Placement de capteurs pour le recouvrement de l'observabilité des systèmes à commutations

Sinuhé MARTINEZ MARTINEZ[1,2]
Nadhir MESSAI[1], Noureddine MANAMANNI[1],
Taha BOUKHOBZA[2], Frédéric HAMELIN[2]

CReSTIC[1] - CRAN[2]

Réunion GT S3 - 18 janvier 2013

CReSTIC   GIS 3SGS   CRAN

Outline

Structural analysis?

$$\text{Approaches} \begin{cases} \text{Algebraic} \rightarrow \text{rank of pencil matrices} \\ \text{Gometric} \rightarrow \text{dimension of vectorial subspaces} \\ \text{Structural} \rightarrow \text{structural properties} \end{cases}$$

STRUCTURAL APPROACH

- Systems in state space or transfer function representation
- Entries of matrices are fixed to zero or free non-zero parameters

Advantages

- Systems non specified numerically, in the stage of design or uncertain systems
- Properties are structural and generic

Covered problems

- Basic properties of systems (controllability, observability, invariant zeros, subspace dimensions, ...)
- Classical problems of the control theory (input-output decoupling, disturbance rejection, detection and locations of faults, sensor placement, ...)

Structural analysis for linear systems

We study the systems in the state space form:

$$\Sigma_\Lambda = \begin{cases} \dot{x}(t) & = & A_\lambda x(t) + B_\lambda u(t) \\ y(t) & = & C_\lambda x(t) + D_\lambda u(t) \end{cases}$$

$x(t) \in \mathbb{R}^n$ state vector,
$u(t) \in \mathbb{R}^m$ input vector
$y(t) \in \mathbb{R}^p$ output vector

- $A_\lambda, B_\lambda, C_\lambda$ and D_λ are real matrices,
- the only knowledge is whether each entry is fixed to zero or an unknown real value represented by a parameter λ_i,
- the vector of such parameters is $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_h\}$ and it can take any value in $\mathbb{R}^h$,
- a property is true generically for (Σ^Λ) if it is true for almost all parameters values λ_i.

Graphic representation of a structured linear system

Structured Linear System		Directed Graph
Σ_Λ	$\rightarrow$	$\mathcal{G}(\Sigma_\Lambda) = (\mathcal{V}, \mathcal{E})$

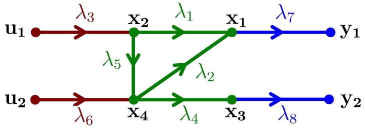
$\mathcal{V} = \mathbf{X} \cup \mathbf{U} \cup \mathbf{Y} \Rightarrow$ Vertex subset

$\mathcal{E} = A\text{-edges} \cup B\text{-edges} \cup C\text{-edges} \cup D\text{-edges} \Rightarrow$ Edge subset

for every matrix $M \Rightarrow M$-edges $= \{(\mathbf{v_j}, \mathbf{v_i}) \mid M(i,j) \neq 0\}$ où $\mathbf{v_j}$ is the beginning vertex and $\mathbf{v_i}$ is the end vertex

Example

$$
\begin{aligned}
\dot{x}_1 &= \lambda_1 x_2 + \lambda_2 x_4 \\
\dot{x}_2 &= \lambda_3 u_1 \\
\dot{x}_3 &= \lambda_4 x_4 \\
\dot{x}_4 &= \lambda_5 x_2 + \lambda_6 u_2 \\
y_1 &= \lambda_7 x_1 \\
y_2 &= \lambda_8 x_3
\end{aligned}
$$



Generic controllability for SISO Systems

Lin identified two structural forms where the system is not generically controllable ([Lin, 1974]):

Form I. $A = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}; b = \begin{bmatrix} 0 \\ b_2 \end{bmatrix}$

Example of Form I:

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}; b = \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix}$$

which corresponds to a graph of type

Form II. $(Ab) = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$

where P_2 is $(n-k) \times (n+1)$ and P_1 is $k \times (n+1)$ with $k \geq 1$ with not more than $k-1$ non-zero columns.

Example of Form II: $A = \begin{bmatrix} 0 & a_{12} & 0 \\ 0 & a_{22} & 0 \\ 0 & a_{32} & 0 \end{bmatrix}; b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

which corresponds to a graph of type

Generic controllability for SISO Systems

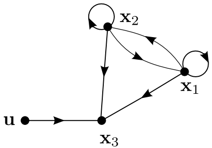
Lin identified two structural forms where the system is not generically controllable ([Lin, 1974]):

Form I. $A = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}; b = \begin{bmatrix} 0 \\ b_2 \end{bmatrix}$

Example of Form I:

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}; b = \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix}$$

which corresponds to a graph of type



Form II. $(Ab) = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$

where P_2 is $(n-k) \times (n+1)$ and P_1 is $k \times (n+1)$ with $k \geq 1$ with not more than $k-1$ non-zero columns.

Example of Form II: $A = \begin{bmatrix} 0 & a_{12} & 0 \\ 0 & a_{22} & 0 \\ 0 & a_{32} & 0 \end{bmatrix}; b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

which corresponds to a graph of type

Generic controllability for SISO Systems

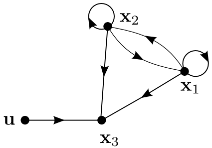
Lin identified two structural forms where the system is not generically controllable ([Lin, 1974]):

Form I. $A = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}; b = \begin{bmatrix} 0 \\ b_2 \end{bmatrix}$

Example of Form I:

$$A = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}; b = \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix}$$

which corresponds to a graph of type

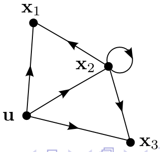


Form II. $(Ab) = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$

where P_2 is $(n-k) \times (n+1)$ and P_1 is $k \times (n+1)$ with $k \geq 1$ with not more than $k-1$ non-zero columns.

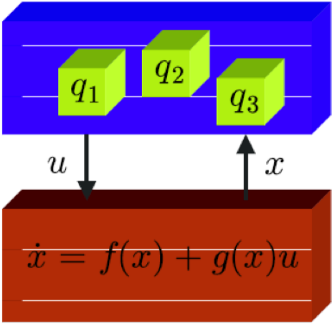
Example of Form II: $A = \begin{bmatrix} 0 & a_{12} & 0 \\ 0 & a_{22} & 0 \\ 0 & a_{32} & 0 \end{bmatrix}; b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

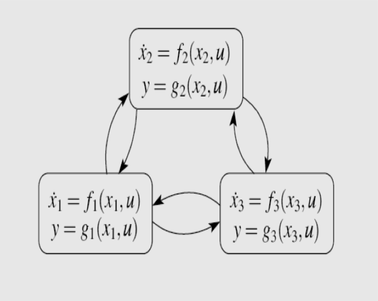
which corresponds to a graph of type



Switched Linear Systems

consist of finite number of continuous-time subsystems and a rule that orchestrates the switching among them [Savkin and Evans, 2002];





Observability recovering by switching: example

Let us consider the SLS with three modes

mode 1	mode 2	mode 3
$\dot{x}_1 = \lambda_1 x_3 + \lambda_2 x_4 + \lambda_3 x_5$	$\dot{x}_1 = 0$	$\dot{x}_1 = \lambda_1 x_3 + \lambda_2 x_4 + \lambda_3 x_5$
$\dot{x}_2 = \lambda_4 x_5$	$\dot{x}_2 = \lambda_5 x_5$	$\dot{x}_2 = \lambda_5 x_5$
$y_1 = \lambda_6 x_1 + \lambda_7 x_2$	$y_1 = \lambda_6 x_1 + \lambda_7 x_2$	$y_1 = \lambda_6 x_1 + \lambda_8 x_2$
$y_2 = 0$	$y_2 = \lambda_9 x_3$	$y_2 = \lambda_{10} x_3$

(1)

If the mode sequence is
mode 2 | mode 1 | mode 3

Observability recovering by switching: example

Let us consider the SLS with three modes

mode 1	mode 2	mode 3
$\dot{x}_1 = \lambda_1 x_3 + \lambda_2 x_4 + \lambda_3 x_5$	$\dot{x}_1 = 0$	$\dot{x}_1 = \lambda_1 x_3 + \lambda_2 x_4 + \lambda_3 x_5$
$\dot{x}_2 = \lambda_4 x_5$	$\dot{x}_2 = \lambda_5 x_5$	$\dot{x}_2 = \lambda_5 x_5$
$y_1 = \lambda_6 x_1 + \lambda_7 x_2$	$y_1 = \lambda_6 x_1 + \lambda_7 x_2$	$y_1 = \lambda_6 x_1 + \lambda_8 x_2$
$y_2 = 0$	$y_2 = \lambda_9 x_3$	$y_2 = \lambda_{10} x_3$

(1)

If the mode sequence is

mode 2	mode 1	mode 3
$y_{1,2} \Rightarrow f_1(x_1, x_2)$ $\dot{y}_{1,2} \Rightarrow f_2(x_5)$		
$y_{2,2} \Rightarrow f_3(x_3)$		

Observability recovering by switching: example

Let us consider the SLS with three modes

mode 1	mode 2	mode 3
$\dot{x}_1 = \lambda_1 x_3 + \lambda_2 x_4 + \lambda_3 x_5$	$\dot{x}_1 = 0$	$\dot{x}_1 = \lambda_1 x_3 + \lambda_2 x_4 + \lambda_3 x_5$
$\dot{x}_2 = \lambda_4 x_5$	$\dot{x}_2 = \lambda_5 x_5$	$\dot{x}_2 = \lambda_5 x_5$
$y_1 = \lambda_6 x_1 + \lambda_7 x_2$	$y_1 = \lambda_6 x_1 + \lambda_7 x_2$	$y_1 = \lambda_6 x_1 + \lambda_8 x_2$
$y_2 = 0$	$y_2 = \lambda_9 x_3$	$y_2 = \lambda_{10} x_3$

(1)

If the mode sequence is

mode 2	mode 1	mode 3
$y_{1,2} \Rightarrow f_1(x_1, x_2)$ $\dot{y}_{1,2} \Rightarrow f_2(x_5)$	$y_{1,1} \Rightarrow f_1(x_1, x_2)$ $\dot{y}_{1,1} \Rightarrow f_4(x_3, x_4, x_5)$	
$y_{2,2} \Rightarrow f_3(x_3)$	$\ddot{y}_{2,1} = 0$	

Observability recovering by switching: example

Let us consider the SLS with three modes

mode 1	mode 2	mode 3
$\dot{x}_1 = \lambda_1 x_3 + \lambda_2 x_4 + \lambda_3 x_5$	$\dot{x}_1 = 0$	$\dot{x}_1 = \lambda_1 x_3 + \lambda_2 x_4 + \lambda_3 x_5$
$\dot{x}_2 = \lambda_4 x_5$	$\dot{x}_2 = \lambda_5 x_5$	$\dot{x}_2 = \lambda_5 x_5$
$y_1 = \lambda_6 x_1 + \lambda_7 x_2$	$y_1 = \lambda_6 x_1 + \lambda_7 x_2$	$y_1 = \lambda_6 x_1 + \lambda_8 x_2$
$y_2 = 0$	$y_2 = \lambda_9 x_3$	$y_2 = \lambda_{10} x_3$

(1)

If the mode sequence is

mode 2	mode 1	mode 3
$y_{1,2} \Rightarrow f_1(x_1, x_2)$	$y_{1,1} \Rightarrow f_1(x_1, x_2)$	$y_{1,3} \Rightarrow f_5(x_1, x_2)$
$\dot{y}_{1,2} \Rightarrow f_2(x_5)$	$\dot{y}_{1,1} \Rightarrow f_4(x_3, x_4, x_5)$	$\dot{y}_{1,3} \Rightarrow f_6(x_3, x_4, x_5)$
$y_{2,2} \Rightarrow f_3(x_3)$	$\ddot{y}_{2,1} = 0$	$y_{2,3} = f_7(x_3)$

the SLS is globally observable

Structured Switched Linear Systems (SSLS)

Switched Linear System — Directed Graph

$$\Sigma^{\Lambda} : \begin{cases} \dot{x}(t) = & A(q)x(t), \quad x(t_0) = x_0 \\ y(t) = & C(q)x(t) \end{cases} \quad \Rightarrow \quad \mathcal{G}(\Sigma_{\Lambda}) = (\mathcal{V}, \mathcal{E}) \tag{2}$$

- A parameter can be common to all modes or to some modes only,
- subset of parameters can be common

Matrix decomposition

$$A(1) = \begin{bmatrix} 0 & 0 & \lambda_1 & \lambda_2 & \lambda_3 \\ 0 & 0 & 0 & 0 & \lambda_4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad A(2) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad A(3) = \begin{bmatrix} 0 & 0 & \lambda_1 & \lambda_2 & \lambda_3 \\ 0 & 0 & 0 & 0 & \lambda_5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

For the general case, using binary notation, matrix $A(q)$ can be spread out for mode 1 as:

$$A(1) = \underbrace{A_{0,0,1}}_{\text{specific elements to mode 1}} + \underbrace{A_{0,1,1}}_{\text{common elements between mode 1 and mode 2}} + \underbrace{A_{1,0,1}}_{\text{common elements between mode 1 and mode 3}} + \underbrace{A_{1,1,1}}_{\text{common elements in 3 modes}}$$

For the particular case of matrix $A(1)$ of equation (3),
$A(1) = A_{0,0,1} + A_{1,0,1}$

Matrix decomposition

$$A(1) = \begin{bmatrix} 0 & 0 & \lambda_1 & \lambda_2 & \lambda_3 \\ 0 & 0 & 0 & 0 & \lambda_4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad A(2) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad A(3) = \begin{bmatrix} 0 & 0 & \lambda_1 & \lambda_2 & \lambda_3 \\ 0 & 0 & 0 & 0 & \lambda_5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

For the general case, using binary notation, matrix $A(q)$ can be spread out for mode 2 as:

$$A(2) = \underbrace{A_{0,1,0}}_{\text{specific elements to mode 2}} + \underbrace{A_{0,1,1}}_{\text{common elements between mode 1 and mode 2}} + \underbrace{A_{1,1,0}}_{\text{common elements between mode 2 and mode 3}} + \underbrace{A_{1,1,1}}_{\text{common elements in 3 modes}}$$

For the particular case of matrix $A(2)$ of equation (3),

$A(1) = A_{0,0,1} + A_{1,0,1} \quad A(2) = A_{1,1,0}$

Matrix decomposition

$$A(1) = \begin{bmatrix} 0 & 0 & \lambda_1 & \lambda_2 & \lambda_3 \\ 0 & 0 & 0 & 0 & \lambda_4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad A(2) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad A(3) = \begin{bmatrix} 0 & 0 & \lambda_1 & \lambda_2 & \lambda_3 \\ 0 & 0 & 0 & 0 & \lambda_5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

For the general case, using binary notation, matrix $A(q)$ can be spread out for mode 3 as:

$$A(2) = \underbrace{A_{1,0,0}}_{\text{specific elements to mode 3}} + \underbrace{A_{1,0,1}}_{\text{common elements between mode 1 and mode 3}} + \underbrace{A_{1,1,0}}_{\text{common elements between mode 2 and mode 3}} + \underbrace{A_{1,1,1}}_{\text{common elements in 3 modes}}$$

For the particular case of matrix $A(3)$ of equation (3),

$A(1) = A_{0,0,1} + A_{1,0,1}$ $\quad A(2) = A_{1,1,0}$ $\quad A(3) = A_{1,0,1} + A_{1,1,0}$

Graphic representation of SSLS

$$\begin{array}{c} \\ A_{1,0,1} = \end{array} \begin{array}{c} \begin{array}{ccccc} \mathbf{x_1} & \mathbf{x_2} & \mathbf{x_3} & \mathbf{x_4} & \mathbf{x_5} \end{array} \\ \begin{bmatrix} 0 & 0 & \lambda_1 & \lambda_2 & \lambda_3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Graphic representation of SSLS

$$
\begin{array}{ll}
 & \qquad\qquad\quad \mathbf{x_1} \quad \mathbf{x_2} \quad \mathbf{x_3} \quad \mathbf{x_4} \quad \mathbf{x_5} \\
\begin{array}{c}\mathbf{z_1}\\ \mathbf{z_2}\\ \mathbf{z_3}\\ \mathbf{z_4}\\ \mathbf{z_5}\end{array} & A_{1,0,1} = \begin{bmatrix} 0 & 0 & \lambda_1 & \lambda_2 & \lambda_3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{array}
$$

Graphic representation of SSLS

$$\begin{array}{ccc} & & \begin{array}{ccccc} \mathbf{x_1} & \mathbf{x_2} & \mathbf{x_3} & \mathbf{x_4} & \mathbf{x_5} \end{array} \\ \begin{array}{c} \mathbf{z_1} \\ \mathbf{z_2} \\ \mathbf{z_3} \\ \mathbf{z_4} \\ \mathbf{z_5} \end{array} & A_{1,0,1} = & \begin{bmatrix} 0 & 0 & \mathbf{w}^{\mathbf{1,0,1}}_{\mathbf{1,3}} & \mathbf{w}^{\mathbf{1,0,1}}_{\mathbf{1,4}} & \mathbf{w}^{\mathbf{1,0,1}}_{\mathbf{1,5}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Graphic representation of SSLS

$$\mathbf{v}_1^{1,0,1}$$

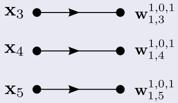
	$\mathbf{x_1}$	$\mathbf{x_2}$	$\mathbf{x_3}$	$\mathbf{x_4}$	$\mathbf{x_5}$
$\mathbf{z_1}$	0	0	$\mathbf{w}_{1,3}^{1,0,1}$	$\mathbf{w}_{1,4}^{1,0,1}$	$\mathbf{w}_{1,5}^{1,0,1}$
$\mathbf{z_2}$	0	0	0	0	0
$\mathbf{z_3}$	0	0	0	0	0
$\mathbf{z_4}$	0	0	0	0	0
$\mathbf{z_5}$	0	0	0	0	0

$$A_{1,0,1} = \begin{bmatrix} 0 & 0 & \mathbf{w}_{1,3}^{1,0,1} & \mathbf{w}_{1,4}^{1,0,1} & \mathbf{w}_{1,5}^{1,0,1} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Graphic representation of SSLS

$$
\begin{array}{l}
\mathbf{z_1} \\ \mathbf{z_2} \\ \mathbf{z_3} \\ \mathbf{z_4} \\ \mathbf{z_5}
\end{array}
\quad \mathbf{v_1^{1,0,1}} \quad
A_{1,0,1} =
\begin{array}{c}
\begin{array}{ccccc} \mathbf{x_1} & \mathbf{x_2} & \mathbf{x_3} & \mathbf{x_4} & \mathbf{x_5} \end{array} \\
\begin{bmatrix}
0 & 0 & \mathbf{w_{1,3}^{1,0,1}} & \mathbf{w_{1,4}^{1,0,1}} & \mathbf{w_{1,5}^{1,0,1}} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

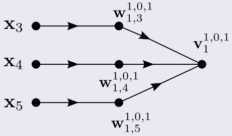
Digraph



Graphic representation of SSLS

$$\begin{array}{c} \mathbf{z_1} \\ \mathbf{z_2} \\ \mathbf{z_3} \\ \mathbf{z_4} \\ \mathbf{z_5} \end{array} \quad \mathbf{v}_1^{1,0,1} \leftarrow \quad A_{1,0,1} = \begin{array}{c} \begin{array}{ccccc} \mathbf{x_1} & \mathbf{x_2} & \mathbf{x_3} & \mathbf{x_4} & \mathbf{x_5} \end{array} \\ \begin{bmatrix} 0 & 0 & \mathbf{w}_{1,3}^{1,0,1} & \mathbf{w}_{1,4}^{1,0,1} & \mathbf{w}_{1,5}^{1,0,1} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Digraph

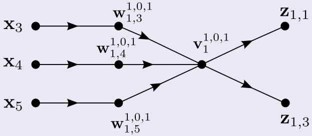


Graphic representation of SSLS

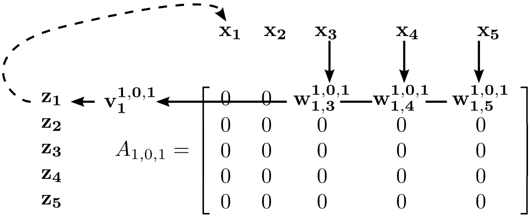
$\mathbf{x_1}$ $\mathbf{x_2}$ $\mathbf{x_3}$ $\mathbf{x_4}$ $\mathbf{x_5}$; $\mathbf{z_1} \leftarrow \mathbf{v_1^{1,0,1}}$; $\mathbf{z_2}$ $\mathbf{z_3}$ $\mathbf{z_4}$ $\mathbf{z_5}$

$$A_{1,0,1} = \begin{bmatrix} 0 & 0 & \mathbf{w_{1,3}^{1,0,1}} & \mathbf{w_{1,4}^{1,0,1}} & \mathbf{w_{1,5}^{1,0,1}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

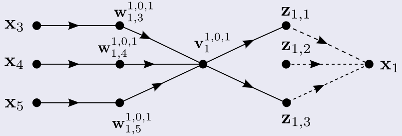
Digraph



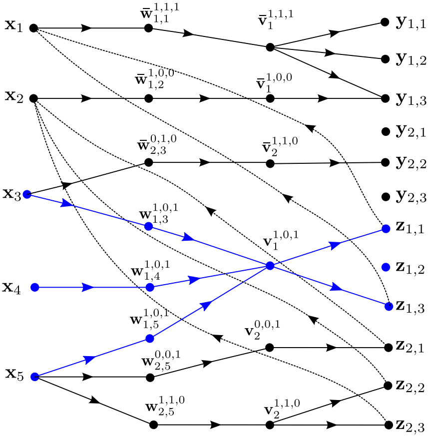
Graphic representation of SSLS



Digraph



Graphic representation of SSLS



Graphic conditions for observability

Necessary and sufficient conditions for the existence of sequences guaranteeing the observability of the SSLS Σ_{SLS}^{Λ} are given in [Martinez-Martinez et al., 2011].

$\Rightarrow$ For a SSLS Σ_{SLS}^{Λ} with associated graph $\mathcal{G}(\Sigma_{SLS}^{\Lambda})$ two conditions have to be verified:

- Connectivity and
- Maximal linking

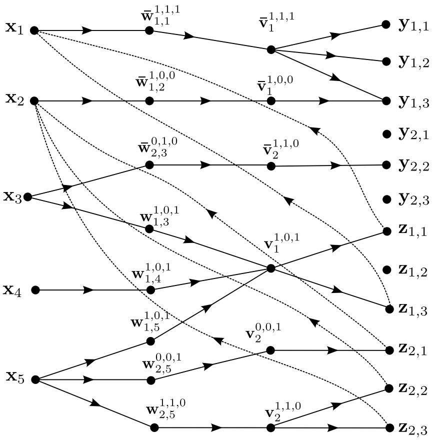
Connectivity condition

Connectivity

Every $\mathbf{x_i}$ is covered by a $\mathbf{Y}$-topped path

Some useful definitions

- Covered vertex → A vertex which forms part of some path,
- $\mathbf{Y}$-topped path → A path which final vertex belongs to $\mathbf{Y}$



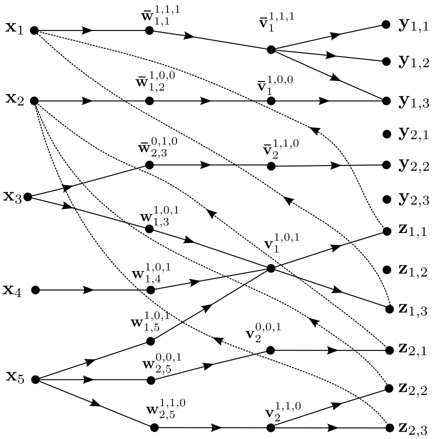
Maximal linking condition

Maximal linking

The maximal number of direct disjoint paths between $\mathbf{X}$ and $\mathbf{Y} \cup \mathbf{Z}$ is equal to n.

Some useful definitions

- Direct path $\rightarrow$ A path between vertex subset $\mathbf{X}$ and $\mathbf{Y} \cup \mathbf{Z}$
- Direct disjoint paths $\rightarrow$ Direct paths subset without common vertices



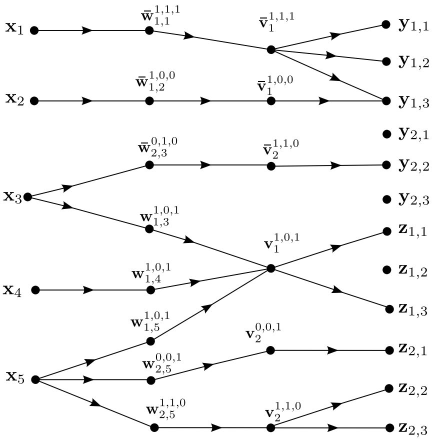
Maximal linking condition

Maximal linking

The maximal number of direct disjoint paths between $\mathbf{X}$ and $\mathbf{Y} \cup \mathbf{Z}$ is equal to n.

Some useful definitions

- Direct path $\rightarrow$ A path between vertex subset $\mathbf{X}$ and $\mathbf{Y} \cup \mathbf{Z}$
- Direct disjoint paths $\rightarrow$ Direct paths subset without common vertices



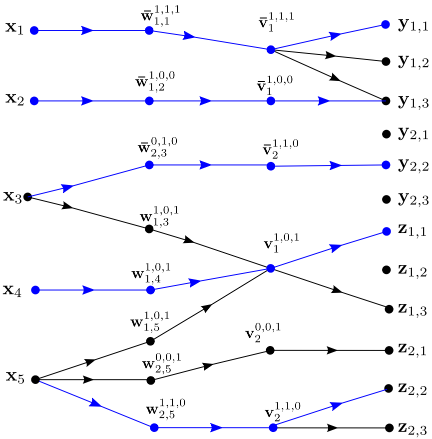
Maximal linking condition

Maximal linking

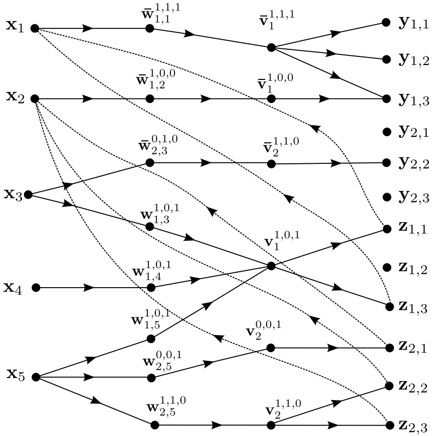
The maximal number of direct disjoint paths between $\mathbf{X}$ and $\mathbf{Y} \cup \mathbf{Z}$ is equal to n.

Some useful definitions

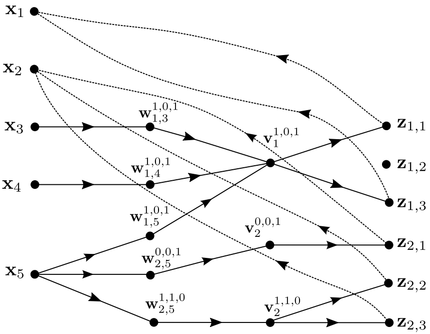
- Direct path → A path between vertex subset $\mathbf{X}$ and $\mathbf{Y} \cup \mathbf{Z}$
- Direct disjoint paths → Direct paths subset without common vertices



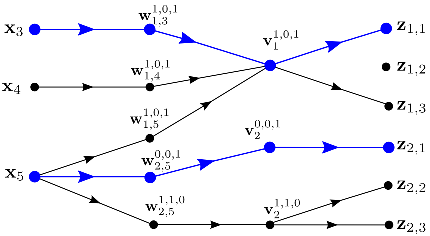
Observability conditions not verified



Observability conditions not verified



Observability conditions not verified



Observability recovering via sensor placement

Solution of the localisation and minimal number of sensor in order to recover de observability based on a completely graphic framework ([Martinez-Martinez et al., 2012])

- $\Rightarrow$ Sensor placement to recover the connectivity condition
- $\Rightarrow$ Sensor placement to recover maximal linking condition

How many sensors and where?

Sensor placement for recovering the connectivity condition

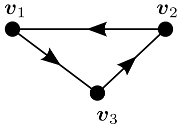
To recover the output connectivity condition, additional sensors must measure at least one state x_i in any mode for each **strongly connected component** constituting a **minimal unconnected element**.

Strongly connected components...

- equivalent elements
- a vertex is equivalent to itself

... and minimal unconnected components

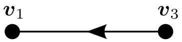
- Example
- Strongly connected components
- Partial order : $\mathbf{C}_1$ is the infimal unconnected element



Sensor placement for recovering the connectivity condition

To recover the output connectivity condition, additional sensors must measure at least one state x_i in any mode for each **strongly connected component** constituting a **minimal unconnected element**.

Strongly connected components...



- equivalent elements
- a vertex is equivalent to itself

... and minimal unconnected components

- Example
- Strongly connected components
- Partial order : $\mathbf{C}_1$ is the infimal unconnected element

Sensor placement for recovering the connectivity condition

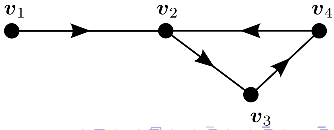
To recover the output connectivity condition, additional sensors must measure at least one state x_i in any mode for each **strongly connected component** constituting a **minimal unconnected element**.

Strongly connected components...

- equivalent elements
- a vertex is equivalent to itself

... and minimal unconnected components

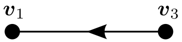
- Example
- Strongly connected components
- Partial order : C_1 is the infimal unconnected element



Sensor placement for recovering the connectivity condition

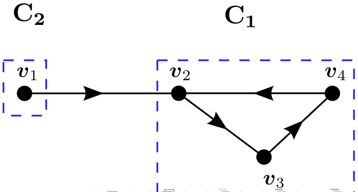
To recover the output connectivity condition, additional sensors must measure at least one state x_i in any mode for each **strongly connected component** constituting a **minimal unconnected element**.

Strongly connected components...



- equivalent elements
- a vertex is equivalent to itself

... and minimal unconnected components



- Example
- Strongly connected components
- Partial order : $\mathbf{C_1}$ is the infimal unconnected element

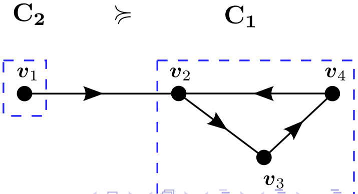
Sensor placement for recovering the connectivity condition

To recover the output connectivity condition, additional sensors must measure at least one state x_i in any mode for each **strongly connected component** constituting a **minimal unconnected element**.

Strongly connected components...

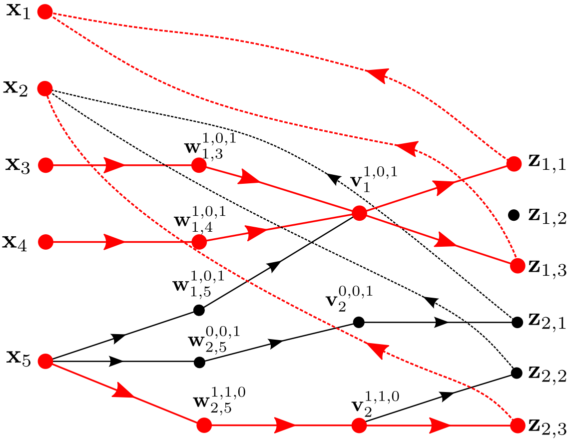
- equivalent elements
- a vertex is equivalent to itself

... and minimal unconnected components

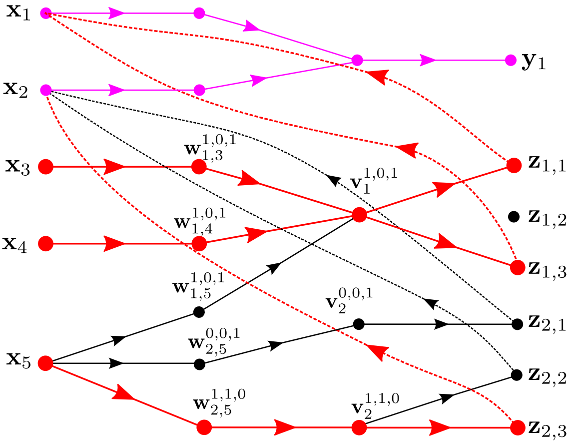


- Example
- Strongly connected components
- Partial order : $\mathbf{C_1}$ is the infimal unconnected element

Guaranteeing connectivity condition



Guaranteeing connectivity condition

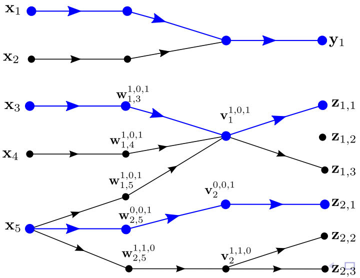


Sensor placement for recovering the maximal linking condition

Minimal number of sensors

To recover the maximal linking condition, the minimal number of additional sensors is equal to $n - \theta(\mathcal{V}^+, \mathcal{V}^-)$.

$\theta(\mathcal{V}^+, \mathcal{V}^-) \rightarrow$ number of edges in maximal subset of direct disjoint paths from $\mathcal{V}^+$ to $\mathcal{V}^-$



Sensor placement for recovering the maximal linking condition

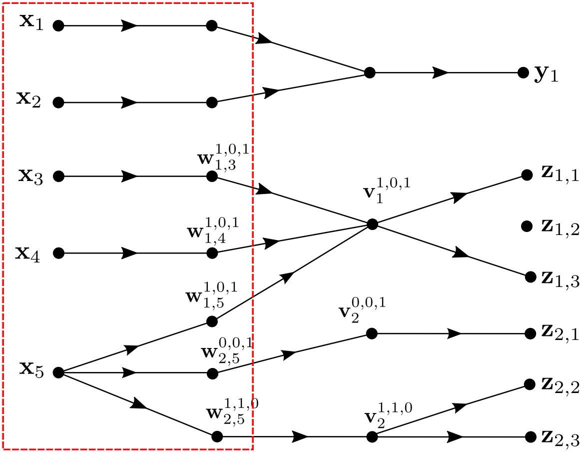
...but where?

localisation

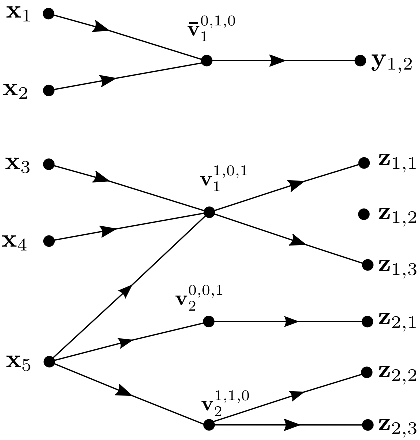
Such additional sensors must measure $n - \theta(\mathcal{V}^+, \mathcal{V}^-)$ states in $\mathcal{V}_0^+$ such that a maximal matching of size n is obtained in the bipartite graph $\mathcal{B}(\Sigma_{SLS}^{\Lambda})$.

Direct graph $\Rightarrow$ Bipartite graph

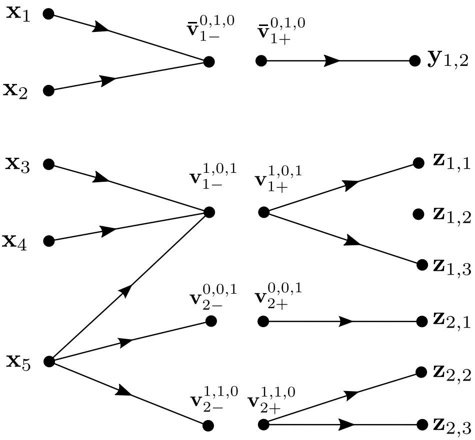
Simplifications



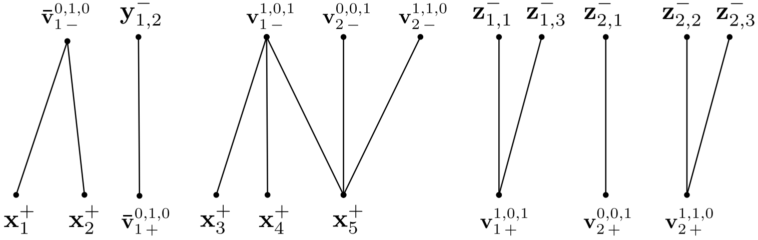
Simplifications



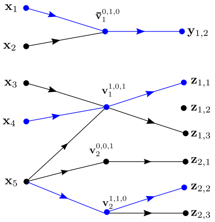
Bipartite graph



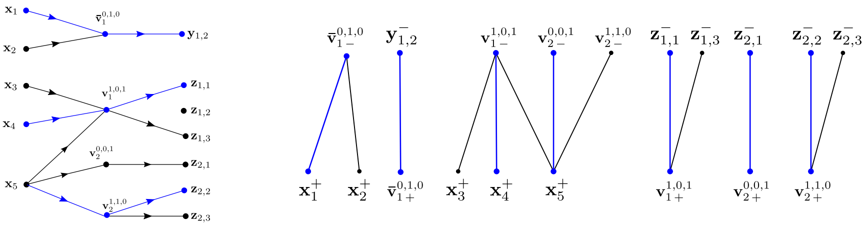
Bipartite graph



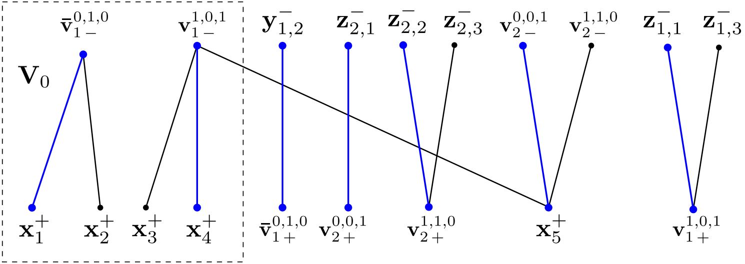
Guaranteeing maximal linking condition



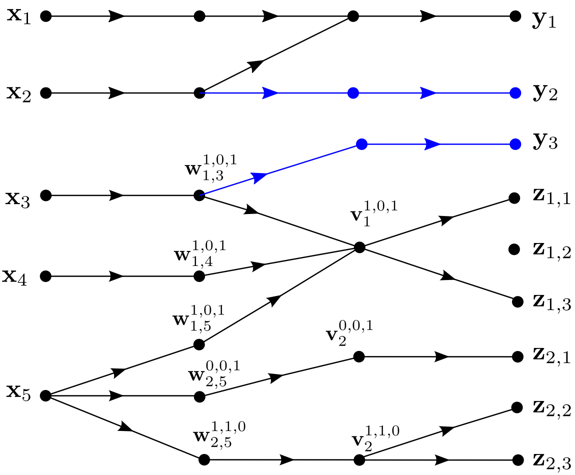
Guaranteeing maximal linking condition



Guaranteeing maximal linking condition



Guaranteeing maximal linking condition



Conclusions

Structural approach

- Sensor placement to recover the observability has been extended to the SLS case
- The strong relation between the structure and the observability of SLS has been shown

Graph approach

- A new graph which represents SSLS has been introduced
- Simple algorithms of graph theory are required to solve the problem of sensor placement (low complexity)
  - Strongly connected components
  - output connected paths
  - Disjoint paths
  - Maximal number of edges/vertices belonging to a particular path

Merci pour votre attention

Bibliography I

Lin, C. (1974).
Structural controllability.
IEEE Trans. on Automatic Control, AC 19:201– 208.

Martinez-Martinez, S., Messai, N., Hamelin, F., Manamanni, N., and Boukhobza, T. (2011).
Graphic approach for the determination of the existence of sequences guaranteeing observability of switched linear systems.
Internal repport CReSTIC, september 2011.

Martinez-Martinez, S., Messai, N., Manamanni, N., Boukhobza, T., and Hamelin, F. (2012).
Observability recovering by sensor placement for switched linear systems : a graphic approach.
In *SAFEPROCESS 2012*, Mexico.

Bibliography II

Savkin, A. V. and Evans, R. J. (2002).
Hybrid Dynamical Systems. Controller and Sensor Switching Problems.
Birkhauser, Boston, U.S.A.