^{\mathbb{T}}$ be a controller to be designed. Let $\mathcal{D} \subset (\mathbb{R}^{q})^{\mathbb{T}}$ be a desired behavior. Then there exists $\mathcal{C}$ such that $\mathcal{P} \wedge_c \mathcal{C} = \mathcal{D}$ if $\mathcal{D} \subset \mathcal{P}_w$.

desired behavior is a restricted behavior in the manifest behavior, $(\mathcal{P})_w$, therefore

$$\Rightarrow \exists\, L(\xi) : \quad \begin{array}{l} \mathrm{L}(\xi)R(\xi) = D(\xi) \\ \mathrm{L}(\xi)M(\xi)c(t) = 0 \end{array}$$

Above equation induces the kernel representation of a canonical controller acting only on the variable c

Canonical Control

$$\begin{pmatrix} C_e & C_u \end{pmatrix} \begin{pmatrix} D_r & 0 \\ 0 & (D_r - D_y) \end{pmatrix} \begin{pmatrix} u \\ y \end{pmatrix} = 0$$

Desired behavior , $T_d = D_y^{-1} D_r$, Controller behavior , $C = C_e^{-1} C_u$

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for constructing the canonical controller, assign structure to polynomials

$$C_u(\theta) = \sum_{i=0}^{m} \theta_i \xi^i, \; C_e(\rho) = \sum_{i=1}^{n} \rho_i \xi^i$$

with unknown parameters θ and ρ. Consider a finite interval data of length N,

$$C_e(\rho) D_r u_{[k,k+N]} = C_u(\theta)(D_r - D_y) y_{[k,k+N]}$$

$$\bar{u}_{[k,k+N-n_a]} := D_r u_{[k,k+N]}, \bar{y}_{[k,k+N-n_b]} := (D_r - D_y) y_{[k,k+N]}$$

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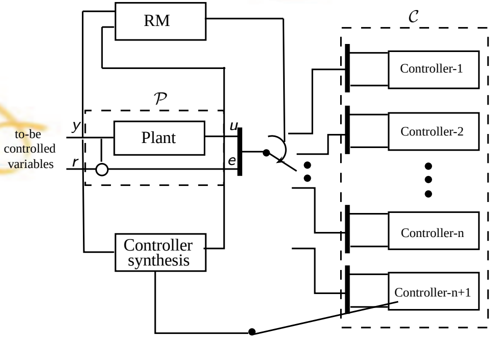
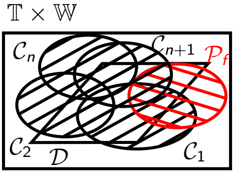
where n_a and n_b is the degree of D_r and $D_r - D_y$. In matrix form

$$\begin{pmatrix} \bar{y}_k & \cdots & \bar{y}_{k+n} & -\bar{u}_{k+1} & \cdots & -\bar{u}_{m+k} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \bar{y}_{k+N-n_b-n-1} & \cdots & \bar{y}_{k+N-n_b-1} & -\bar{u}_{k+N-n_a-m} & \cdots & -\bar{u}_{k+N-n_a-1} \end{pmatrix} \begin{pmatrix} \theta \\ \rho \end{pmatrix} = \begin{pmatrix} \bar{u}_k \\ \vdots \\ \bar{u}_{k+N-n_a-m-1} \end{pmatrix}$$

Fault tolerant control scheme





Functioning

- Reconfiguration mechanism (RM) unfalsify the controllers which exists in the controller bank.
- Controller synthesis block synthesize a set of new controllers using canonical control concept for a set of considered desired trajectories.

Features of the scheme

- For the N potential controllers in the bank, N performance indexes $J(r_{C_i}^{fict}, u, y)$ are computed and only those controllers are marked unfalsified which satisfies $J(w_i) \leq \gamma$.
- The control selection algorithm has input $J(w_i)_{i=1}^{N}$ and output σ, the switching signal.

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- This lower bound during which a controller is active in the loop is called dwell time and the measured data is collected in this time interval $[t_n, t_n + \tau_D]$.

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- This lower bound during which a controller is active in the loop is called dwell time and the measured data is collected in this time interval $[t_n, t_n + \tau_D]$.
- The logic is then realized through

$$\sigma(t) = \sigma(t_n), \text{ for } t_n \leq t < t_{n+1}$$

with the updating rule

$$\sigma(t_{n+1}) = \begin{cases} \sigma(t_n), \text{ if } C_{\sigma(t_n)} \text{ is not invalidated} \\ \widehat{i} = \arg\min\{J(w_i) \mid J(w_i) \leq \gamma\}_{i \neq \sigma(t_n)}, \end{cases}$$

Features of the scheme

- After the time interval $[t, t+\tau_D]$ if $J(w_i)_{i=1}^{N} > \gamma$, a new set of controller is generated using

$$\begin{pmatrix} C_e & C_u \end{pmatrix} \begin{pmatrix} D_r & 0 \\ 0 & (D_r - D_y) \end{pmatrix} \begin{pmatrix} u_{\tau_D} \\ y_{\tau_D} \end{pmatrix} = 0$$

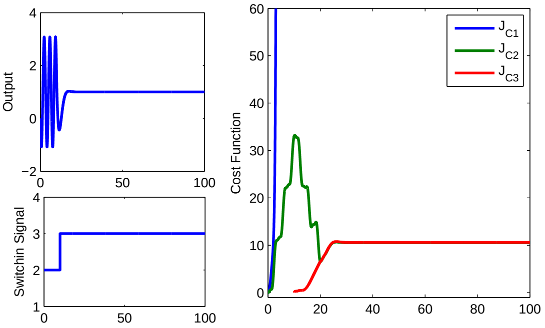
  where (u_{τ_D}, y_{τ_D}) are the plant trajectories collected in the time interval $[t, t+\tau_D]$.

- For N' desired trajectory, N' new controllers are generated and added into the controller bank.
- If one of N' desired trajectory achieves the desired behavior, then that particular controller get switched into the feedback loop by RM.

Simulation

$$P = \text{ unknown plant }, C_1 = \frac{-s+1}{0.3s+1}, C_2 = \frac{-s-1}{0.3s+1}, C_3 = \frac{-1.049s-1.176}{s+2.978}$$

$$\text{Desired trajectory} : D_y(\xi) = \xi^2 + 1.6\xi + 1, D_r(\xi) = -\xi + 1, J_{C_i} = \int_t^{t+\tau_D} (r_{C_i} - y)^2 dt$$



$\implies \gamma = 31, \tau_D = 10sec$
$\implies J(w_i) \leq \gamma = 31$
$\implies C_2$ is in the feedback loop. Fault occurs at t=0sec.
$\implies$ Initially two controllers C_1 and C_2 are installed in the bank.
$\implies$ At t=10sec, $J(r_{C_i,u,y})_{i=1}^2 > \gamma$. Therefore, a new controller is designed from the collected trajectories.
$\implies$ The new controller C_3 is now installed in the bank and tested for its feasibility.
$\implies$ RM block unfalsify the new controller during the next dwell time.

Conclusions

- Fault tolerant control problem is studied in the behavioral theoretic framework.
- Real time model free reconfiguration mechanism is suggested to achieve fault tolerance.
- The limitation for the presence of right controller in a pre-determined controller bank is released.
- Future work includes the consideration of different types of fault for the MIMO system.

Conclusions

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Publications

- A model based 2-DOF fault tolerant control strategy, 18th IEEE Mediterranean Conf. Control and Automation, June 2010.
- A real time router fault accommodation, IEEE SysTol'10, Oct.2010
- Synergy of canonical control and unfalsified control concept to achieve fault tolerance, IFAC World Congress, 2011 (submitted)

Thank you for your attention!!