Model-Based Fault Diagnosis Observer Design for Descriptor Linear Parameter Varying System with Unmeasurable Gain Scheduling Functions

Francisco R. LÓPEZ ESTRADA $^{1,2,3}$, Didier THEILLIOL $^{1,2}$, Jean C. PONSART $^{1,2}$, and, Carlos M. ASTORGA ZARAGOZA $^{3}$

$^{1}$University of Lorraine, CRAN, UMR 7039, Campus Sciences, France
$^{2}$CNRS, CRAN, UMR 7039
$^{3}$Department of Electronic Systems, CENIDET, México

25 June 2014
1 Context
2 Introduction
3 Applications
4 Problem formulation
5 Observer design
6 Illustrative example
7 Conclusions
National Center of Research and Technological Development

[Map of Mexico with Morelos highlighted]
Focus: Modeling and identification, analysis, control and observation, and diagnosis.
Introduction

Dependability and safety

In order to respect the growing of economic demand for high plant availability, and system safety, dependability is becoming an essential need in industrial automation.
DLP-Systems

Modeling as:

- Nonlinear system

\[
\begin{align*}
\dot{x} &= F(x, u, t), \\
y &= g(x, u, t),
\end{align*}
\]

\[
\begin{align*}
0 &= F(\dot{x}, x, u, t) \\
y &= g(x, u, t)
\end{align*}
\]
DLP-Systems

Modeling as:

- Nonlinear system

\[
\begin{align*}
\dot{x} &= F(x, u, t), \\
y &= g(x, u, t),
\end{align*}
\]

- Descriptor LTI (DLTI)

\[
\begin{align*}
Ex(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{align*}
\]

 Descriptor LPV (Polytopic)

\[
\begin{align*}
\dot{E}x(t) &= \sum_{i=1}^{\rho} \rho_i(x(t)) \left[A_i x(t) + B_i u(t)\right] \\
y(t) &= Cx(t)
\end{align*}
\]

Singular Matrix:

\[
\tilde{E} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

Ordinary Differential Equations (Dynamic)
Algebraic Equations (Static)
Modeling as:

- **Nonlinear system**
  
  \[
  \begin{align*}
  \dot{x} &= F(x, u, t), & 0 &= F(\dot{x}, x, u, t) \\
  y &= g(x, u, t), & y &= g(x, u, t)
  \end{align*}
  \]

- **Descriptor LTI (DLTI)**
  
  \[
  Ex(t) = Ax(t) + Bu(t) \quad (1)
  \]
  
  \[
  y(t) = Cx(t)
  \]

- **Descriptor LPV (Polytopic)**
  
  \[
  E\dot{x}(t) = \sum_{i=1}^{h} \rho_i(x(t)) [A_i x(t) + B_i u(t)]
  \quad (2)
  \]
  
  \[
  y(t) = Cx(t)
  \]
Modeling as:

- Nonlinear system

\[
\dot{x} = F(x, u, t), \quad 0 = F(\dot{x}, x, u, t)
\]
\[
y = g(x, u, t), \quad y = g(x, u, t)
\]

- Descriptor LTI (DLTI)

\[
E \dot{x}(t) = Ax(t) + Bu(t) \quad (1)
\]
\[
y(t) = Cx(t)
\]

- Descriptor LPV (Polytopic)

\[
E \dot{x}(t) = \sum_{i=1}^{h} \rho_i(x(t)) [A_i x(t) + B_i u(t)]
\]
\[
y(t) = Cx(t) \quad (2)
\]

Convex Scheduling Functions

\[
\forall i \in [1, 2..., h], \quad \rho_i(x(t)) \geq 0 \quad (3)
\]
\[
\sum_{i=1}^{h} \rho_i(x(t)) = 1, \quad \forall t
\]

\(x(t)\) is unmeasurable.
Application: Flight vehicle control

\[ E \dot{x} = \sum_{i=1}^{h} \rho_i(x(t)) [A_i x(t) + B_i u(t)] \]

with

\[ A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & F_0 + VF_V & Vl & 0 \\ 0 & -a & 0 & 0 \\ 0 & VF_2 & -l & 0 \end{bmatrix} \]

\[ B_i = \begin{bmatrix} 0 \\ 0 \\ a \\ 0 \end{bmatrix} \]

\[ E = \text{diag}(I_4, 0) \]

\(^1\) The model is valid for airspeed \(45 \leq V \leq 80\) m/s

\(^1\) Masubuchi et al. [Gain-scheduled controller design based on descriptor representation of LPV systems: Application to flight vehicle control 2004, 43rd IEEE Conference on Decision and Control (CDC) (2004).]
Application: Simple Electrical Circuit

\[ E\dot{x}(t) = \sum_{i=1}^{h} \rho_i(x(t)) [A_i x(t) + B_i u(t)] \]

with

\[
A_i = \begin{bmatrix}
\frac{-R_1 + \theta_1(t)}{L_1} & 0 & \frac{R_{13}}{L_2} & \frac{R_{14}}{L_2} \\
0 & -\frac{R_{22} + \theta_2}{L_2} & \frac{R_{23}}{L_2} & 0 \\
R_{31} & R_{32} & \frac{-R_{33}}{L_2} & 0 \\
R_{41} & R_{42} & 0 & \frac{-R_{44}}{L_2}
\end{bmatrix}
\]

\[
B_i = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix}, \quad E = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[ R_2 \text{ and } R_6 \text{ are variable resistors of } [-0.5, 0.5]\Omega \text{ and } [-1, 1]\Omega, \text{ respectively} \]

---

2 Rodrigues et al. [Fault Diagnosis Based on Adaptive Polytopic Observer for LPV Descriptor Systems 2012, Safeprocess, 2012]
Application: Anaerobic Bio-reactor

\[
E \dot{x} = \sum_{i=1}^{h} \rho_i(x(t)) [A_i x(t) + B_i u(t)]
\]

with

\[
A = \begin{bmatrix}
Y_1 x_4 - \alpha D - k_d & 0 & 0 & Y_1 x_1 \\
-x_4 & -D & 0 & -x_1 \\
(1 - Y_1) Y_{CH4} x_4 & 0 & -1 & 0 \\
0 & \frac{\partial x}{\partial x_4} & 0 & -1
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
-\alpha x_1 & 0 \\
s_1^i - s_1 & D \\
0 & 0 \\
0 & 0
\end{bmatrix},
E = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

with

\[
\frac{\partial x}{\partial x_4} = \frac{l_{PH} k_{m1} k_{s1}}{(k_{s1} + s_1)^2}
\]

The DLPV system is obtained by linearization in different operation points.
Application: Waste-water treatment plant

\[
\dot{E}x = \sum_{i=1}^{16} \rho_i(x(t)) \left[ A_i x(t) + B_i u(t) + B_{di} d(t) \right]
\]

Sixteen modes are obtained by considering the sector non-linearity approach \(^3\)

\(^3\)Nagy Kiss et al. [State estimation of two-time scale multiple models. Application to wastewater treatment plant 2011, Control Engineering Practice (2011)]
Application: Waste-water treatment plant

\[ E\dot{x} = \sum_{i=1}^{4} \rho_i(x(t)) \left[ A_i x(t) + B_i u(t) + B_{di} d(t) \right] \]

Four modes are obtained by considering parameter variations on the liquid molar flow and volume.

---

Aguilera-González et al. [Singular linear parameter-varying observer for composition estimation in a binary distillation column 2013]
Application to sensor fault estimation

Consider a LPV system under sensor faults and disturbances given by

\[ \mathcal{T}_- := \begin{cases} \hat{x}(t) &= \sum_{i=1}^{h} \rho_i(\hat{x}(t)) \left[ \tilde{A}_i \hat{x}(t) + \tilde{B}_i u(t) + \tilde{B}_d d(t) \right] \\ y(t) &= \tilde{C} \hat{x}(t) + \tilde{D}_d d(t) + f(t) \end{cases} \]  

(4)
Application to sensor fault estimation

Consider a LPV system under sensor faults and disturbances given by

$$\mathcal{F}_\theta := \begin{cases} 
\dot{x}(t) &= \sum_{i=1}^{h} \rho_i(x(t)) \left[ \bar{A}_i x(t) + \bar{B}_i u(t) + \bar{B}_d d(t) \right] \\
y(t) &= Cx(t) + \bar{D}_d d(t) + f(t) 
\end{cases}$$  \hspace{1cm} (4)

An augmented system $x(t) = [\bar{x}^T(t) \quad f^T(t)]^T$, can be obtained as

$$F_x := \begin{cases} 
\dot{E}x(t) &= \sum_{i=1}^{h} \rho_i \left[ A_i x(t) + B_i u(t) + B_d d(t) + B_f f(t) \right] \\
y(t) &= Cx(t) + D_d d(t) 
\end{cases}$$  \hspace{1cm} (5)

where:

$$\begin{align*}
E &= \begin{bmatrix} I & 0 \\
0 & 0_p \end{bmatrix}, & A_i &= \begin{bmatrix} \bar{A}_i & 0 \\
0 & -I_p \end{bmatrix}, & B_i &= \begin{bmatrix} \bar{B}_i \\
0_p \end{bmatrix} \\
B_d &= \begin{bmatrix} \bar{B}_d \\
0 \end{bmatrix}, & B_f &= \begin{bmatrix} 0 \\
I_p \end{bmatrix}, & C &= \begin{bmatrix} \bar{C} & I_p \end{bmatrix}, & D_d &= \bar{D}_d.
\end{align*}$$
Application to sensor fault estimation

Consider a LPV system under sensor faults and disturbances given by

$$
\dot{x}(t) = \sum_{i=1}^{h} \rho_i(\tilde{x}(t)) \left[ \tilde{A}_i \tilde{x}(t) + \tilde{B}_i u(t) + \tilde{B}_d d(t) \right] \\
y(t) = \tilde{C} \tilde{x}(t) + \tilde{D}_d d(t) + f(t)
$$

(4)

An augmented system $\mathbf{x}(t) = [\tilde{x}^T(t) \ f^T(t)]^T$, can be obtained as

$$
\dot{F}_x := \begin{cases} \\
E \dot{x}(t) = \sum_{i=1}^{h} \rho_i [A_i x(t) + B_i u(t) + B_d d(t) + B_f f(t)] \\
y(t) = C x(t) + D_d d(t) 
\end{cases}
$$

(5)

Consider the following FD observer

$$
\dot{z}(t) = \sum_{j=1}^{h} \rho_j(\hat{x}(t)) \left[ N_j z(t) + G_j u(t) + L_j y(t) \right] \\
\hat{x}(t) = z(t) + T_2 y(t) \\
r(t) = y(t) - C \hat{x}(t)
$$

(6)

The problem is reduced to find $N_j$, $G_j$, $L_j$, $T_2$ (5) such that observer can estimate the states and the augmented fault vector.
Objective

Detec and isolate sensor faults in nonlinear systems modeled as descriptor-linear parameter varying systems (D-LPV) with unmeasurable gain scheduling functions.
Objective

Detec and isolate sensor faults in nonlinear systems modeled as descriptor-linear parameter varying systems (D-LPV) with unmeasurable gain scheduling functions.

Impact

1. The unmeasurable scheduling problem has been addressed in the literature mainly for state-space LPV systems, e.i. Ichalal et al. [2008, 2010a], Nagy-Kiss et al. [2011].

2. The design of observers for descriptor systems is more complex due to the fact that the observer must be designed to be robustly stable, regular and impulse-free.

3. We have considered the problem of D-LPV with unmeasurable scheduling functions while more of the reported works are consider the measurable scheduling problem.

4. As result of the problem formulation, a solution is provided based on novels LMI equations.
Problem formulation

*The D-LPV system*

\[ \mathcal{F} = \begin{cases} 
E \dot{x}(t) = \sum_{i=1}^{h} \rho_i(x(t)) [A_i x(t) + B_i u(t) + B_{di} d(t)] \\
 y(t) = C x(t) 
\end{cases} \]

(7)

**Assumptions**

- System (7) is admissible
- System (7) is R-observable
- System (7) is I-Observable
Problem formulation

The D-LPV system

\[
\mathcal{F} = \begin{cases} 
E\dot{x}(t) = \sum_{i=1}^{h} \rho_i(x(t)) \left[ A_i x(t) + B_i u(t) + B_{di} d(t) \right] \\
y(t) = C x(t) 
\end{cases} 
\]  
\tag{7}

Assumptions

- System (7) is admissible
- System (7) is R-observable
- System (7) is I-Observable

Fault diagnosis observer

\[
\mathcal{O} = \begin{cases} 
\dot{z}(t) = \sum_{j=1}^{h} \rho_j(\hat{x}(t)) \left[ N_j z(t) + G_j u(t) + L_j y(t) \right] \\
\hat{x}(t) = z(t) + T_2 y(t) 
\end{cases} 
\]  
\tag{8}

Residual vector

\[
r(t) = M (y(t) - C\hat{x}(t)) \]  
\tag{9}

The problem is reduced to find \( N_j, G_j, L_j, \) and \( T_2 \) and \( M \) of (8) such that the residual converges to zero in fault-free case and different in faulty case.
Problem formulation

*The D-LPV system*

\[ \mathcal{F} = \begin{cases} E\dot{x}(t) = \sum_{i=1}^{h} \rho_i(x(t)) [A_i x(t) + B_i u(t) + B_{di} d(t)] \\ y(t) = C x(t) \end{cases} \] (7)

**Assumptions**

- System (7) is admissible
- System (7) is R-observable
- System (7) is I-Observable

**Fault diagnosis observer**

\[ \mathcal{O} = \begin{cases} \dot{z}(t) = \sum_{j=1}^{h} \rho_j(\hat{x}(t)) [N_j z(t) + G_j u(t) + L_j y(t)] \\ \hat{x}(t) = z(t) + T_2 y(t) \end{cases} \] (8)

**The state error**

\[ e(t) = x(t) - \hat{x}(t) \approx 0 \]

**The dynamic of the state error**

\[ \dot{e}(t) = \sum_{i=1}^{h} \rho_i(x(t)) [\mathcal{F}] - \sum_{j=1}^{h} \rho_j(\hat{x}(t)) [\mathcal{O}] \]

**Residual vector**

\[ r(t) = M(y(t) - C \hat{x}(t)) \] (9)

*The problem is reduced to find* \( N_j, G_j, L_j, \) and \( T_2 \)
Problem formulation

The D-LPV system

\[ \mathcal{F} = \left\{ \begin{array}{l}
E \dot{x}(t) = \sum_{i=1}^{\infty} \rho_i(x(t)) \left[ A_i x(t) + B_i u(t) + B_{di} d(t) \right] \\
y(t) = C x(t)
\end{array} \right. \]  

(7)

Fault diagnosis observer

\[ \phi = \left\{ \begin{array}{l}
\dot{z}(t) = \sum_{j=1}^{\infty} \rho_j(\hat{x}(t)) \left[ N_j z(t) + G_j u(t) + L_j y(t) \right] \\
\hat{x}(t) = z(t) + T_2 y(t)
\end{array} \right. \]  

(8)

Residual vector

\[ r(t) = M(y(t) - C \hat{x}(t)) \]  

(9)

The problem is reduced to find \( N_j, G_j, L_j, \) and \( T_2 \) and \( M \) of (8) such that the residual converges to zero in fault free case and different in faulty case.

Assumptions

- System (7) is admissible
- System (7) is R-observable
- System (7) is I-Observable

Notations

\( \rho_i = \rho_i(x(t)) \)  
\( \hat{\rho}_j = \rho_j(\hat{x}(t)) \)
Observer design: Uncertain error system

The state error is given by

\[ e(t) = x(t) - \hat{x}(t) = (I - T_2 C)x(t) - z(t) \]
Observer design: Uncertain error system

The state error is given by

\[ e(t) = x(t) - \hat{x}(t) = (I - T_2 C)x(t) - z(t) \]

The error system becomes

\[
\dot{e}(t) = \sum_{i=1}^{h} \rho_i T_1 [A_i x(t) + B_i u(t) + B_d d(t)] - \sum_{i=1}^{h} \hat{\rho}_i [N_i T_1 E x(t) - N_i e(t) + G_i u(t) + L_i y(t)]
\]

(10)
Observer design: Uncertain error system

The state error is given by

\[ e(t) = x(t) - \hat{x}(t) = (1 - T_2 C)x(t) - z(t) \]

The error system becomes

\[
\dot{e}(t) = \sum_{i=1}^{h} \rho_i T_1 \left[ A_i x(t) + B_i u(t) + B_d d(t) \right] - \sum_{i=1}^{h} \hat{\rho}_i \left[ N_i T_1 E x(t) - N_i e(t) + G_i u(t) + L_i y(t) \right]
\]

By considering the convex property of the scheduling functions, the term \(-\sum_{i=1}^{h} \hat{\rho}_i N_i T_1 E x(t)\) can be handled as

\[
\sum_{i=1}^{h} \rho_i \left[ \sum_{j=1}^{h} [(\rho_j - \hat{\rho}_j) N_j T_1 E] \right] x(t) - \sum_{i=1}^{h} \rho_i N_i T_1 E x(t),
\]

(11)
Observer design: Uncertain error system

The state error is given by

$$e(t) = x(t) - \hat{x}(t) = (I - T_2C)x(t) - z(t)$$

The error system becomes

$$\dot{e}(t) = \sum_{i=1}^{h} \rho_i T_1 [A_i x(t) + B_i u(t) + B_d d(t)] - \sum_{i=1}^{h} \hat{\rho}_i [N_i T_1 E x(t) - N_i e(t) + G_i u(t) + L_i y(t)]$$

(10)

By considering the convex property of the scheduling functions, the term $- \sum_{i=1}^{h} \hat{\rho}_i N_i T_1 E x(t)$ can be handled as

$$\sum_{i=1}^{h} \rho_i \left[ \sum_{j=1}^{h} [ (\rho_j - \hat{\rho}_j) ] N_j T_1 E \right] x(t) - \sum_{i=1}^{h} \rho_i N_i T_1 E x(t),$$

(11)

The residual state error system becomes

$$\dot{\hat{e}}(t) = \sum_{i=1}^{h} \rho_i \left\{ N_i e(t) + \sum_{j=1}^{h} [ (\rho_j - \hat{\rho}_j) ] \left[ (T_1 A_j) x(t) + G_j u(t) - N_j e(t) \right] \right\}$$

(12)

$$r(t) = M(y(t) - C\hat{x}(t))$$

(13)
Uncertain error system

The residual state-space error system is rewritten in compact form with augmented states \( x_e(t) = [e(t)^T \ x(t)^T]^T \) as

\[
\bar{E} \dot{x}_e(t) = \sum_{i=1}^{h} \rho_i \left[ (\bar{A}_i + \Delta \bar{A}_i) x_e(t) + (\bar{B}_i + \Delta \bar{B}_i) u(t) \right]
\]

\[
r(t) = \bar{C} x_e(t)
\]

with

\[
\bar{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \quad \bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & N_i \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 0 & MC \end{bmatrix}
\]

and, matrices \( \Delta \bar{A}_i \) and \( \Delta \bar{B}_i \) defined by

\[
\Delta \bar{A}_i = H_A F_A \Phi_A, \quad \Delta \bar{B}_i = H_B F_B \Phi_B,
\]

with

\[
H_A = \begin{bmatrix} T_1 A_1 & 0 & \cdots & T_1 A_h \\ \vdots & \ddots & \ddots & \vdots \\ N_1 & \cdots & N_h \end{bmatrix}, \quad F_A = \begin{bmatrix} F \\ 0 \\ F \end{bmatrix},
\]

\[
\Phi_A = \begin{bmatrix} I_A \\ 0 \\ -I_A \end{bmatrix}^T, \quad I_A = \begin{bmatrix} I_{n_1} & \cdots & I_{n_h} \end{bmatrix}^T,
\]

\[
H_B = \begin{bmatrix} G_1 & 0 & \cdots & G_h \end{bmatrix}, \quad F_B = \begin{bmatrix} F \end{bmatrix}, \quad \Phi_B = I_B.
\]

\[
I_B = \begin{bmatrix} I_{m_1} & \cdots & I_{m_h} \end{bmatrix}^T, \quad F = \begin{bmatrix} (\rho_1 - \hat{\rho}_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & (\rho_h - \hat{\rho}_h) \end{bmatrix}
\]
The residual state-space error system is rewritten in compact form with augmented states \( x_e(t) = [e(t)^T \ x(t)^T]^T \) as

\[
\dot{E}x_e(t) = \sum_{i=1}^{h} \rho_i \left[ (\bar{A}_i + \Delta \bar{A}_i) x_e(t) + (\bar{B}_i + \Delta \bar{B}_i) u(t) \right]
\]

\( r(t) = \bar{C}x_e(t) \)

Solution is obtained by considering the \( H_\infty \) performance criterion

\[
J_1 = \text{Performance} + \text{Asymptotic stability}
\]

\[
J_1 = r^T(t)r(t) - \gamma^2 u^T(t)u(t) + V(x_e(t)) \leq 0
\]

\( \gamma \) is the attenuation level.
Theorem 1 (To be presented in World IFAC 2014.)

There exists a robust state estimation observer (8) for the D-LPV system (7) with $H_{\infty}$ attenuation level $\gamma > 0$, if there exist scalars $\epsilon_A > 0$, $\epsilon_B > 0$, matrices $X = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix}$ with $P > 0$, $Q = Q^T > 0$, and gain matrices $K_i = Q^{-1}\Xi_i$, $\forall i \in [1, 2, \ldots, h]$, such that there exists a solution to the following optimization problem:

$$\min_{P, Q, \Xi_i, \epsilon_A, \epsilon_B} \gamma$$

s.t.

$$E^T P = P^T E > 0$$

$$\begin{cases}
\begin{bmatrix}
\text{He}\{A_i^T P\} & 0 \\
0 & \text{He}\{(T_1 A_i)^T Q + C \Xi_i\}
\end{bmatrix} \\
\begin{bmatrix}
0 & PB_i & 0 & 0 & 0 \\
0 & 0 & [Q T_1 A_1 \ldots Q T_1 A_h] & [Q T_1 A_1 + \Xi_1 C \ldots Q T_1 A_h + \Xi_h C] & 0 \\
-\gamma^2 I_m & 0 & -\epsilon A^T \eta \times h & 0 & 0 \\
-\epsilon A^T \eta \times h & 0 & 0 & -\epsilon A^T \eta \times h & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-\epsilon A^T \eta \times h & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \leq 0
\end{cases}$$

$$LÓPEZ ESTRADA et al (CRAN/CENIDET)$$
Illustrative example
Considers the following descriptor-LPV system under disturbances as:

\[
E \dot{x}(t) = \sum_{i=1}^{3} \rho_i(x(t)) \left[ A_i x(t) + B_i u(t) + B_{di} d(t) \right]
\]
\[
y(t) = C x(t) + D_{di} d(t) + f(t)
\]

with: \( E = \text{diag}(1, 1, 0) \),

\[
A_1 = \begin{bmatrix}
-10 & 5 & 6.5 \\
2 & -5.5 & -1.25 \\
-9 & 4 & 8.5
\end{bmatrix},
A_2 = \begin{bmatrix}
-10 & 5 & 6.5 \\
5 & -4 & -1.25 \\
-2 & 4 & 7
\end{bmatrix}
\]

\[
A_3 = \begin{bmatrix}
-8 & 5 & 6.5 \\
5 & -4 & -1.25 \\
-5 & 4 & 6
\end{bmatrix},
B_1 = \begin{bmatrix}
0 \\
1 \\
0.5
\end{bmatrix},
B_2 = \begin{bmatrix}
0 \\
1 \\
0.7
\end{bmatrix}
\]

\[
B_3 = \begin{bmatrix}
0.5 \\
0.6
\end{bmatrix},
B_{d1} = \begin{bmatrix}
1 \\
0
\end{bmatrix},
B_{d2} = \begin{bmatrix}
0.5 \\
0.5
\end{bmatrix},
B_{d3} = \begin{bmatrix}
0 \\
0.5
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 1 \\
0 & 0 & 1
\end{bmatrix},
D_{d} = \begin{bmatrix}
0.5 \\
0.3 \\
0
\end{bmatrix}
\]

\textbf{Scheduling functions}

\[
\rho_i(x(t)) = \frac{\mu_i(x(t))}{\sum_{i=1}^{3} \mu_i(x)}
\]

\[
\mu_1(x) = \exp \left[ \frac{1}{2} \left( \frac{x_3(t) + 0.4}{0.5} \right)^2 \right]
\]

\[
\mu_2(x) = \exp \left[ \frac{1}{2} \left( \frac{x_3(t) - 0.4}{0.1} \right)^2 \right]
\]

\[
\mu_3(x) = \exp \left[ \frac{1}{2} \left( \frac{x_3(t) - 1}{0.5} \right)^2 \right]
\]

Models = 3; Outputs = 3; States = 3; \( x \) is unmeasurable.
Simulation conditions

Fault-free case

\[
\begin{align*}
\mathbf{x}(0) &= [0, 1, -1]^T, \\
\mathbf{x}(0) &= [0, 1, -2, -3]^T, \\
u(t) &= \sin(t)
\end{align*}
\]

\text{is chosen as zero-mean noise with standard deviation 0.3.}

Faulty-case

\begin{align*}
f_1 &= \begin{cases} 
\sin(t) & \text{on } 0 \leq t \leq 20 \\
0 & \text{otherwise}
\end{cases} \\
f_2 &= \begin{cases} 
-1.5 & \text{on } 30 \leq t \leq 40 \\
0 & \text{otherwise}
\end{cases} \\
f_3 &= \begin{cases} 
-0.05t - 2 & \text{on } 40 \leq t \leq 60 \\
0 & \text{otherwise}
\end{cases}
\end{align*}
Simulation conditions

Fault-free case

\[ x(0) = [0, 1, -1]^T \]
\[ \hat{x}(0) = [0.1, -2, -3]^T \]
\[ u(t) = \sin(t) \]

\( d(t) \) is chosen as zero-mean noise with standard deviation 0.3.
Simulation conditions

Fault-free case

\[ x(0) = [0, 1, -1]^T \]
\[ \hat{x}(0) = [0.1, -2, -3]^T \]
\[ u(t) = \sin(t) \]

\( d(t) \) is chosen as zero-mean noise with standard deviation 0.3.

Faulty-case

\[ f_1 = \begin{cases} 
\sin(t) & \text{on } y_1 \\
0 & \text{otherwise}
\end{cases} \quad 10 \text{s} \leq t \leq 20 \text{s} \]

\[ f_2 = \begin{cases} 
-1.5 & \text{on } y_2 \\
0 & \text{otherwise}
\end{cases} \quad 30 \text{s} \leq t \leq 35 \text{s} \]

\[ f_3 = \begin{cases} 
-0.05t - 2 & \text{on } y_3 \\
0 & \text{otherwise}
\end{cases} \quad 40 \text{s} \leq t \leq 60 \text{s} \]
Fault free scenario: Estimation errors

The gains of the observers are found by solving Theorem 1 with $\gamma = 2.4 \times 10^{-4}$. 
Fault free scenario: Estimation errors

The gains of the observers are found by solving Theorem 1 with $\gamma = 2.4 \times 10^{-4}$.

Figure: Estimation errors $e_x$. a) $e_{x_1}$, b) $e_{x_2}$, c) $e_{x_3}$. 
Estimated gain scheduling functions

Figure: Gains scheduling functions errors and estimated scheduling functions.
Fault scenario

Figure: Generalized observer scheme (GOS)

Table: Performance comparison

<table>
<thead>
<tr>
<th>Method</th>
<th>Obsv 1</th>
<th>Obsv 2</th>
<th>Obsv 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>$3.5683 \times 10^{-4}$</td>
<td>$3.022 \times 10^{-4}$</td>
<td>$3.087 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
Fault scenario: Residuals

\[ \| r_1 \| \quad (a) \]

\[ \| r_2 \| \quad (b) \]

\[ \| r_3 \| \quad (c) \]

\[ f_1 = \sin(t) \quad f_2 = -1.5 \quad f_3 = 0.05t - 2 \quad (d) \]
Conclusions

- A method to design fault detection observers for D-LPV systems based $H_\infty$ performance was proposed.
- The main challenge is to deal with disturbances and the error provided by the unmeasurable scheduling functions.
- Sufficient conditions to guarantee the criterion performance were given in terms of LMIs.
- The approach presents good performance and gives a good trademark between disturbance attenuation and residual magnitude.
- Clearly, the method can deal successfully with the unmeasurable scheduling problem and is useful for fault diagnosis.
- Other approaches can be consulted in
Conclusions

● A method to design fault detection observers for D-LPV systems based $H_\infty$ performance was proposed.

● The main challenge is to deal with disturbances and the error provide by the unmeasurable scheduling functions.

● Sufficient conditions to guaranteed the criterion performance were given in terms of LMIs.

● The approach presents good performance and gives a good trademark between disturbance attenuation and residual magnitude.

● Clearly, the method can deal successfully with the unmeasurable scheduling problem and is useful for fault diagnosis.

● Other approaches can be consulted in


Conclusions

- A method to design fault detection observers for D-LPV systems based $H_\infty$ performance was proposed.
- The main challenge is to deal with disturbances and the error provide by the unmeasurable scheduling functions.
- Sufficient conditions to guaranteed the criterion performance were given in terms of LMIs.
- The approach presents good performance and gives a good trademark between disturbance attenuation and residual magnitude.
- Clearly, the method can deal successfully with the unmeasurable scheduling problem and is useful for fault diagnosis.
- Other approaches can be consulted in
Conclusions

- A method to design fault detection observers for D-LPV systems based $H_\infty$ performance was proposed.
- The main challenge is to deal with disturbances and the error provide by the unmeasurable scheduling functions.
- Sufficient conditions to guaranteed the criterion performance were given in terms of LMIs.
- The approach presents good performance and gives a good trademark between disturbance attenuation and residual magnitude.
- Clearly, the method can deal successfully with the unmeasurable scheduling problem and is useful for fault diagnosis.
- Other approaches can be consulted in
Conclusions

- A method to design fault detection observers for D-LPV systems based $H_{\infty}$ performance was proposed.
- The main challenge is to deal with disturbances and the error provided by the unmeasurable scheduling functions.
- Sufficient conditions to guarantee the criterion performance were given in terms of LMIs.
- The approach presents good performance and gives a good trademark between disturbance attenuation and residual magnitude.
- Clearly, the method can deal successfully with the unmeasurable scheduling problem and is useful for fault diagnosis.
- Other approaches can be consulted in
A method to design fault detection observers for D-LPV systems based $H_{\infty}$ performance was proposed.

The main challenge is to deal with disturbances and the error provide by the unmeasurable scheduling functions.

Sufficient conditions to guaranteed the criterion performance were given in terms of LMIs.

The approach presents good performance and gives a good trademark between disturbance attenuation and residual magnitude.

Clearly, the method can deal successfully with the unmeasurable scheduling problem and is useful for fault diagnosis.

Other approaches can be consulted in

Model-Based Fault Diagnosis Observer Design for Descriptor Linear Parameter Varying System with Unmeasurable Gain Scheduling Functions

Francisco R. LÓPEZ ESTRADA $^{1,2,3}$, Didier THEILLIOL $^{1,2}$, Jean C. PONSART $^{1,2}$, and, Carlos M. ASTORGA ZARAGOZA $^3$

$^1$University of Lorraine, CRAN, UMR 7039, Campus Sciences, France
Systems Diagnosis/Reconfiguration: Analysis and Safe Design Group, CRAN

$^2$CNRS, CRAN, UMR 7039

$^3$Department of Electronic Systems, CENIDET, México

25 June 2014
References I


