Model-Based Fault Diagnosis Observer Design for Descriptor Linear Parameter Varying System with Unmeasurable Gain Scheduling Functions

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25 June 2014

National Center of Research and Technological Development

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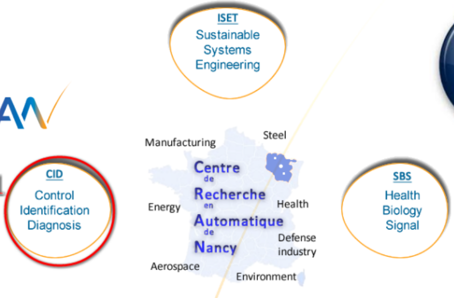
CRAN organization



- Focus: Modeling and identification, analysis, control and observation, and diagnosis.

Introduction

AZF Toulouse
France

Three Mile Island - USA

Bhopal Union Carbide
India

Flight Rio_Paris
AF447 – A330

BP Mexico Gulf - USA

Dependability and safety

In order to respect the growing of economic demand for high plant availability, and system safety, dependability is becoming an essential need in industrial automation

DLP-Systems

Modeling as:

- Nonlinear system

ODE

$$\dot{x} = F(x, u, t), \qquad y = g(x, u, t),$$

DAE

$$0 = F(\dot{x}, x, u, t) \qquad y = g(x, u, t)$$

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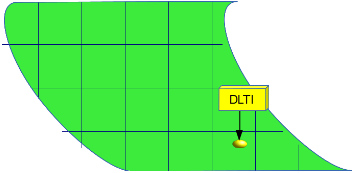
- Descriptor LTI (DLTI)

$$\begin{aligned} Ex(t) &= Ax(t) + Bu(t) \qquad (1) \\ y(t) &= Cx(t) \end{aligned}$$



Singular Matrix:

$$\tilde{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Ordinary Differential Equations (Dynamic)
Algebraic Equations (Static)

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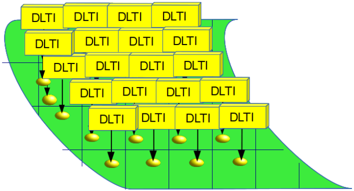
- Descriptor LTI (DLTI)

$$Ex(t) = Ax(t) + Bu(t) \qquad (1)$$
$$y(t) = Cx(t)$$

- Descriptor LPV (Polytopic)

$$E\dot{x}(t) = \sum_{i=1}^{h} \rho_i(x(t))\,[A_i x(t) + B_i u(t)] \qquad (2)$$

$$y(t) = Cx(t)$$



DLP-Systems

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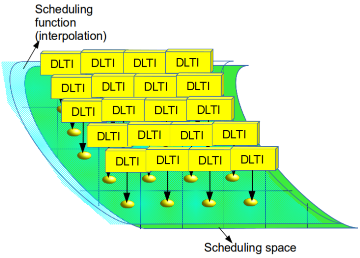


Convex Scheduling Functions

$$\forall i \in [1, 2..., h], \ \rho_i(x(t)) \geq 0 \ (3)$$
$$\sum_{i=1}^{h} \rho_i(x(t)) = 1, \ \forall t$$

$x(t)$ is unmeasurable.

Application: Flight vehicle control

$$E\dot{x} = \sum_{i=1}^{h} \rho_i(x(t)) \left[A_i x(t) + B_i u(t)\right]$$

with

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & F_0 + VF_V & VI & \\ 0 & -a & 0 & 0 \\ 0 & VF_2 & -I & 0 \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0 \\ 0 \\ a \\ 0 \end{bmatrix}, \; E = \mathrm{diag}(I_4, 0)$$

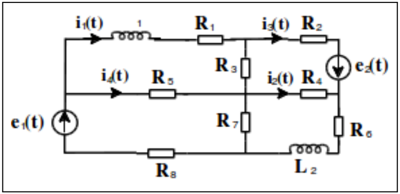
[1] The model is valid for airspeed $45 \leq V \leq 80\mathrm{m/s}$

[1] Masubuchi et al. [Gain-scheduled controller design based on descriptor representation of LPV systems: Application to flight vehicle control 2004, 43rd IEEE Conference on Decision and Control (CDC) (2004).]

Application: Simple Electrical Circuit

$$E\dot{x}(t) = \sum_{i=1}^{h} \rho_i(x(t)) \left[A_i x(t) + B_i u(t)\right]$$

with



$$A_i = \begin{bmatrix} \frac{-R_1+\theta_1(t)}{L_1} & 0 & \frac{R_{13}}{L_1} & \frac{R_{14}}{L_1} \\ 0 & -\frac{R_{22}+\theta_2}{L_2} & \frac{R_{23}}{L_2} & \frac{R_{24}}{L_2} \\ R_{31} & R_{32} & -R_{33} & 0 \\ R_{41} & R_{42} & 0 & -R_{44} \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \; E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

R_2 and R_6 are variable resistors of $[-0.5, 0.5]\Omega$ and $[-1, 1]\Omega$, respectively [2]

[2] Rodrigues et al. [Fault Diagnosis Based on Adaptive Polytopic Observer for LPV Descriptor Systems 2012, Safeprocess, 2012]

Application: Anaerobic Bio-reactor

$$E\dot{x} = \sum_{i=1}^{h} \rho_i(x(t)) \left[A_i x(t) + B_i u(t)\right]$$

with

$$A = \begin{bmatrix} Y_1 x_4 - \alpha D - k_d & 0 & 0 & Y_1 x_1 \\ -x_4 & -D & 0 & -x_1 \\ (1-Y_1) Y_{CH4} x_4 & 0 & -1 & 0 \\ 0 & \frac{\partial x}{\partial x_4} & 0 & -1 \end{bmatrix}$$

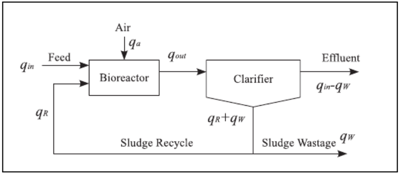
$$B = \begin{bmatrix} -\alpha x_1 & 0 \\ s_1^i - s_1 & D \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \; E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

with

$$\frac{\partial x}{\partial x_4} = \frac{I_{pH} k_{m1} k_{s1}}{(k_{s1} + s_1)^2}$$

The DLPV system is obtained by linearization in different operation points

Application: Waste-water treatment plant

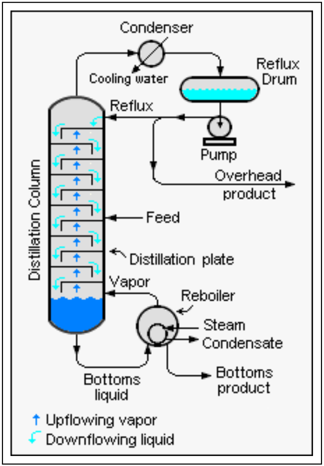


$$E\dot{x} = \sum_{i=1}^{16} \rho_i(x(t)) \left[A_i x(t) + B_i u(t) + B_{di} d(t)\right]$$

Sixteen modes are obtained by considering the sector non-linearity approach [3]

[3] Nagy Kiss et al. [State estimation of two-time scale multiple models. Application to wastewater treatment plant 2011, Control Engineering Practice (2011)]

Application: Waste-water treatment plant



$$E\dot{x} = \sum_{i=1}^{4} \rho_i(x(t)) \left[A_i x(t) + B_i u(t) + B_{di} d(t)\right]$$

Four modes are obtained by considering parameter variations on the liquid molar flow and volume [4]

[4] Aguilera-González et al. [Singular linear parameter-varying observer for composition estimation in a binary distillation column 2013]

Application to sensor fault estimation

Consider a LPV system under sensor faults and disturbances given by

$$\mathscr{F}_{\sim} := \begin{cases} \dot{\tilde{x}}(t) & = \sum_{i=1}^{h} \rho_i(\tilde{x}(t)) \left[\tilde{A}_i \tilde{x}(t) + \tilde{B}_i u(t) + \tilde{B}_d d(t) \right] \\ y(t) & = \tilde{C}\tilde{x}(t) + \tilde{D}_d d(t) + f(t) \end{cases} \tag{4}$$

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An augmented system $x(t) = \begin{bmatrix}\tilde{x}^T(t) & f^T(t)\end{bmatrix}^T$, *can be obtained as*

$$F_x := \begin{cases} E\dot{x}(t) = \sum_{i=1}^{h} \rho_i \left[A_i x(t) + B_i u(t) + B_d d(t) + B_f f(t)\right] \\ y(t) = Cx(t) + D_d d(t) \end{cases} \quad (5)$$

where:

$$E = \begin{bmatrix} I & 0 \\ 0 & 0_p \end{bmatrix}, \; A_i = \begin{bmatrix} \tilde{A}_i & 0 \\ 0 & -I_p \end{bmatrix}, \; B_i = \begin{bmatrix} \tilde{B}_i \\ 0_p \end{bmatrix}$$

$$B_d = \begin{bmatrix} \tilde{B}_d \\ 0 \end{bmatrix}, \; B_f = \begin{bmatrix} 0 \\ I_p \end{bmatrix}, \; C = \begin{bmatrix} \tilde{C} & I_p \end{bmatrix}, \; D_d = \tilde{D}_d.$$

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Consider the following FD observer

$$\mathscr{O} := \begin{cases} \dot{z}(t) = & \sum_{j=1}^{h} \rho_j(\hat{x}(t)) \left[N_j z(t) + G_j u(t) + L_j y(t)\right] \\ \hat{x}(t) & = z(t) + T_2 y(t) \\ r(t) & = y(t) - C\hat{x}(t) \end{cases} \quad (6)$$

The problem is reduced to find N_j, G_j, L_j, T_2 (5) such that observer can estimate the states and the augmented fault vector.

Objective

Detec and isolate sensor faults in nonlinear systems modeled as descriptor-linear parameter varying systems (D-LPV) with unmeasurable gain scheduling functions.

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Detec and isolate sensor faults in nonlinear systems modeled as descriptor-linear parameter varying systems (D-LPV) with unmeasurable gain scheduling functions.

Impact

1. The unmeasurable scheduling problem has been addressed in the literature mainly for state-space LPV systems, e.i. Ichalal et al. [2008, 2010a], Nagy-Kiss et al. [2011].
2. The design of observers for descriptor systems is more complex due to the fact that the observer must be designed to be robustly stable, regular and impulse-free:,
3. We have considered the problem of D-LPV with unmeasurable scheduling functions while more of the reported works are consider the measurable scheduling problem
4. As result of the problem formulation, a solution is provided based on novels LMI equations.

Problem formulation

The D-LPV system

$$\mathscr{F} = \begin{cases} E\dot{x}(t) = \sum_{i=1}^{h} \rho_i(x(t))\left[A_i x(t) + B_i u(t) + B_{di} d(t)\right] \\ y(t) = Cx(t) \end{cases} \tag{7}$$

Assumptions

- System (7) is admissible
- System (7) is R-observable
- System (7) is I-Observable

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Fault diagnosis observer

$$\mathscr{O} = \begin{cases} \dot{z}(t) = & \sum_{j=1}^{h} \rho_j(\hat{x}(t)) \left[N_j z(t) + G_j u(t) + L_j y(t)\right] \\ \hat{x}(t) = & z(t) + T_2 y(t) \end{cases} \tag{8}$$

Residual vector

$$r(t) = M(y(t) - C\hat{x}(t)) \tag{9}$$

The problem is reduced to find N_j, G_j, L_j, and T_2 and M of (8) such that the residual converges to zero in fault free case and different in faulty case.

Problem formulation

The D-LPV system

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The problem is reduced to find N_j, G_j, L_j, and

The state error

$$e(t) = x(t) - \hat{x}(t) \approx 0$$

The dynamic of the state error

$$\dot{e}(t) = \sum_{i=1}^{h} \rho_i(x(t))\left[\mathscr{F}\right] - \sum_{j=1}^{h} \rho_j(\hat{x}(t))\left[\mathscr{O}\right]$$

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Notations

$\rho_i = \rho_i(x(t))$
$\hat{\rho}_j = \rho_j(\hat{x}(t))$

Residual vector

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The problem is reduced to find N_j, G_j, L_j, and T_2 and M of (8) such that the residual converges to zero in fault free case and different in faulty case.

Observer design: Uncertain error system

The state error is given by

$$e(t) = x(t) - \hat{x}(t) = (I - T_2 C)x(t) - z(t)$$

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The error system becomes

$$\dot{e}(t) = \sum_{i=1}^{h} \rho_i T_1 \left[A_i x(t) + B_i u(t) + B_d d(t)\right] - \sum_{i=1}^{h} \hat{\rho}_i \left[N_i T_1 E x(t) - N_i e(t) + G_i u(t) + L_i y(t)\right] \quad (10)$$

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By considering the convex property of the scheduling functions, the term $-\sum_{i=1}^{h} \hat{\rho}_i N_i T_1 E x(t)$ can be handled as

$$\sum_{i=1}^{h} \rho_i \left[\sum_{j=1}^{h} \left[(\rho_j - \hat{\rho}_j)\right] N_j T_1 E\right] x(t) - \sum_{i=1}^{h} \rho_i N_i T_1 E x(t), \tag{11}$$

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Gain synthesis

$$\begin{aligned} G_j &= T_1 B_j, \\ N_j &= T_1 A_j + K_j C \\ K_j &= N_j T_2 - L_j, \end{aligned}$$

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} = \begin{bmatrix} I_n & 0 \end{bmatrix} \begin{bmatrix} E & B_d \\ C & 0 \end{bmatrix}^{\dagger}$$

The residual state error system becomes

$$\dot{e}(t) = \sum_{i=1}^{h} \rho_i \left\{ N_i e(t) + \sum_{j=1}^{h} \left[(\rho_j - \hat{\rho}_j)\right] \left[\left(T_1 A_j\right) x(t) + G_j u(t) - N_j e(t)\right] \right\} \tag{12}$$

$$r(t) = M(y(t) - C\hat{x}(t)) \tag{13}$$

Uncertain error system

The residual state-space error system is rewritten in compact form with augmented states $x_e(t) = \begin{bmatrix} e(t)^T & x(t)^T \end{bmatrix}^T$ as

$$\bar{E}\dot{x}_e(t) = \sum_{i=1}^{h} \rho_i \left[\left(\bar{A}_i + \Delta\bar{A}_i \right) x_e(t) + \left(\bar{B}_i + \Delta\bar{B}_i \right) u(t) \right] \qquad (14)$$

$$r(t) = \bar{C} x_e(t)$$

with

$$\bar{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & N_i \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} 0 & MC \end{bmatrix}$$

and, matrices $\Delta\bar{A}_i$ and $\Delta\bar{B}_i$ defined by

$$\Delta\bar{A}_i = H_A F_A \Phi_A, \ \Delta\bar{B}_i = H_B F_B \Phi_B,$$

with

$$H_A = \begin{bmatrix} [T_1 A_1 & \overset{0}{\dots} & T_1 A_h] & [N_1 & \overset{0}{\dots} & N_h] \end{bmatrix}, F_A = \begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix},$$

$$\Phi_A = \begin{bmatrix} I_A & 0 \\ 0 & -I_A \end{bmatrix}^T, I_A = \begin{bmatrix} I_{n_1} & \cdots & I_{n_h} \end{bmatrix}^T,$$

$$H_B = \begin{bmatrix} [G_1 & \overset{0}{\dots} & G_h] \end{bmatrix}, F_B = \begin{bmatrix} & F \end{bmatrix}, \Phi_B = I_B,$$

$$I_B = \begin{bmatrix} I_{m_1} & \cdots & I_{m_h} \end{bmatrix}^T, F = \begin{bmatrix} (\rho_1 - \hat{\rho}_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & (\rho_h - \hat{\rho}_h) \end{bmatrix}$$

Uncertain error system

The residual state-space error system is rewritten in compact form with augmented states $x_e(t) = \begin{bmatrix} e(t)^T & x(t)^T \end{bmatrix}^T$ *as*

$$\begin{aligned} \bar{E}\dot{x}_e(t) &= \sum_{i=1}^{h} \rho_i \left[\left(\bar{A}_i + \Delta\bar{A}_i \right) x_e(t) + \left(\bar{B}_i + \Delta\bar{B}_i \right) u(t) \right] \\ r(t) &= \bar{C} x_e(t) \end{aligned} \tag{14}$$

Solution is obtained by considering the H_∞ performance criterion

$$J_1 = \text{Performance} + \text{Asymptotic stability}$$

$$J_1 = r^T(t)r(t) - \gamma^2 u^T(t)u(t) + V(x_e(t)) \leq 0$$

γ is the attenuation level.

Theorem 1 (To be presented in World IFAC 2014.)

There exists a robust state estimation observer (8) for the D-LPV system (7) with H_∞ attenuation level $\gamma > 0$, if there exist scalars $\epsilon_A > 0$, $\epsilon_B > 0$, matrices $X = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix}$ with $P > 0$, $Q = Q^T > 0$, and gain matrices $K_i = Q^{-1}\Xi_i$, $\forall i \in [1, 2, ..., h]$, such that there exists a solution to the following optimization problem:

$$\min_{P,\, Q,\, \Xi_i,\, \epsilon_A,\, \epsilon_B} \quad \gamma$$

$$s.t.$$

$$E^T P = P^T E > \quad 0 \tag{15}$$

$$\left[\begin{array}{cccccccccc}
\mathrm{He}\{A_i^T P\} & 0 & PB_i & 0 & 0 & \epsilon_A I_A^T & 0 & 0 & 0 & 0 \\
* & \mathrm{He}\left\{(T_1 A_i)^T Q + C\Xi_i\right\} & 0 & [QT_1A_1 \ldots QT_1A_h] & [QT_1A_1 + \Xi_1 C \ldots QT_1A_h + \Xi_h C] & 0 & -\epsilon_A I_A^T & [QG_1 \ldots QG_h] & 0 & (MC)^T \\
* & * & -\gamma^2 I_m & 0 & 0 & 0 & 0 & 0 & \epsilon_B \Phi_B^T & 0 \\
* & * & * & -\epsilon_A I_{n\times h} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\epsilon_A I_{n\times h} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\epsilon_A I_{n\times h} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\epsilon_A I_{n\times h} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\epsilon_B I_{m\times h} & 0 & 0 \\
* & * & * & * & * & * & * & * & -\epsilon_B I_{m\times h} & 0 \\
* & * & * & * & * & * & * & * & * & -I_p
\end{array}\right] \leq 0 \tag{16}$$

Illustrative example

Considers the following descriptor-LPV system under disturbances as:

$$E\dot{x}(t) = \sum_{i=1}^{3} \rho_i(x(t)) \left[A_i x(t) + B_i u(t) + B_{di} d(t)\right]$$
$$y(t) = Cx(t) + D_d d(t) + f(t)$$

with: $E = \text{diag}(1, 1, 0)$,

$$A_1 = \begin{bmatrix} -10 & 5 & 6.5 \\ 2 & -5.5 & -1.25 \\ -9 & 4 & 8.5 \end{bmatrix},\ A_2 = \begin{bmatrix} -10 & 5 & 6.5 \\ 5 & -4 & -1.25 \\ -2 & 4 & 7 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -8 & 5 & 6.5 \\ 5 & -4 & -1.25 \\ -5 & 4 & 6 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ 1 \\ 0.5 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 0.7 \\ 1 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0 \\ 0.5 \\ 0.6 \end{bmatrix},\ B_{d1} = \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix},\ B_{d2} = \begin{bmatrix} 0.5 \\ 0 \\ 0.5 \end{bmatrix},\ B_{d3} = \begin{bmatrix} 0.2 \\ 0 \\ 0.5 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix},\ D_d = \begin{bmatrix} 0.5 \\ 0.3 \\ 0 \end{bmatrix}$$

Scheduling functions

$$\rho_i(x(t)) = \frac{\mu_i(x(t))}{\sum_{i=1}^{3} \mu_i(x)} \tag{17}$$

$$\mu_1(x) = exp\left[\frac{1}{2}\left(\frac{x_3(t) + 0.4}{0.5}\right)^2\right]$$

$$\mu_2(x) = exp\left[\frac{1}{2}\left(\frac{x_3(t) - 0.4}{0.1}\right)^2\right]$$

$$\mu_3(x) = exp\left[\frac{1}{2}\left(\frac{x_3(t) - 1}{0.5}\right)^2\right]$$

Models=3; Outputs=3; States=3; x is unmeasurable.

Simulation conditions

Simulation conditions

Fault-free case

$$
\begin{aligned}
x(0) &= [0,\ 1, -1]^T \\
\hat{x}(0) &= [0.1,\ -2,\ -3]^T \\
u(t) &= \sin(t)
\end{aligned}
$$

$d(t)$ is chosen as zero-mean noise with standard deviation 0.3.

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Faulty-case

$$f_1 = \begin{cases} \sin(t) & \text{on } y_1 \quad 10s \le t \le 20s \\ 0 & \text{otherwise} \end{cases}$$

$$f_2 = \begin{cases} -1.5 & \text{on } y_2 \quad 30s \le t \le 35s \\ 0 & \text{otherwise} \end{cases}$$

$$f_3 = \begin{cases} -0.05t - 2 & \text{on } y_3 \quad 40s \le t \le 60s \\ 0 & \text{otherwise} \end{cases}$$

Fault free scenario:Estimation errors

The gains of the observers are found by solving Theorem 1 with $\gamma = 2.4 \times 10^{-4}$.

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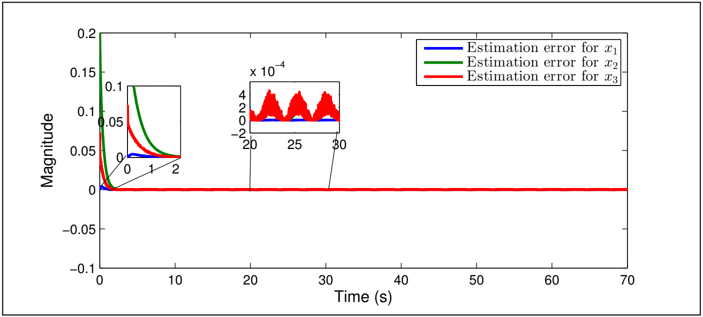


Figure : Estimation errors e_x. a) e_{x_1}, b) e_{x_2}, c) e_{x_3}.

Estimated gain scheduling functions

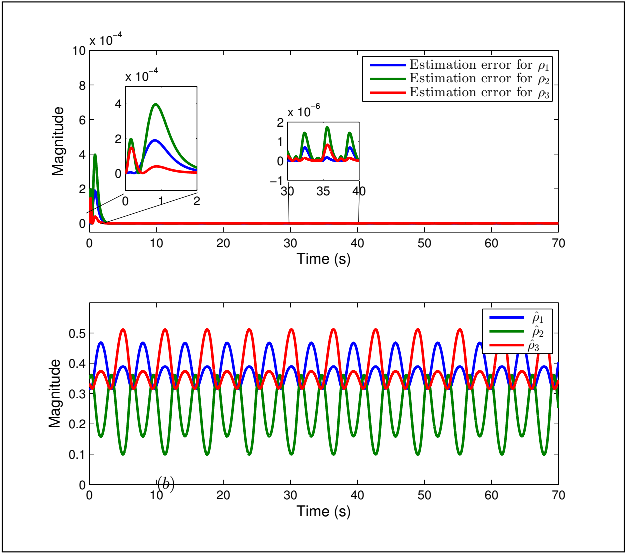


Figure : Gains scheduling functions errors and estimated scheduling functions.

Fault scenario

Figure : Generalized observer scheme (GOS)

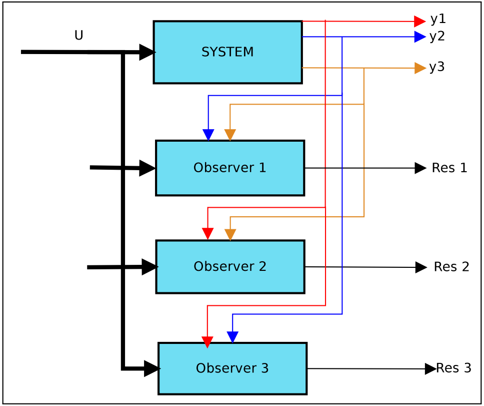
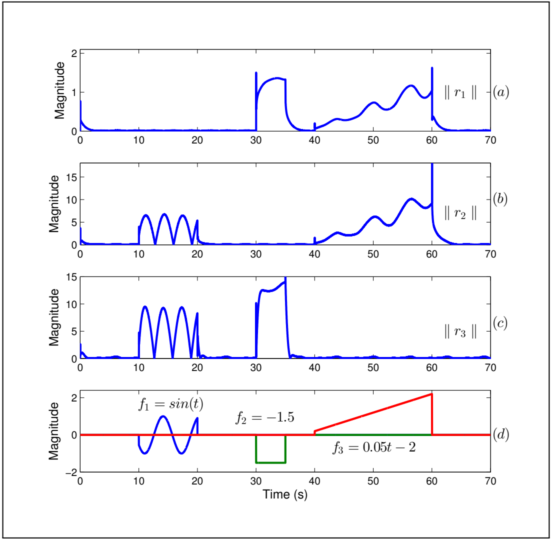


Table : Performance comparison

Method	Obsv 1	Obsv 2	Obsv 3
γ	3.5683×10^{-4}	3.022×10^{-4}	3.087×10^{-4}

Fault scenario:Residuals



Conclusions

- A method to design fault detection observers for D-LPV systems based H_∞ performance was proposed.
- The main challenge is to deal with disturbances and the error provide by the unmeasurable scheduling functions.
- Sufficient conditions to guaranteed the criterion performance were given in terms of LMIs.
- The approach presents good performance and gives a good trademark between disturbance attenuation and residual magnitude.
- Clearly, the method can deal successfully with the unmeasurable scheduling problem and is useful for fault diagnosis.
- Other approaches can be consulted in

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Model-Based Fault Diagnosis Observer Design for Descriptor Linear Parameter Varying System with Unmeasurable Gain Scheduling Functions

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25 June 2014

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