Diagnostic de défaillances des systèmes avec la redondance des mesures : approche statistique

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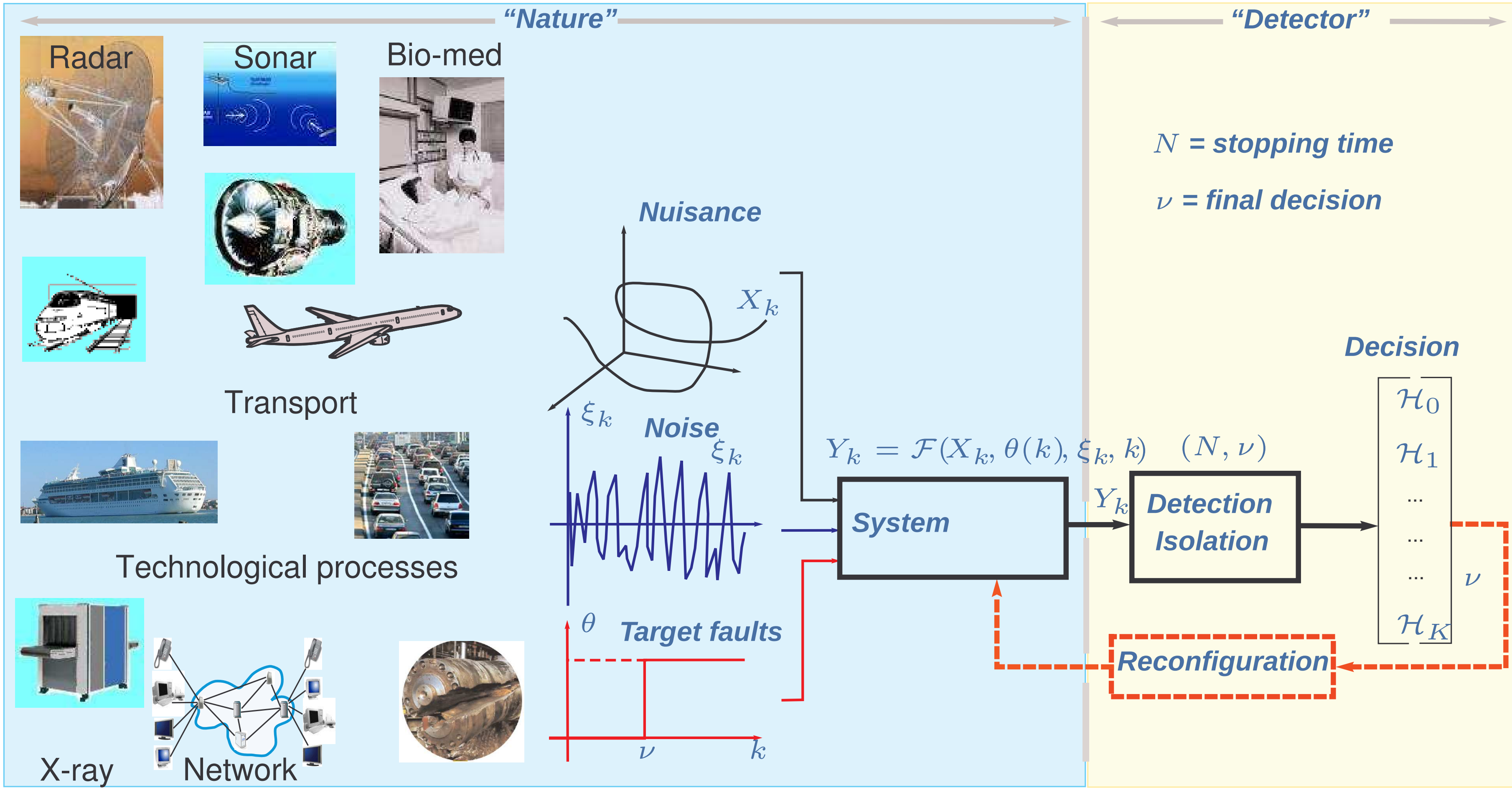
Plan

- Introduction and motivation
- Two inference problems for monitoring
- Hypotheses testing - nuisance parameters
- Off-line FDI with nuisance parameters
- Change diagnosis (detection/isolation)
- Example of application
- Integration of constraints in the decision-making process
- Conclusions

Introduction - Motivation

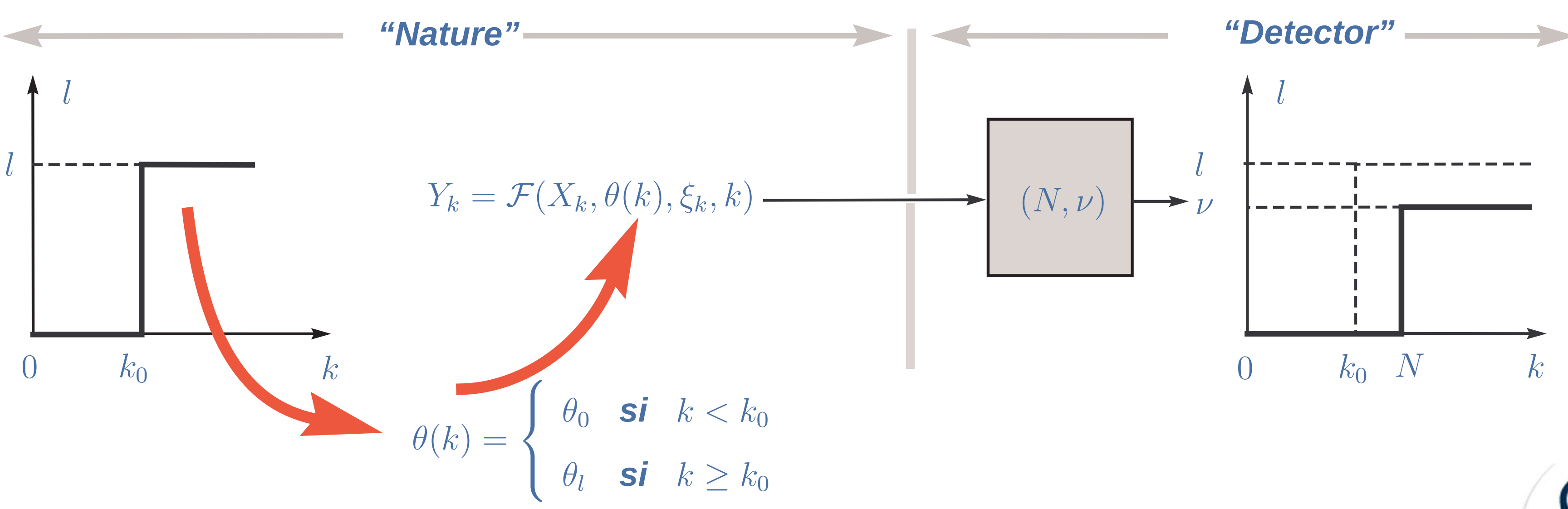


= *unfortunately, it will not be considered in this presentation*

Two inference problems for monitoring

Two following situations are distinguished :

- ***Hypotheses testing : the parameter vector θ is assumed to be*** **constant** ***within the entire data sample $Y_1, \ldots, Y_n$.***
- ***Change detection/isolation : the parameter θ can*** **change** ***within the data sample at an unknown instant (change point) k_0 $(1 \leq k_0 \leq n)$.***



- **Detection** refers to deciding whether the monitored system is in its nominal (safe) state or not :

$$\mathcal{H}_0 : \quad \theta \in \Theta_0, \ \overbrace{X \in \mathbb{R}^q}^{\textbf{nuisance}} \quad \textbf{against} \quad \mathcal{H}_1 \ : \quad \theta \in \Theta_0^c \triangleq \mathbb{R}^m \backslash \Theta_0, \ \overbrace{X \in \mathbb{R}^q}^{\textbf{nuisance}}$$

  If some more information about is available, $\mathcal{H}_1 \ : \quad \theta \in \Theta_1 \subset \Theta_0^c$.

- In case of two fault modes or more, **isolation** refers to deciding which fault mode occurred.

$$\mathcal{H}_0 \quad \textbf{against} \quad \mathcal{H}_i \ : \quad \theta \in \Theta_i \subset \mathbb{R}^m, \ \overbrace{X \in \mathbb{R}^q}^{\textbf{nuisance}} \ (i = 1, \ldots, K-1),$$

  where $\Theta_i \bigcap \Theta_j = \emptyset$ for $i \neq j$.

Informative and nuisance parameters

- *Statistical properties of the tests are functions of both the informative parameters θ and the nuisance parameters X. The desirable relations between the error probabilities or the power of a test and the informative vector θ usually result from the application.*

- *Sometimes, the statistician must define some additional constraints (possibly artificial w.r.t. the application) resulting from the statistical nature of the problem, in order to achieve optimal properties of the test.*

- *The main difference between θ and X is the following : in contrast to the informative parameter θ, the nuisance parameter X has no desirable impact on the performance indexes.*

Off-line FDI with nuisance parameters (1/2)

Static and dynamic models with faults and nuisance parameters

The measured data Y are viewed as the output of a discrete time system [Basseville 1997]

$$\textbf{dynamic} \left\{ \begin{array}{rclclcll} X_{k+1} & = & FX_k & + & GU_k & + & W_k & (+\ \Gamma\,\Upsilon_x) \\ Y_k & = & HX_k & + & JU_k & + & V_k & (+\ \Xi\,\Upsilon_y) \end{array} \right. ,$$

where $W_k \sim \mathcal{N}(0, Q_x)$, $V_k \sim \mathcal{N}(0, Q_y)$, Υ_x and Υ_y are the assumed additive faults, and the fault gains Γ and Ξ are full column rank matrices, or

$$\textbf{static} \quad Y = HX + \xi\ (+M\theta)\ , \quad \xi \sim \mathcal{N}(0, \Sigma)$$

where the matrix M is full column rank (f.c.r.), X is a nuisance parameter, $Y \in \mathbb{R}^r$, $\theta \in \mathbb{R}^m$, with $\mathrm{rang}(H) + m < r$.

dynamic $\Longrightarrow$ static

Off-line FDI with nuisance parameters (2/2)

Goal : *to propose a statistical test to detect/isolate a fault in a linear (dynamical) stochastic system with nuisance parameters :*

$$\mathcal{Y} = \mathcal{F}(X_k, \theta, \xi_k) = HX + \xi + M\theta, \quad \xi_k \sim \mathcal{N}(0, \sigma^2 I)$$

where $X \in \mathbb{R}^m$ *is the vector of nuisance,* $\theta \in \mathbb{R}^k$ *is the vector of fault and* $\dim(\mathcal{Y}) > \operatorname{rang} H + m$.

Results : *UBCP invariant test in the class* $\mathcal{K}_\alpha = \{\delta : \mathbb{P}_0(\delta \neq \mathcal{H}_0) \leq \alpha\}$ *:*

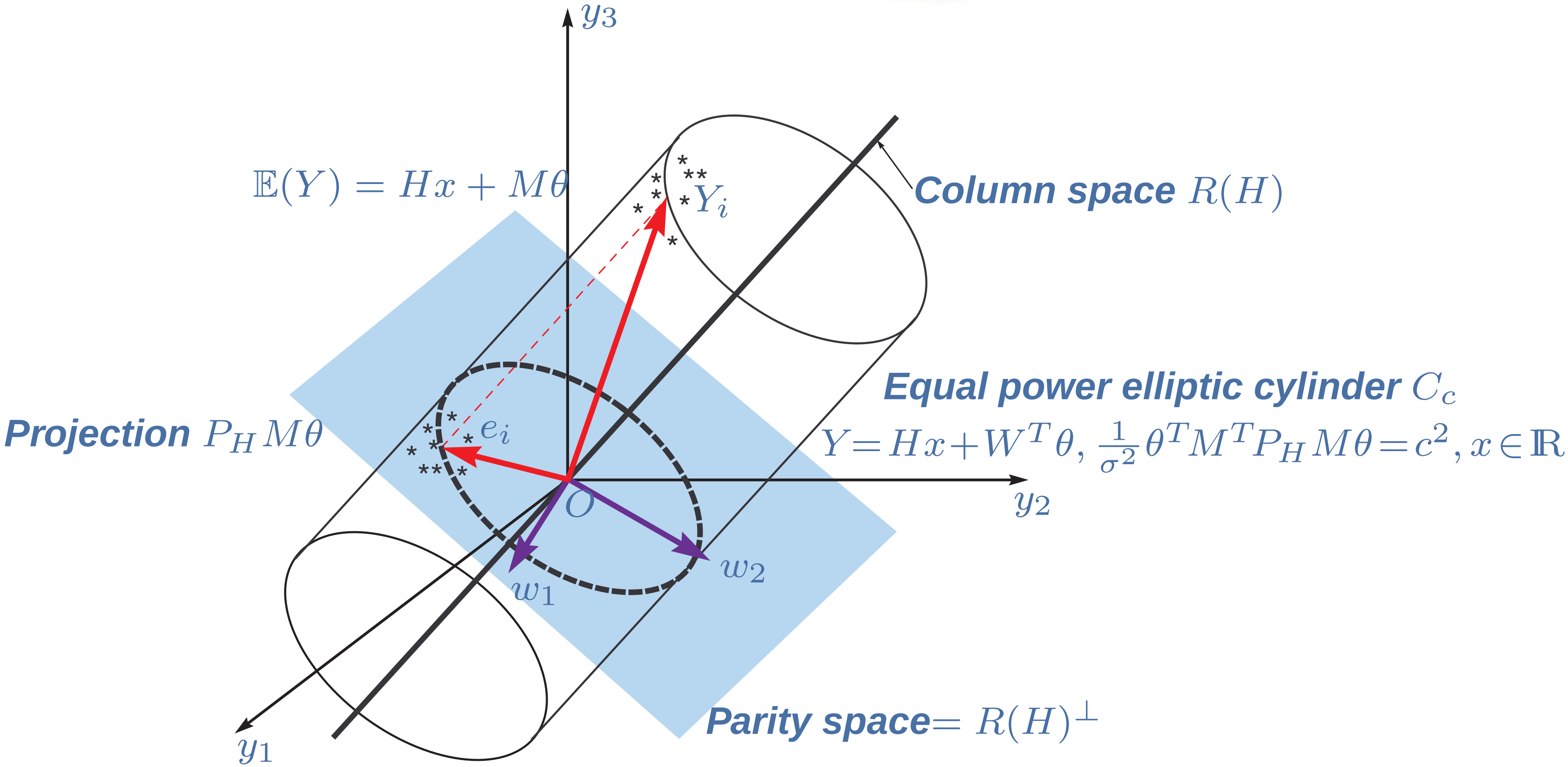
$$\delta^*(Y) = \begin{cases} \mathcal{H}_0 \text{ if } \Lambda(Y) = \frac{1}{\sigma^2} Y^T P_H M (M^T P_H M)^{-1} M^T P_H Y & < \; h(\alpha) \\ \mathcal{H}_1 \text{ if } \Lambda(Y) & \geq \; h(\alpha) \end{cases}$$

over the following family of ellipsoidal cylinders :

$$\mathcal{S} = \left\{ S_c : \theta^T \mathcal{F}_{\widehat{\theta}}\, \theta = \frac{1}{\sigma^2} \theta^T M^T P_H M \theta = c^2 \right\}.$$

Références : [Fouladirad [, Freitag] and Nikiforov : Automatica 2005, Int. J. Adapt. Cont. Signal Process. 2007]

Geometric illustration



$\boxed{\textit{Nuisance rejection} \neq \textit{Nuisance estimation}}$

Fault isolation : Two approaches (1/2)

Again a linear gaussian model :

$$Y = HX + \xi \; (+M\Upsilon) \; , \quad \xi \sim \mathcal{N}(0, \sigma^2 I_r)$$

First approach : several simultaneous faults can be considered.

The approach consists in partitioning Υ in ϕ, and deciding in favor of the fault mode ϕ, while considering the other fault modes collected in ψ as nuisance information. The model writes with $\widetilde{H} \triangleq (H \;\; M_\psi)$:

$$Y = \widetilde{H} \begin{pmatrix} X \\ \Upsilon_\psi \end{pmatrix} + \xi \; (+M_\phi \Upsilon_\phi) = HX + M_\psi \Upsilon_\psi + \xi \; (+M_\phi \Upsilon_\phi)$$

Fault isolation can thus be seen as fault detection in the presence of a nuisance.

Fault isolation : Two approaches (2/2)

Second approach : a single fault at a time is assumed.

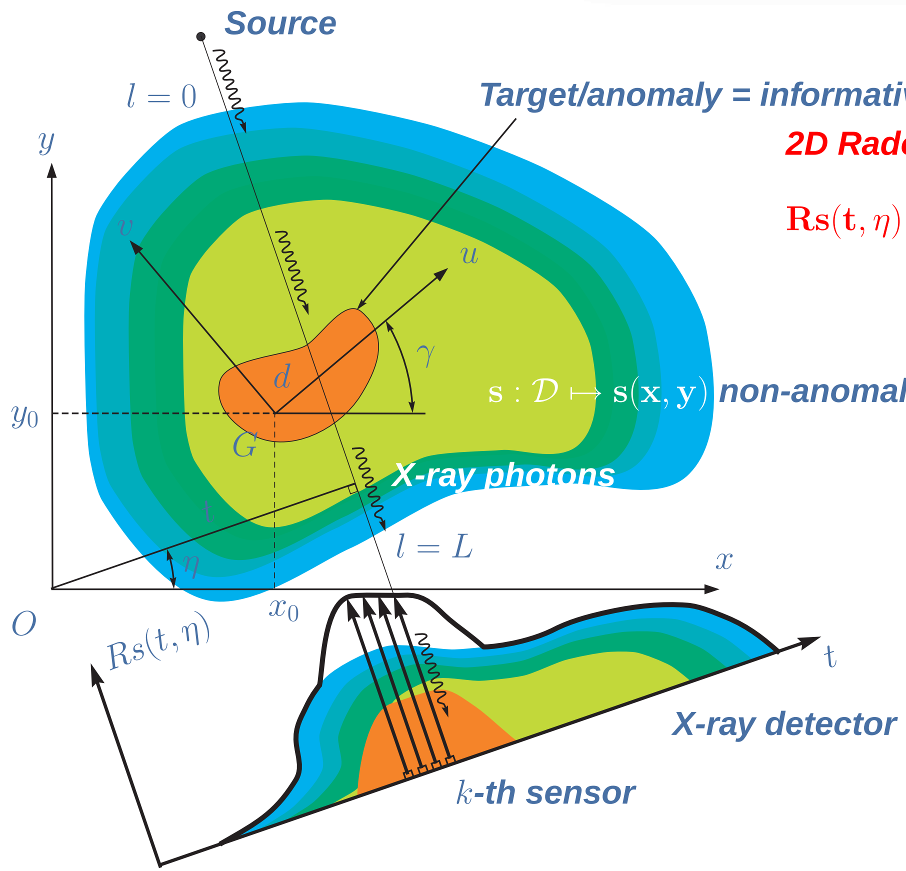
The second approach is multiple hypotheses testing. The problem is to test the null hypothesis $\mathcal{H}_0 : \{Y \sim \mathcal{N}(HX, \sigma^2 I_r);\ X \in \mathbb{R}^p\}$ ***against the alternative hypotheses*** $\mathcal{H}_j : \{Y \sim \mathcal{N}(HX + M_j \Upsilon_j, \sigma^2 I_r);\ X \in \mathbb{R}^p, \theta_j \neq 0\}$ $(j = 1, \ldots, K-1)$***. Unfortunately, this hypotheses testing problem is not invariant under the group of permutations of the*** $\mathcal{H}_j$***'s.***

$$\widehat{\delta}(Y) = \begin{cases} \mathcal{H}_0 & \textit{if} \quad \max_{1 \leq i \leq K-1} \dfrac{f_{WM_i \widehat{\Upsilon}_i}(Z)}{f_0(Z)} < h \\ \mathcal{H}_j & \textit{if} \quad j = \arg\max_{1 \leq i \leq K-1} \dfrac{f_{WM_i \widehat{\Upsilon}_i}(Z)}{f_0(Z)} \geq h \end{cases} \qquad Z = WY$$

where $f_\theta(Z)$ ***is the density of*** $\mathcal{N}(\theta, \sigma^2 I_{r-p})$ ***and***

$$\widehat{\Upsilon}_j = \arg\min \|Z - W M_j \Upsilon_j\|^2 = \frac{M_j^T W^T Z}{M_j^T W^T W M_j}.$$

Target/anomaly detection



2D Radon transform of the function s :

$$\mathbf{Rs}(\mathbf{t}, \eta) = \int_0^{\mathbf{L}} \mathbf{s}(\mathbf{t}\cos\eta - \mathbf{l}\sin\eta, \mathbf{t}\sin\eta + \mathbf{l}\cos\eta)\mathbf{dl}$$

Références : [Fillatre and Nikiforov : JASP 2005, IEEE Trans SP 2007, Fillatre, Nikiforov and Reitrant, JOC 2007, IEEE Trans IP 2008]

Change diagnosis (detection/isolation)

Goal : *to detect and identify a change in the monitored system, pair (N,ν) :*

$$\overline{\mathbb{E}}(N) = \sup_{k_0\geq 1, 1\leq l\leq K} \mathbb{E}^l_{k_0}(N-k_0+1 \mid N\geq k_0) \;\rightarrow\; \min$$

where $1\leq l\leq K$, over the class :

$$\mathcal{K}_{\gamma,\beta} = \left\{ (N,\nu)\, \overbrace{\mathbb{E}_0(N)\geq\gamma,}^{\text{false alarm}}\; \overbrace{\sup_{k_0\geq 1}\mathbb{P}^l_{k_0}(\nu = j\neq l \mid N\geq k_0)\leq\beta}^{\text{false isolation}} \right\}$$

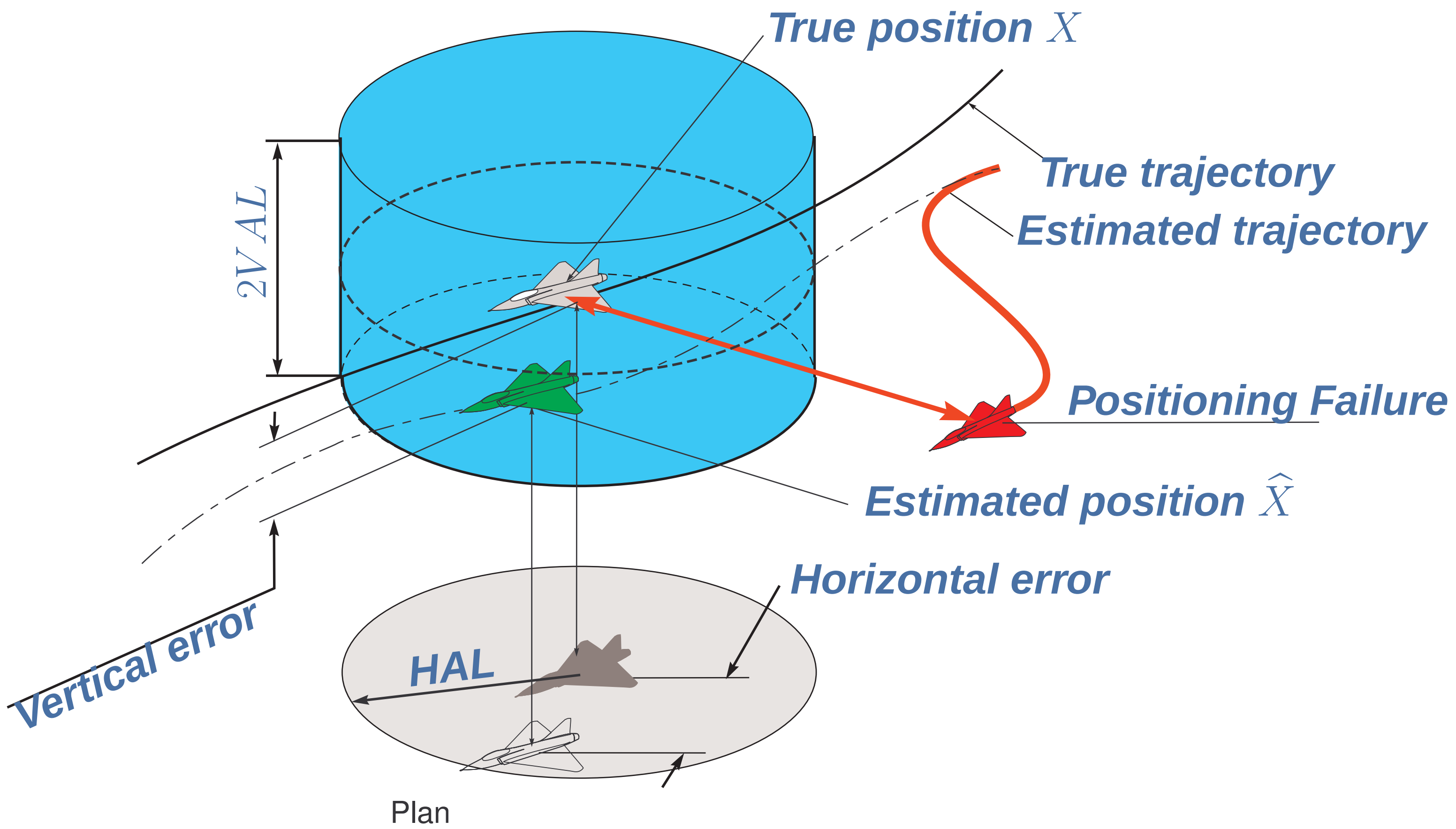
Results : *an optimal test (N,ν) asymptotically reaches a lower bound*

$$\overline{\mathbb{E}}(N;\gamma,\beta) \gtrsim \max\left\{\frac{\log\gamma}{\rho^*_{\mathrm{d}}}, \frac{\log\beta^{-1}}{\rho^*_{\mathrm{i}}}\right\} \text{ when } \min\{\gamma,\beta^{-1}\}\rightarrow\infty,$$

where $\rho^*_{\mathrm{d}} = \min_{1\leq j\leq K}\rho_{j,0}$ and $\rho^*_{\mathrm{i}} = \min_{1\leq l\leq K}\min_{1\leq j\neq l\leq K}\rho_{l,j}$.

Références : [Nikiforov : IEEE Trans IT 1995, 1997, 2000, 2003]

SIRU = Strapdown Inertial Reference Unit



SIRU integrity monitoring

For many safety-critical applications, a major problem of the existing navigation systems consists in its lack of integrity.

The goal of the integrity monitoring is to detect and isolate faults so that they can be removed from the navigation solution before they sufficiently contaminate the output.

$$Y_k = H\mathcal{A}_k + \zeta_k,\ \zeta_k = \zeta_{k-1} + \xi_k + G_l(k, k_0),$$

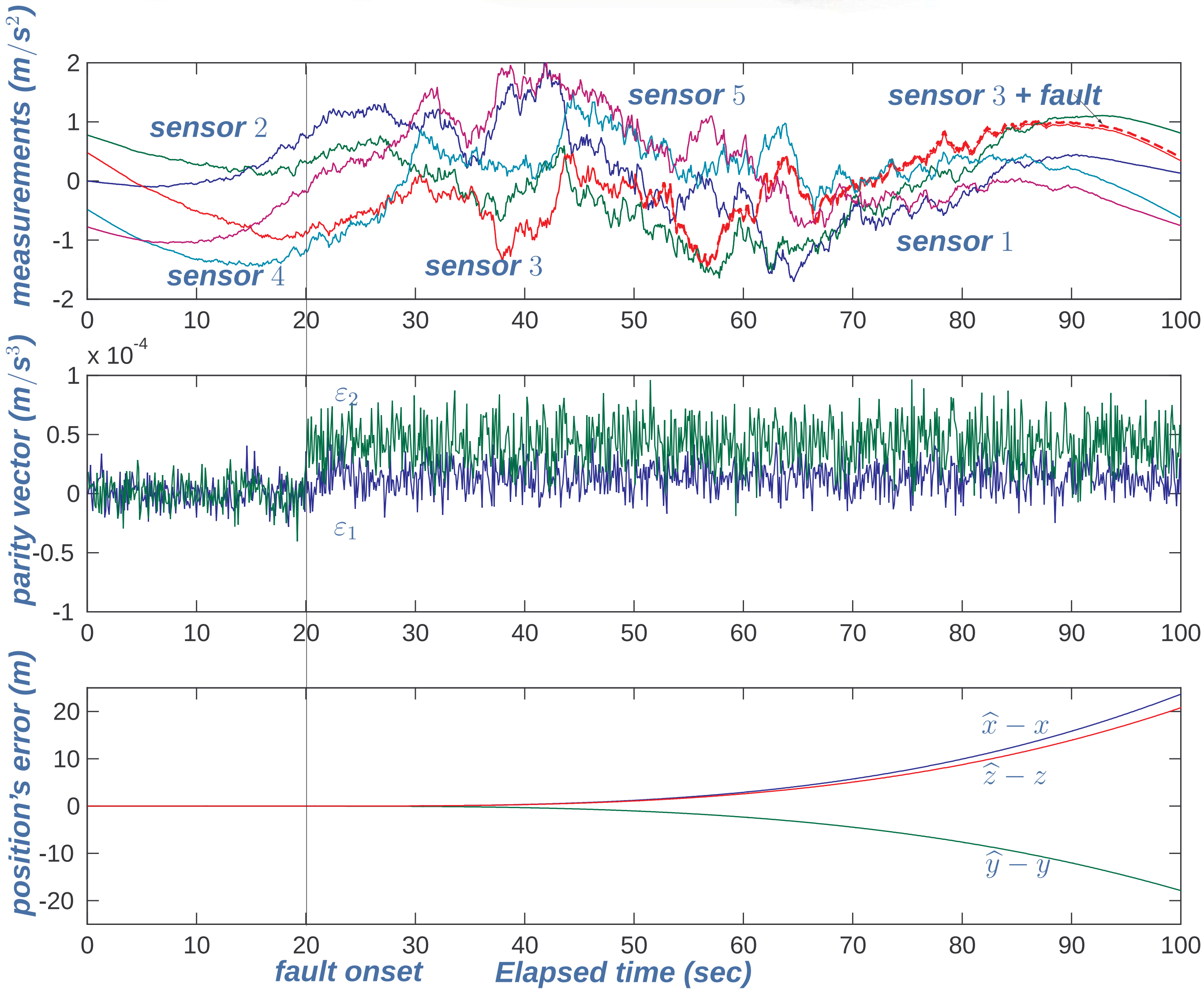
$$\nabla Y_k = H\nabla\mathcal{A}_k + \xi_k + G_l(k, k_0),\ \nabla(.)_k \triangleq (.)_k - (.)_{k-1}.$$

The parity vector sequence $(e_t)_{t\geq 1}$ can be modeled as

$$e_t = W\nabla Y_t = W\xi_t + WG_l(t,k),\ \ l = 1, \ldots, s.$$

where the projection matrix W satisfies the following conditions
$WH = 0,\ \ W^TW = P_H,\ \ WW^T = I_{s-3}$.

SIRU: sensor measurements/fault



The above mentioned criterion can be realized by using the following recursive change detection/isolation algorithm [Nikiforov 2000] :

$$N_r = \min_{1\le l\le K-1}\{N_r(l)\}, \quad \nu_r = \arg\min_{1\le l\le K-1}\{N_r(l)\},$$

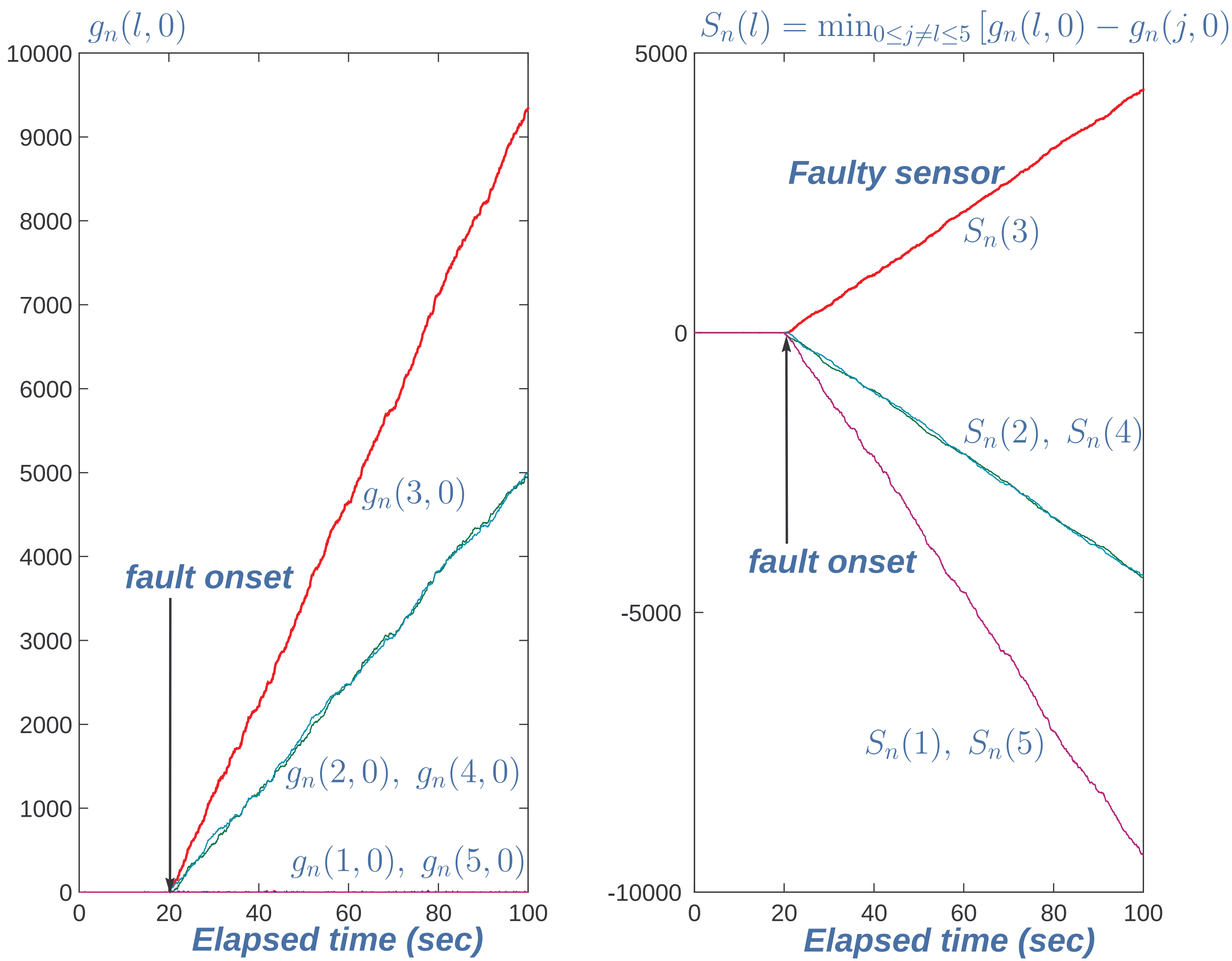
where $N_r(l) = \inf\left\{ n\ge 1 : \overbrace{\min_{0\le j\ne l\le K-1}[S_n(l,j)}^{=S_n(l),\ \textbf{see the next slide}} - h_{l,j}] \ge 0 \right\}$,

$$S_n(l,j) = g_n(l,0) - g_n(j,0),\ g_n(l,0) = (g_{n-1}(l,0) + Z_n(l,0))^+,$$

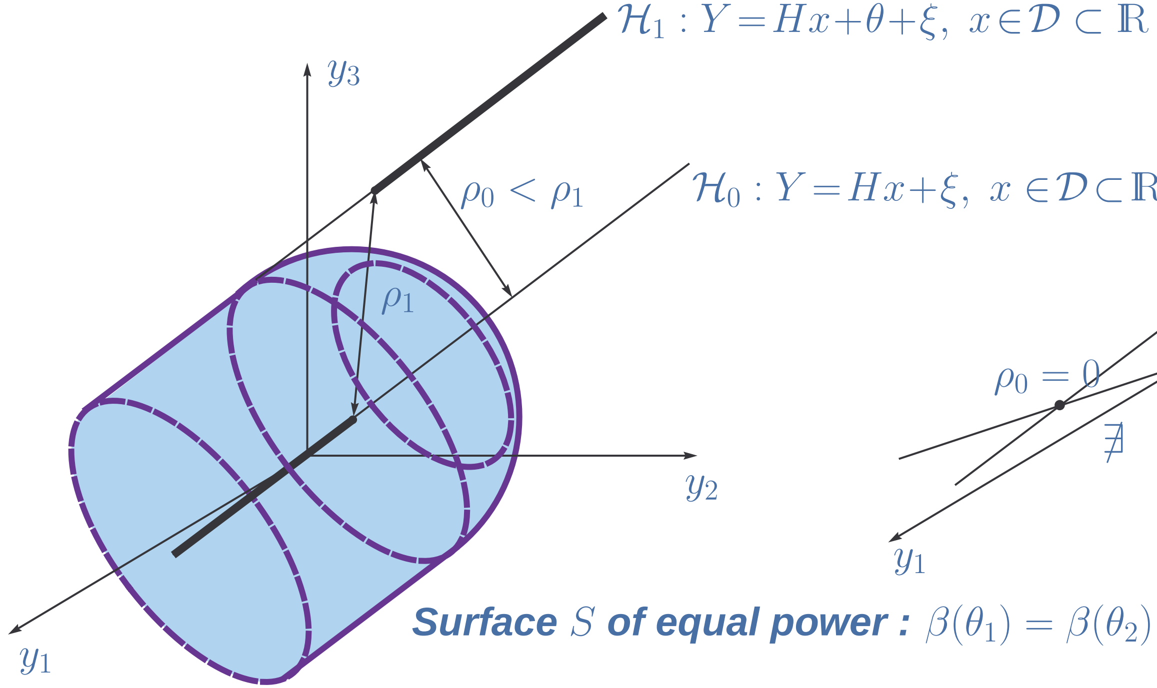
with $Z_n(l,0) = \log f_l(Y_n)/f_0(Y_n)$, $g_0(l,0) = 0$ ***for every*** $1 \le l \le K-1$ ***and*** $g_n(0,0)\equiv 0$,

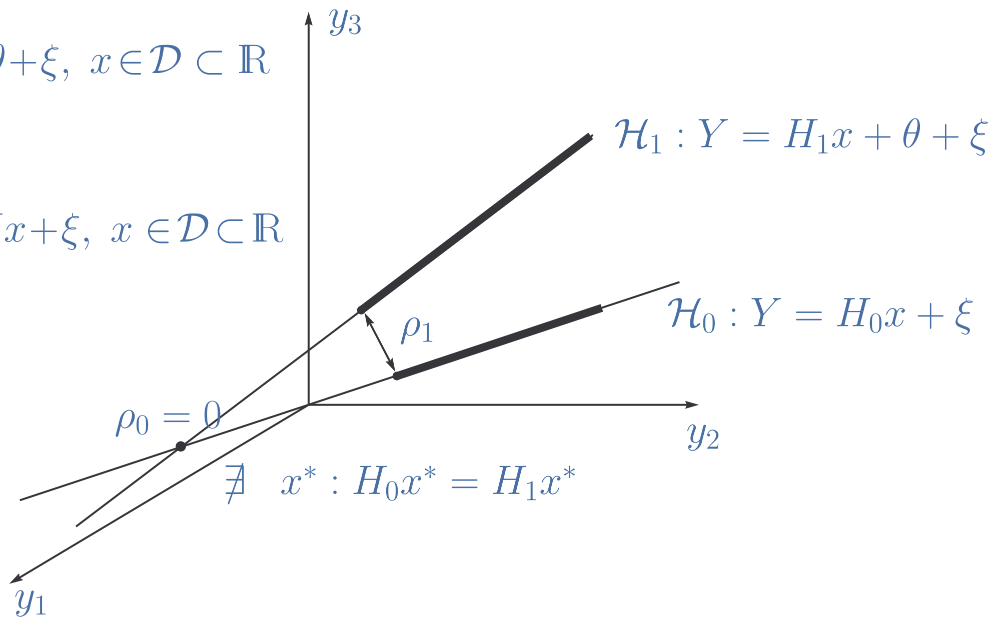
$$h_{l,j} = \begin{cases} h_d \ \textbf{[detection]} & \textbf{if} \quad 1\le l\le K-1 \quad \textbf{and} \quad j=0 \\ h_i \ \textbf{[isolation]} & \textbf{if} \quad 1\le j,l\le K-1 \quad \textbf{and} \quad j\ne l \end{cases}.$$

SIRU : decision functions



Two geometric illustrations of constraints-inequalities





Surface** S **of equal power : $\beta(\theta_1) = \beta(\theta_2)$ ***for any*** $\theta_1, \theta_2 \in S$

Optimality ?

Constrained GLR test

The constrained GLR test is given by

$$\delta(Y) = \begin{cases} \mathcal{H}_0 \text{ if } \Lambda(Y) < h(\alpha) \\ \mathcal{H}_1 \text{ if } \Lambda(Y) \geq h(\alpha) \end{cases}, \quad \Lambda(Y) = 2\log \frac{\sup_{\theta\in\mathbb{R}^n, X\in D} f_{\theta,X}(Y)}{\sup_{X\in D} f_{\theta=0,X}(Y)}$$

where $X \in \mathcal{D}$ is a bounded nuisance parameter and

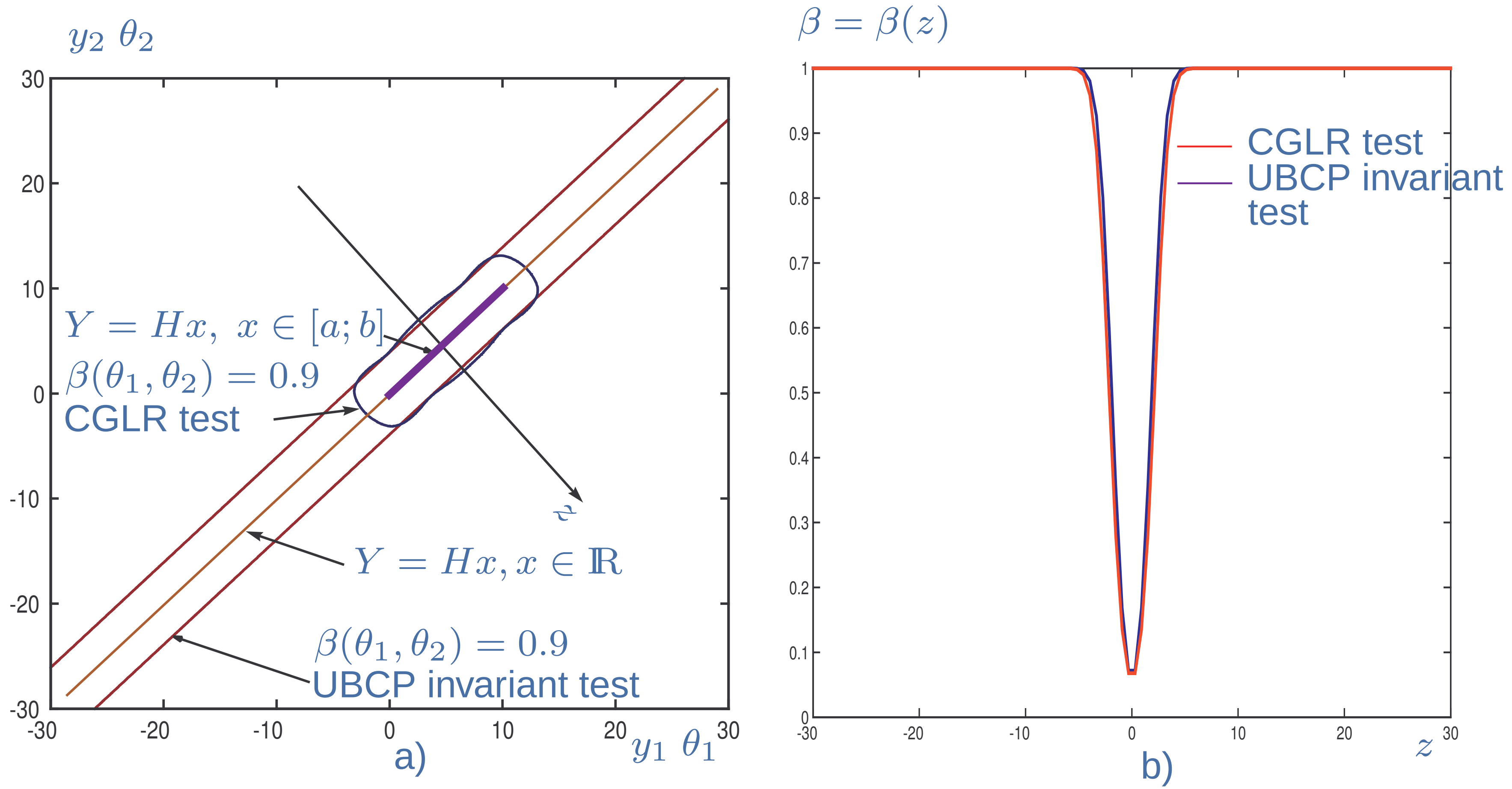
$$f_{\theta,X}(Y) = \frac{1}{(2\pi)^{\frac{n}{2}}\sigma^n} \exp\left\{-\frac{1}{2\sigma^2} \parallel Y - HX - \theta \parallel_2^2\right\}$$

Let us assume that $x \in \mathcal{D} \subset \mathbb{R}$.

$$\Lambda(\xi; x, \theta) = \begin{cases} \frac{1}{\sigma^2} \parallel Hx + \theta + \xi - Ha \parallel_2^2 & \text{if} \quad \widehat{x}_0 < a \\ \frac{1}{\sigma^2} \parallel P_H^{\perp}(\theta + \xi) \parallel_2^2 & \text{if} \quad a \leq \widehat{x}_0 \leq b \\ \frac{1}{\sigma^2} \parallel Hx + \theta + \xi - Hb \parallel_2^2 & \text{if} \quad \widehat{x}_0 > b \end{cases}$$

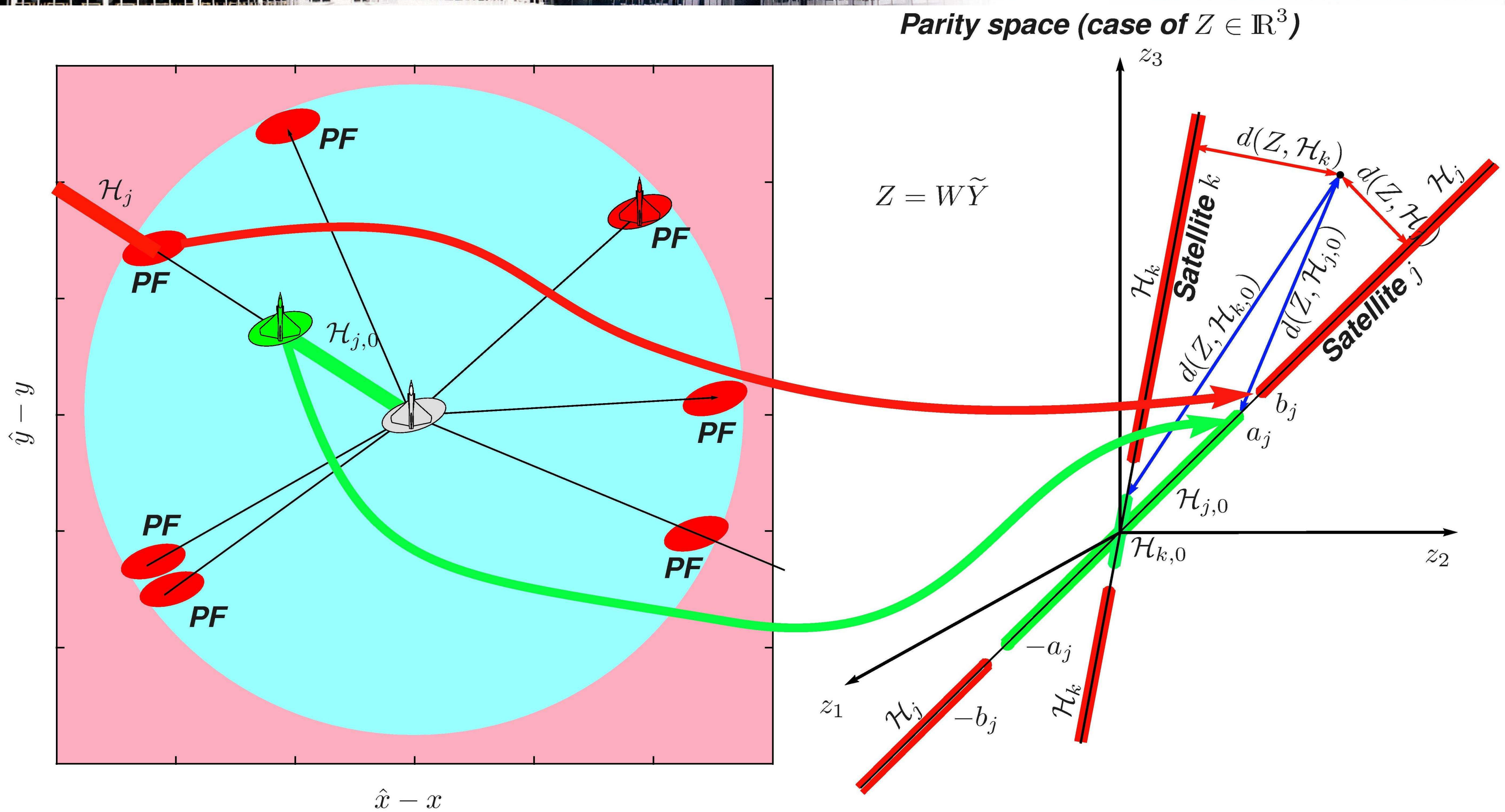
where $\widehat{x}_0 = x + (H^T H)^{-1} H^T (\xi + \theta)$.

Impact of the bounded nuisance



Références : [Harrou, Fillatre and Nikiforov : ICARV 2008, Qualita 2009]

Impact of the bounded hypotheses



Bounded hypotheses improve the quality of decision

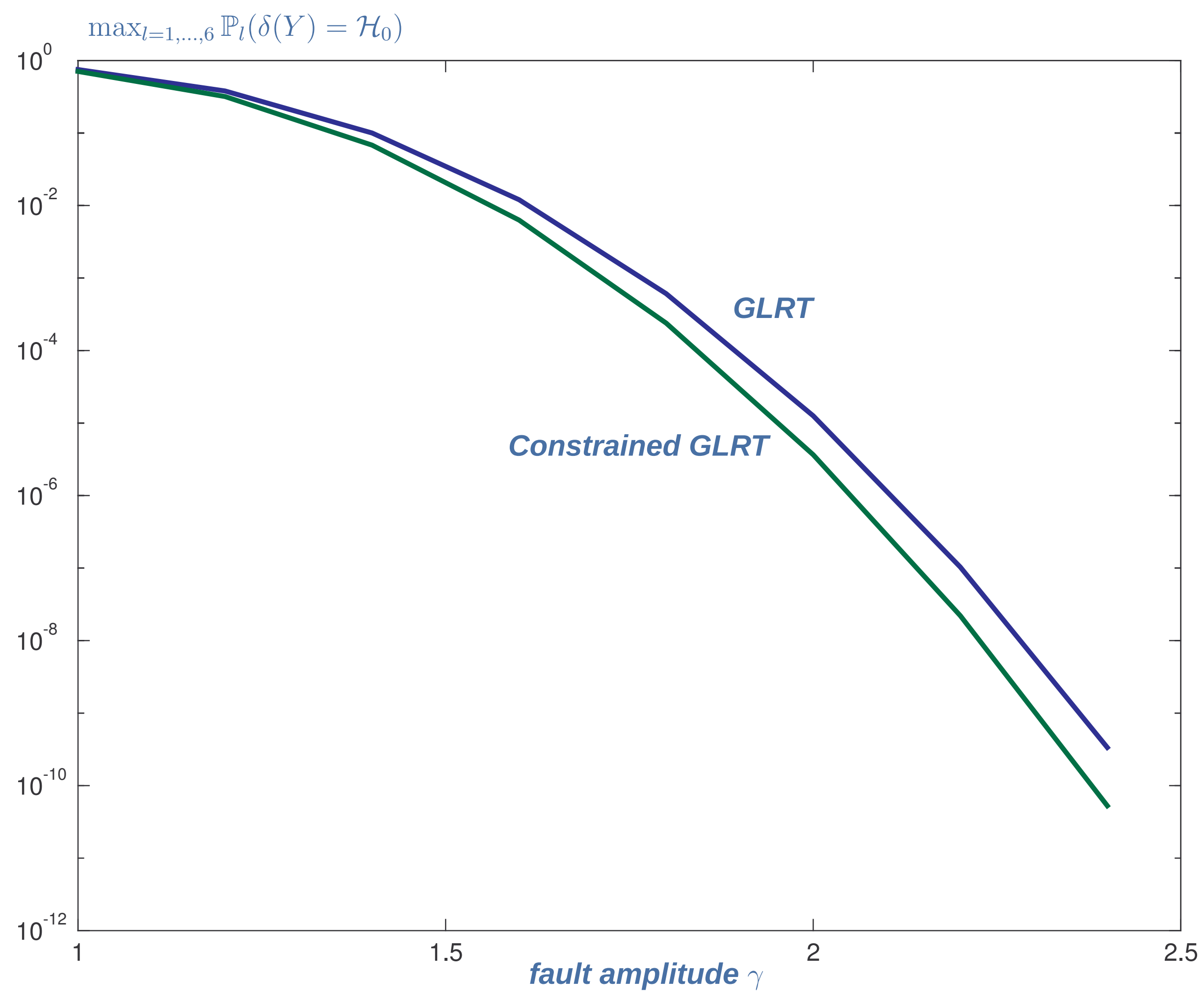
Constrained *GLRT* :

$$\delta(Y) = \begin{cases} \mathcal{H}_0 \textbf{ \textit{if}} & \max_{1\le l\le n} \dfrac{f_{W\tilde{\Upsilon}_l}(Z)}{f_{W\tilde{\Upsilon}_0}(Z)} < h \\ \mathcal{H}_\nu \textbf{ \textit{if}} & \nu = \arg\max_{1\le l\le n} \left\{ \dfrac{f_{W\tilde{\Upsilon}_l}(Z)}{f_{W\tilde{\Upsilon}_0}(Z)} \ge h \right\} \end{cases}$$

where $\widetilde{\upsilon}_l = \arg\max_{|\upsilon_l|\ge b_l} \{f_{W\Upsilon_l}(Z)\} = \arg \underbrace{\min_{|\upsilon_l| \ge b_l}}_{\textit{NEW!}} \left\{ \left\| Z - W_l \dfrac{\upsilon_l}{\sigma_l} \right\|_2^2 \right\},$

$$\begin{aligned} (\widetilde{\upsilon}_j, j) &= \arg \max_{1\le i\le n} \max_{|\upsilon_i|\le a_i} \left\{ f_{W\widehat{\Upsilon}_i}(Z) \right\} \\ &= \arg \min_{1\le i\le n} \underbrace{\min_{|\upsilon_i| \le a_i}}_{\textit{NEW!}} \left\{ \left\| Z - W_i \dfrac{\upsilon_i}{\sigma_i} \right\|_2^2 \right\} \end{aligned}$$

Impact of the bounded hypotheses



Conclusions/Perspectives

- **Conclusions :**
  - *The on-line and off-line FDI problems have been addressed from the statistical point of view. The theoretical tools are : UMP with constant power, invariance, statistical decision - analytical redundancy, change point detection/isolation.*
  - *Handling nuisance parameters and integration of constraints in the decision-making process.*
  - *Optimality criterion : tradeoff between the practical needs and theoretical results*
- **Perspectives :**
  - *Nonlinear systems or nonlinear hypotheses*
  - *Nonlinear nuisance : estimation or rejection ? **GLR is not stable !***