Fault Tolerant Control Design For Hybrid Systems

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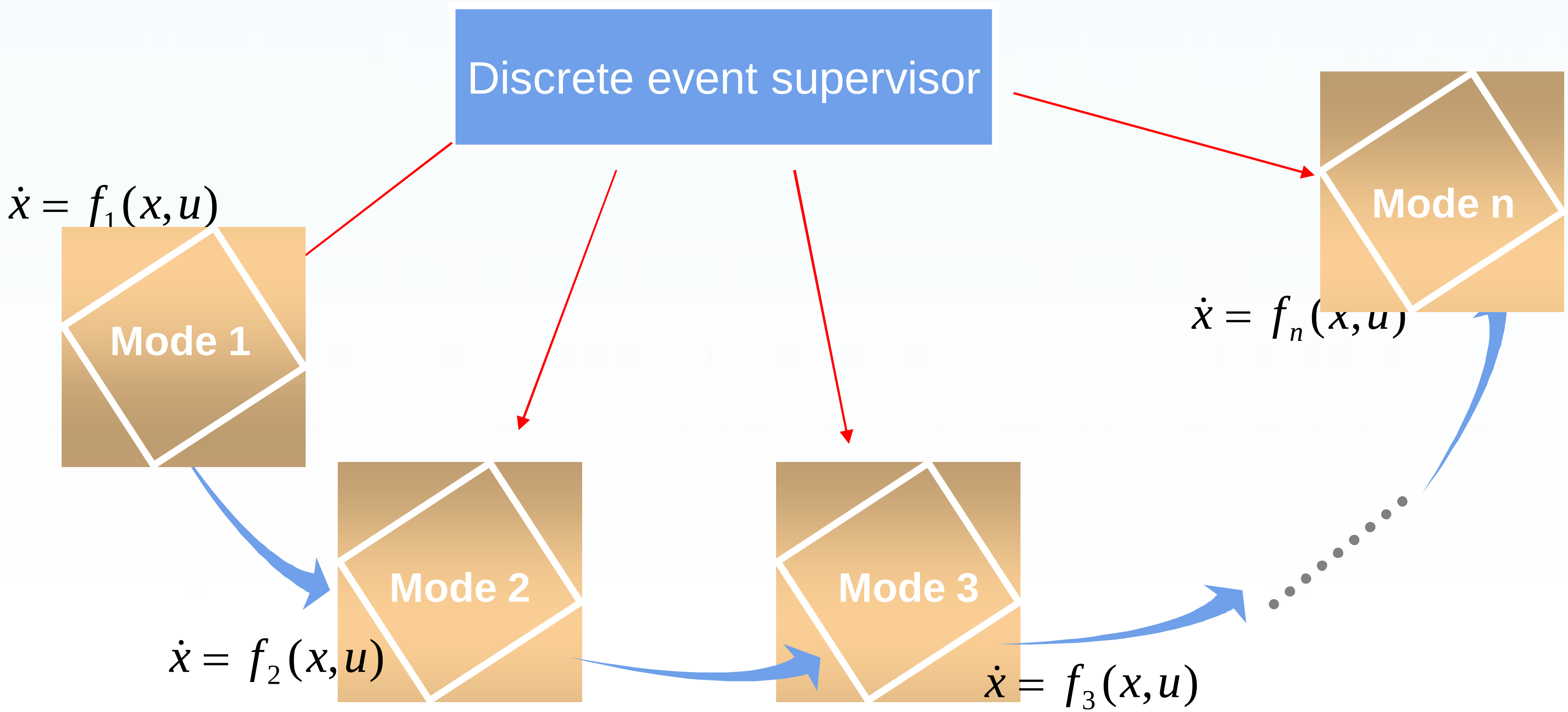
Contents

- Hybrid systems (HS)

  Continuous part ←→ Discrete part

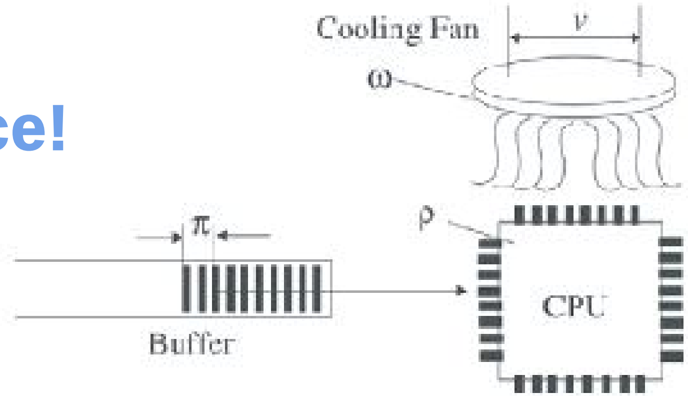


Examples

CPU process control

Balance!

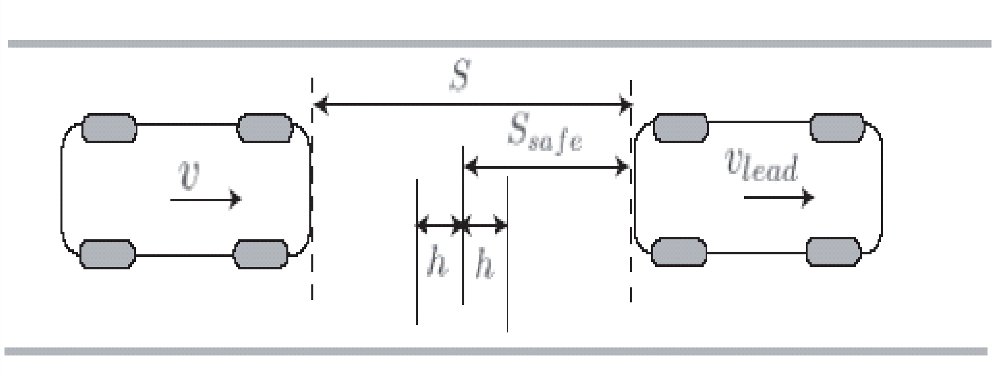
High speed computing (need)
High temperature (hardware trouble)



Mode 1 (busy mode): the amount of CPU tasks is large while CPU temperature is not too high.

Mode 2 (usual mode): the amount of CPU tasks is not large and more effort is made for decreasing the temperature.

Intelligent Transportation System



Control the velocity v of the following car such that the distance S between two cars are within a safe region.

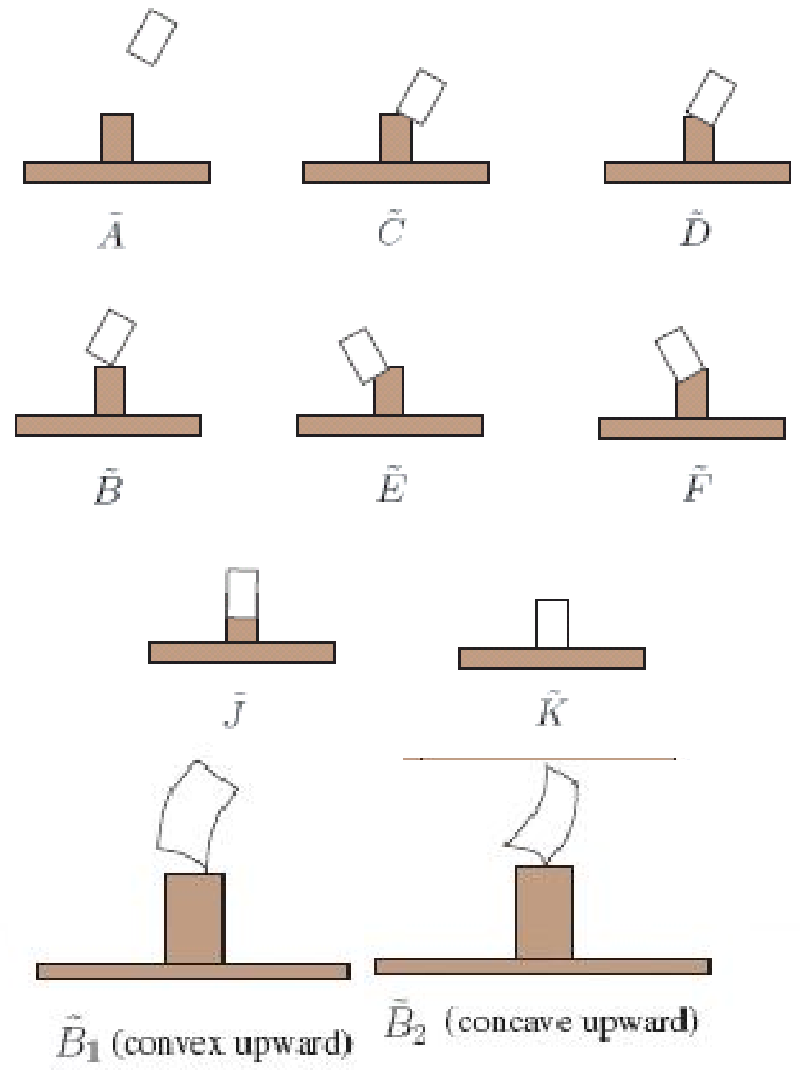
Mode 1 (catching up): The following car accelerates, v tracks a reference signal.

Mode 2 (following): v tracks v_{lead}.

A hose insertion task

The fingertip of the robot arm inserts a deformable hose on the plug

Such task can be modeled as a HS according to different contact configurations between the hose and the plug.



Hybrid system model

A *hybrid automaton with fault*

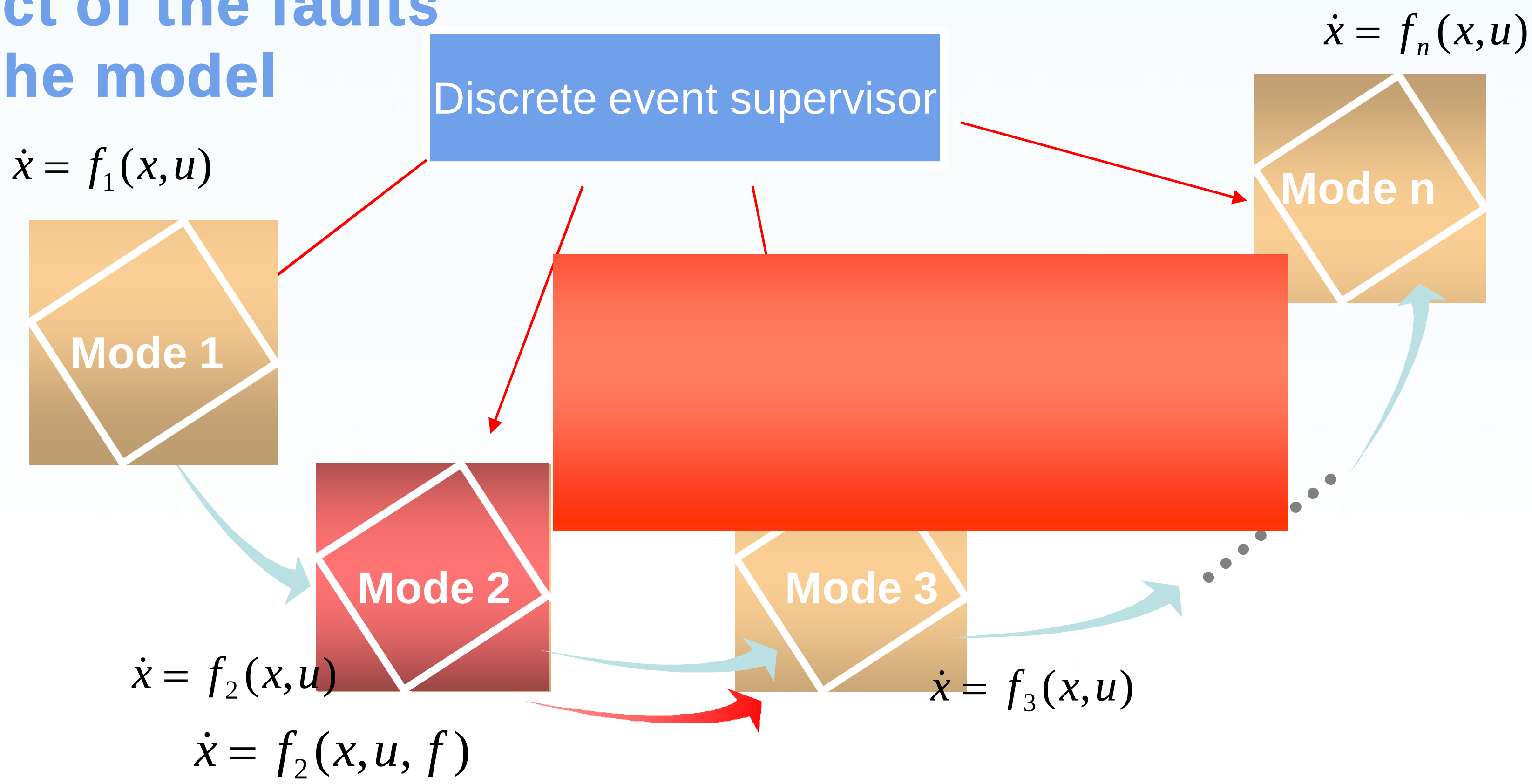
$$\mathcal{H} = (Q, X, U, V, \mathcal{F}, Y, F, Init, Inv, E, G, R) \qquad (1)$$

- $Q = \{1, 2, \ldots, N\}$-Discrete states (modes);
- X-Continuous states;
- U-Continuous inputs;
- V-Discrete inputs;
- $F = F_c \cup F_d$-Faults, F_c-Continuous faults, F_d-Discrete faults;
- $\mathcal{F}$: $Q \times X \times U \times F_c \to X$- Vector fields;
- Y-Continuous outputs;
- $Init \subseteq Q \times X$-Initial states;
- Inv: $Q \to 2^X$-Invariant set;
- $E : V \times F_d \to Q \times Q$-Discrete transitions;
- $G : E \times F_d \to 2^X$-Guard set;
- $R : Q \times Q \times X \to X$-Reset maps.

Lygeros et al, IEEE TAC, 48(1), 2-16, 2003

- Continuous faults corrupt the equality constraints of the related mode.
- Discrete faults change the switching conditions. (special for HS)

Effect of the faults on the model



- The FTC objectives of HS
  concerned with the system requirements

- Continuous performance goal, e.g., the origin of the HS should be stable (Lyapunov stable, asymptotical stable, input-to-state stable) and the output regulation/tracking problem is solvable.

  (Continuous system theories)

  Difficulties: Stability conditions of HS are quite rigorous

- Discrete specification goal, i.e., the HS has to satisfy some constraints on discrete modes, e.g., the prescribed switching sequence.

  (Discrete event system theories)

  Difficulties: Continuous dynamics must be taken into account

- A class of HS named switched systems

$$\begin{aligned} \dot{x} &= g_\sigma(x, u_\sigma, f_\sigma) \\ y &= h_\sigma(x) \end{aligned} \qquad (2)$$

$x \in X,\ u_\sigma \in U,\ y \in Y,\ f_\sigma \in F_c.\quad \sigma(t) : [t_0, \infty) \rightarrow Q$

- The switched system model emphasizes the vector fields, and captures the behaviour of continuous dynamics.
- The affect of the switching on each continuous mode is also clearly represented.

Various Switching properties:

Time-dependent switching; State-dependent switching; Impulsive switching; Stochastic switching.

FTC for continuous performance

- Time-dependent Switching: The switching time is prescribed or is designed by the user
- State-dependent Switching: The switching happens whenever x reaches some surfaces
- Impulsive switching: x abruptly changes due to the impulse effect at each switching time
- Stochastic switching: The switching law follows some stochastic rules, e.g., Markovian Chain.

- HS with time-dependent switching

Two standard sufficient stability conditions of HS without faults:

- Multiple Lyapunov Functions method

Theorem 1. *If each vector field of switched system (2) has an associated Lyapunov function* $V_j(x)$ $(j \in Q)$ *such that*

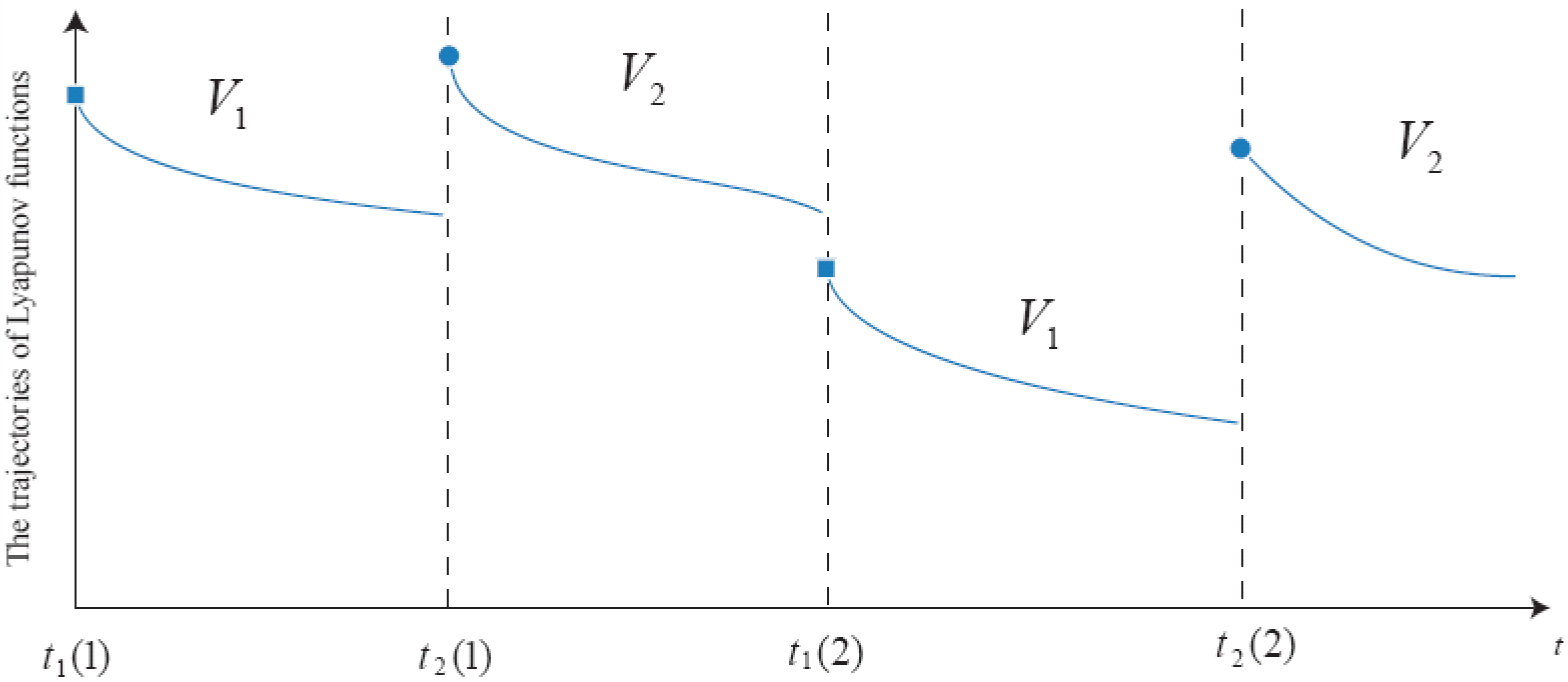
1) $\dot{V}_j(x(t)) < 0$, *i.e., the origin of mode* j *is Lyapunov stable.*

2) $V_j(x(t_j(k+1))) < V_j(x(t_j(k)))$, *where* $t_j(k)$ *is the* k*th time when mode* j *is switched on.*

then the origin of switched system (2) is Lyapunov stable. □

- Multiple Lyapunov Functions method

  - Lyapunov functions using general MLFs method



FTC for continuous performance

- Dwell time scheme

The HS is stable if the interval between any two consecutive switching instants is not smaller than a minimum time interval, which is called dwell time.

The HS is stable if the average interval among switchings is not less than a minimum time interval, which is called average dwell time.

Theorem 2. *Suppose that there exists continuously differentiable functions* V_j *for switched system (2), and constants* $\bar{\alpha}_1, \bar{\alpha}_2, \lambda_0 > 0$, $\mu \geq 1$, *such that* $\forall p, q \in Q$

Switch slowly

$$\bar{\alpha}_1 |x|^2 \leq V_p(x) \leq \bar{\alpha}_2 |x|^2, \quad \dot{V}_p(x) \leq -\lambda_0 V_p(x), \quad V_p(x) \leq \mu V_q(x)$$

Let σ *has an average dwell time* τ_a. *If* $\tau_a > \frac{\ln \mu}{\lambda_0}$, *then the origin of the system (2) is stable.* □

FTC for continuous performance

FTC idea 1 : first design continuous FTC law to stabilize each faulty mode. Then, apply the standard stability conditions of HS.

Stability requirement: During the working period of mode *j*,

$$\dot{V}_j(x(t)) < 0$$

The overall working period: Before fault occurs; During FDI and FTC process; After fault is accommodated

Many classic FTC methods for non-hybrid systems can not be applied to HS

FTC for continuous performance

- Adaptive FDI/FTC scheme

  The diagnosis scheme of mode *j* always works when mode *j* works whatever there is a continuous fault.

  Hao Yang et al, IJC, 82(1), 117-129, 2009

- Transformation based FDI/FTC scheme

  Transform mode *j* into a new form such that the continuous fault can be detected and FTC law can be applied rapidly.

  Hao Yang et al, IET CTA, 3(2), 211-224, 2009.
  IET CTA, 1(5), 1523-1532, 2007.
  IJCAS, 5(6), 707-711, 2007.

FTC for continuous performance

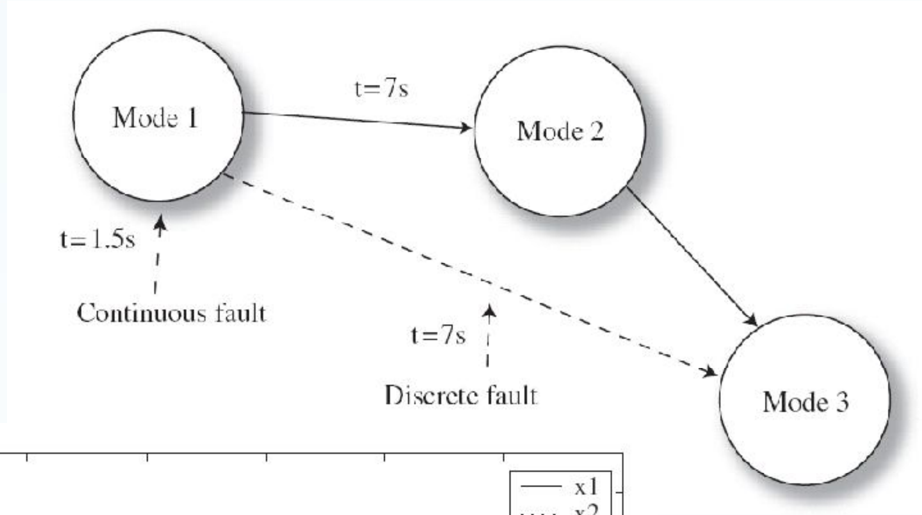
A 3-phase switched reluctance motor

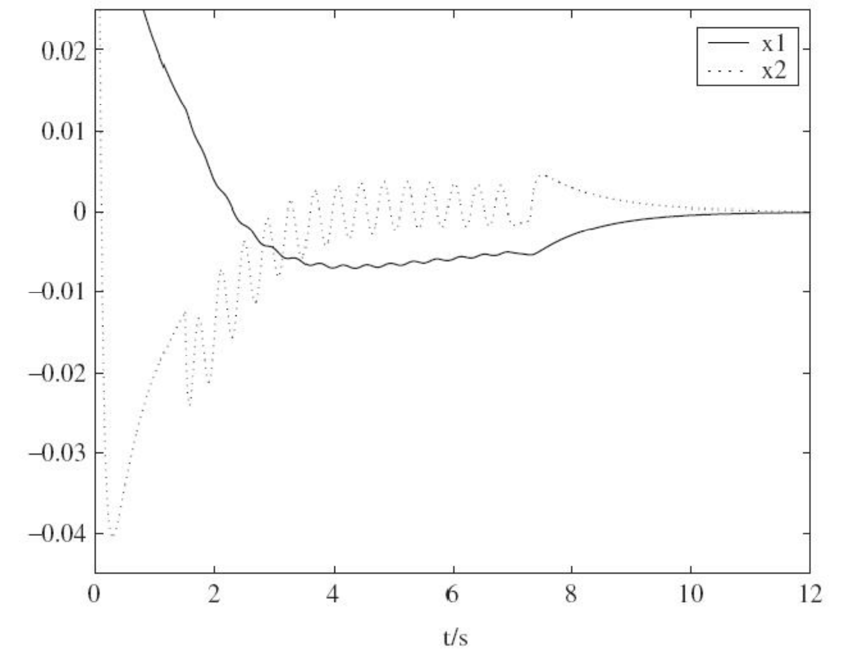
Two states are the angular position and the velocity of the motor.

The normal switching sequence:
mode 1 →mode 2 →mode 3

A continuous fault occurs in mode 1.

A discrete fault forces mode 1 →mode 3.





FTC for continuous performance

FTC idea 2 : research directly the stability of HS without reconfiguring the controller in each unstable faulty mode

- Dwell time scheme

  The dwell time of faulty modes is short enough compared with that of stable modes.

  Hao Yang et al, IJC 2009,

- Energy analysis

  Global energy dissipativit … s less than the *total energ*

  Hao Yang et al, IEEE CSII

- Gain technique

  The function's gain of faulty modes is small enough compared with that of stable modes.

  Hao Yang et al, Systems and Control Letters, revised

The negative effect on the stability resulting from unstable faulty modes can be compensated for by that of stable modes.

A switched RLC circuit

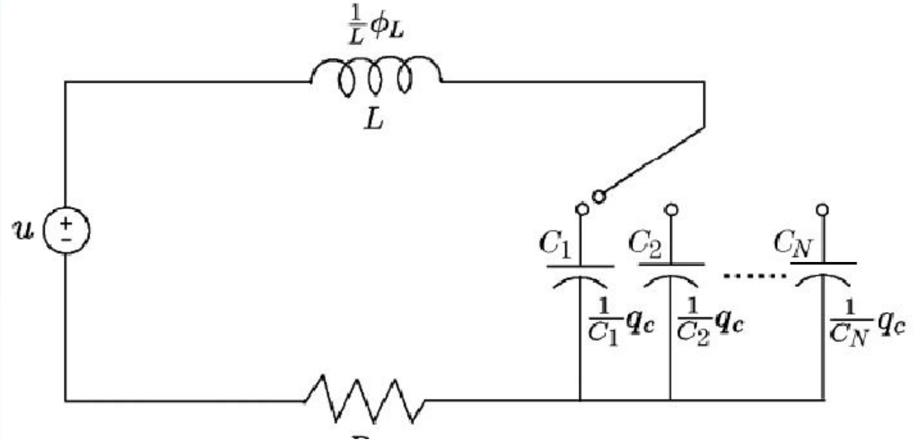
3 capacitors that could be switched between each other.

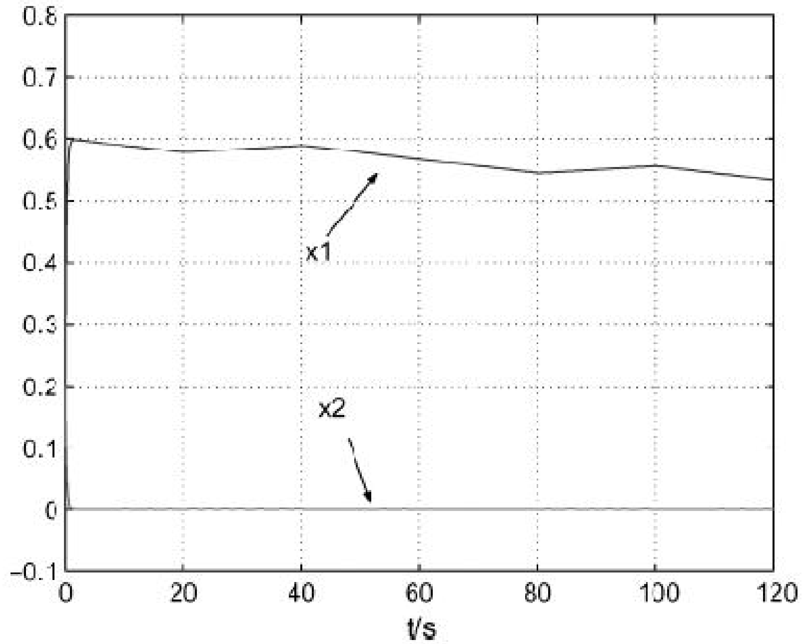
Two states are the charge in the capacitor and the flux in the inductance $x = [q_c, \phi_L]^\top$

The fault occurs in *C*2 (mode 2)

All the dwell periods of three modes are 20s.

Do not have to reconfigure the controller in faulty mode





FTC for continuous performance

- HS with impulsive switching

❑ Dwell time scheme

Trade-off between the frequency of switching, the impulsive magnitude, and the decreasing rate of Lyapunov functions along the solution of the system.

Hao Yang et al, IJRNC 2009, in press

- HS with stochastic switching

❑ Dwell time scheme

Trade-off among the mode transition rate, the frequency of switching, and the decreasing rate of Lyapunov functions.

Hao Yang et al, IJC 2009, in press

- HS with state-dependent switching

The continuous states are not measurable

One challenge of observer design

Distinguish the effects of the continuous faults and mode transitions (may include discrete faults) on the continuous mode.

- Decomposition based FTC design
  1) Design an observer for each mode whose estimation error is not affected by continuous faults and sensitive to switchings.
  2) Design the observer-based FTC law to stabilize each faulty mode.
  3) Apply the standard stability conditions of HS.

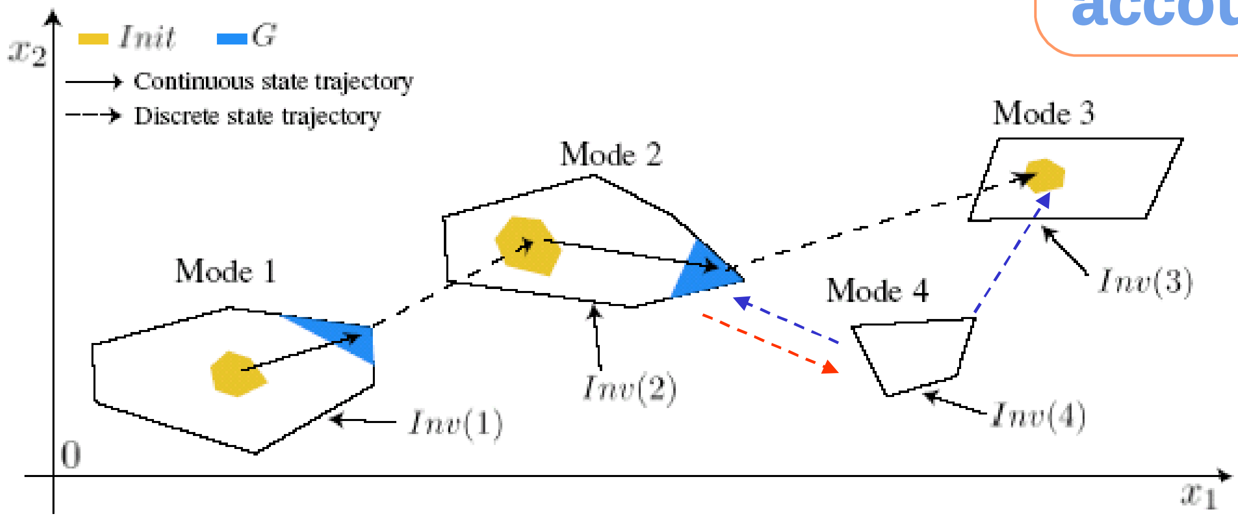
Hao Yang et al, IET CTA, 3(2), 211-224, 2009.

FTC for discrete specification

The continuous system theories are limited for general discrete faults. Discrete event system theories are useful.

Continuous dynamics must be taken into account

- Finite state machine method

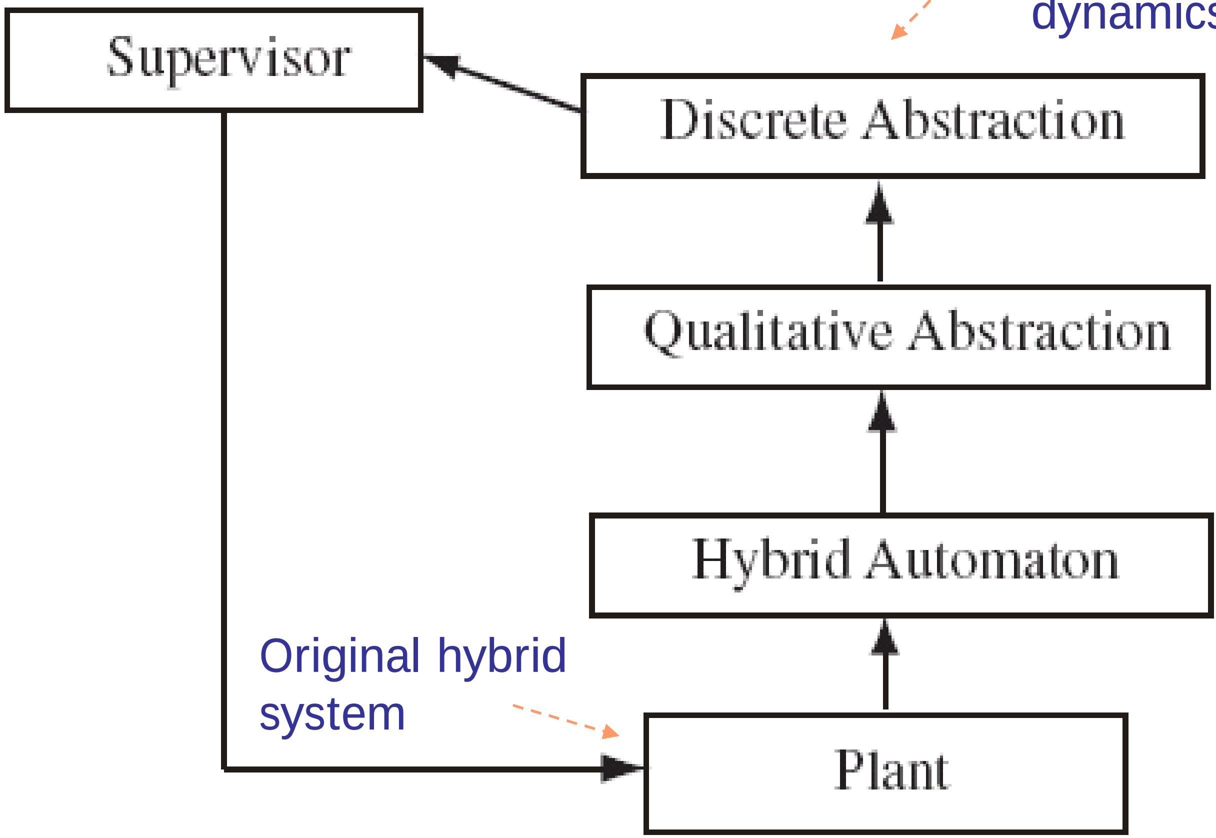


Target mode and marked sequence

Objective (discrete specification): Mode 1->Mode 2->Mode 3

FTC for discrete specification

A finite state machine, which represents the discrete modes and the switchings. The continuous dynamics are removed.



Original hybrid system

A finite state machine, which captures the sign of the time derivatives and the regions of continuous states

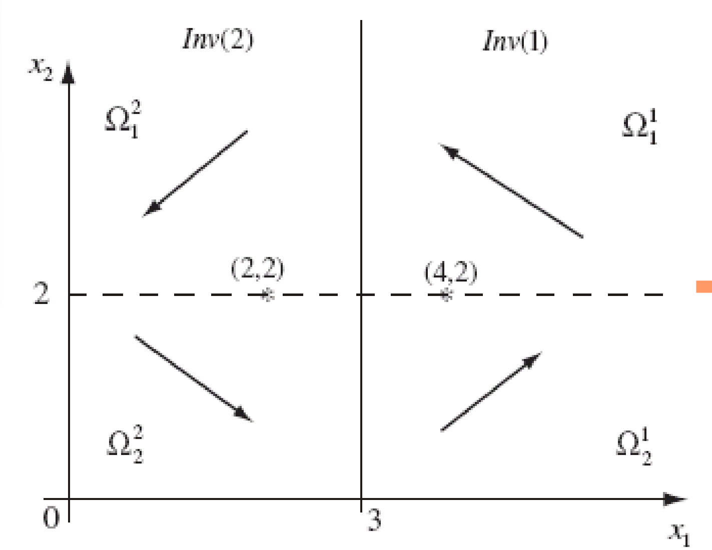
FTC for discrete specification

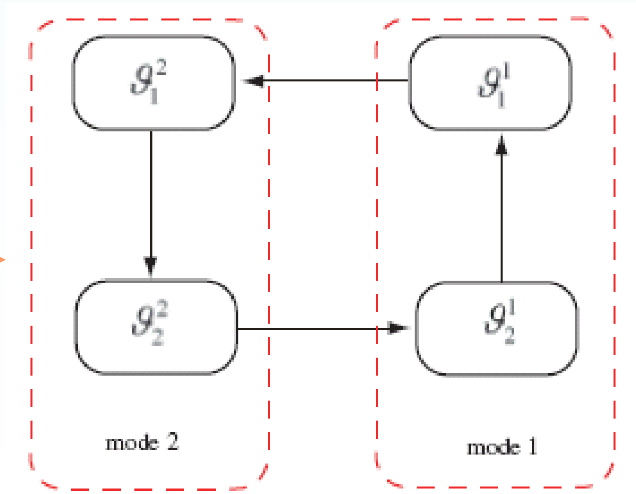
Example of qualitative abstraction and discrete abstraction

$$\text{mode 1: } \begin{cases} \dot{x}_1 = 3x_1 - 1.5x_1x_2 \\ \dot{x}_2 = -2x_2 + x_1x_2 \end{cases}, \quad \text{mode 2: } \begin{cases} \dot{x}_1 = 2x_1 - x_1x_2 \\ \dot{x}_2 = -4x_2 + x_1x_2 \end{cases}$$

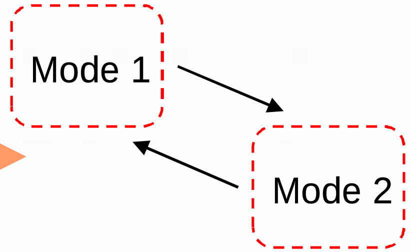
$$Inv(1) = \{x_1 \geq 3\},\ Inv(2) = \{x_1 \leq 3\}$$

$$G(1,2) = \{x \in \Re^2, x_1 \leq 3\} \qquad G(2,1) = \{x \in \Re^2, x_1 \geq 3\}$$





Qualitative abstraction



Discrete abstraction

FTC for discrete specification

FTC steps:

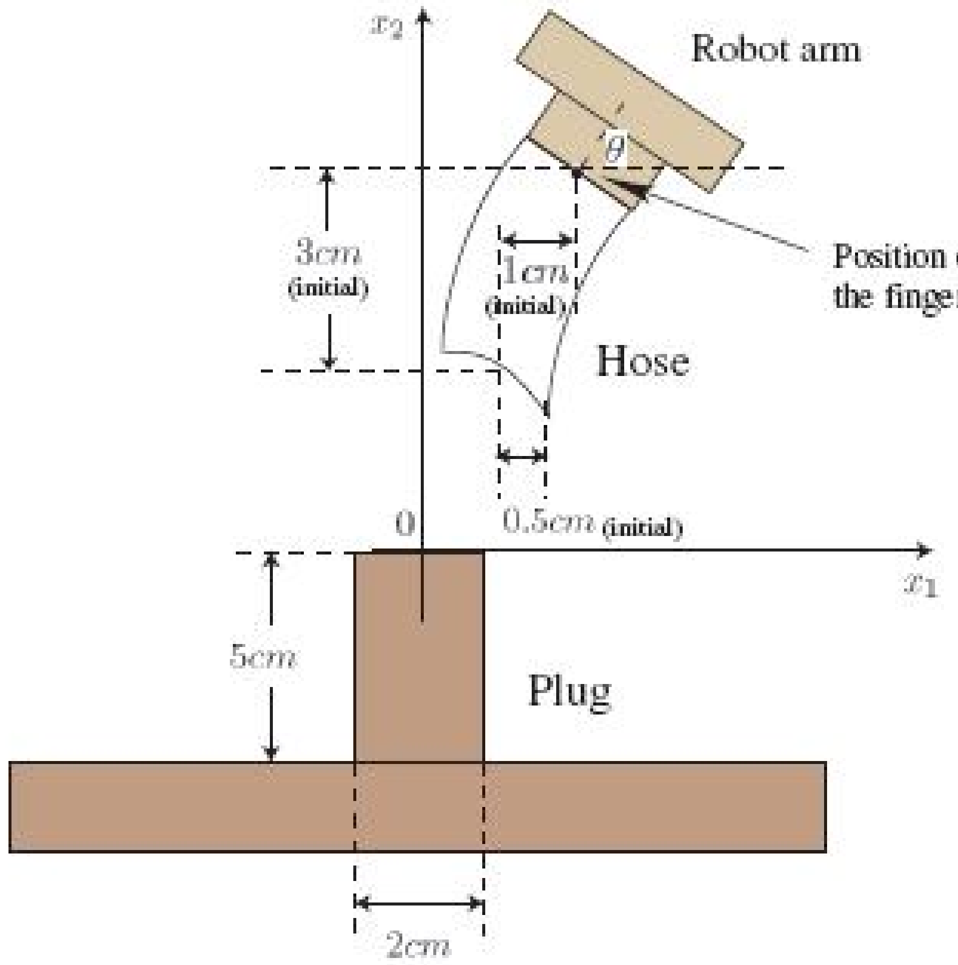
1) Diagnose the fault (*from continuous states*)
2) Check whether the switching sequence after faults occur is controllable, i.e., the system can still be driven to the target mode. (*Discrete abstraction*)
3) Design the new switching sequence. (*Discrete abstraction. DES supervisory control*)
4) Check whether the continuous state behavior is consistent with the new sequence. (*Qualitative abstraction*)

Hao Yang et al, Nonlinear analysis: Hybrid systems, 2(3), 846-861, 2008.

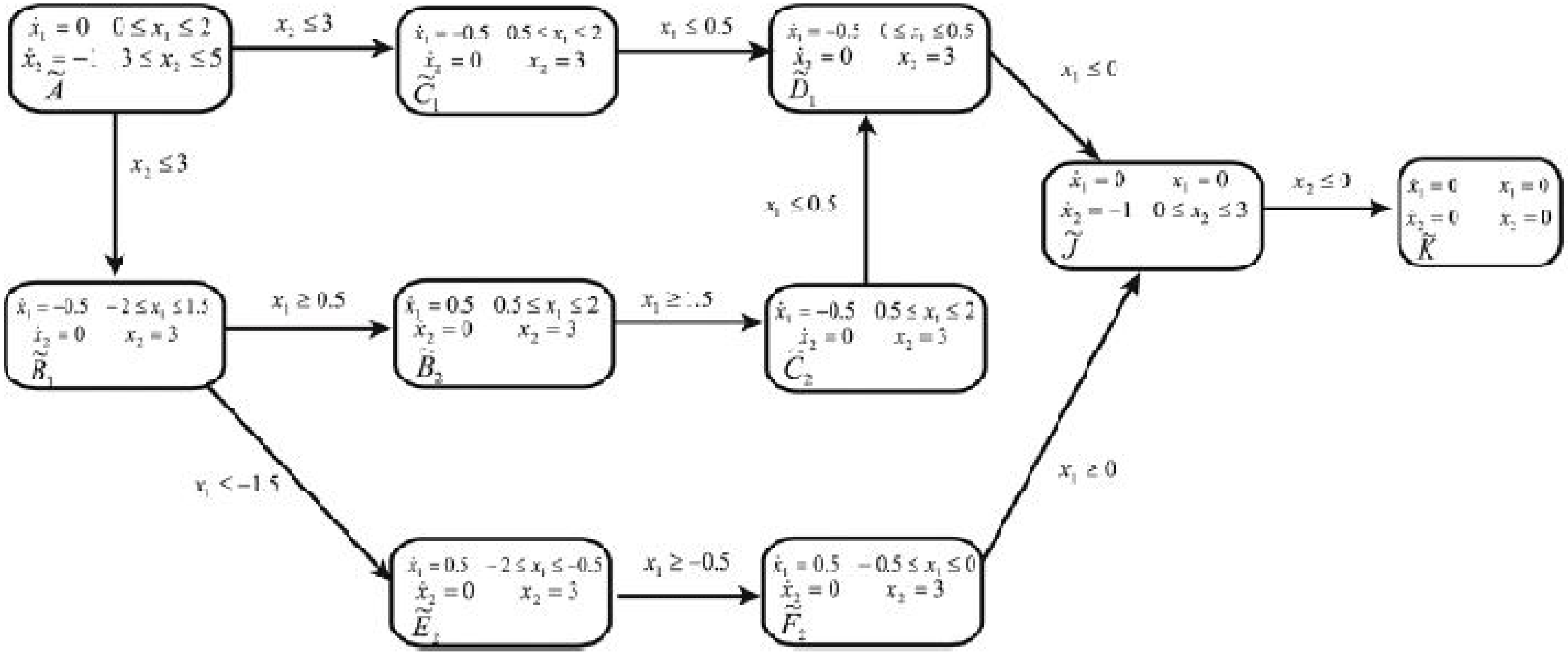
FTC for discrete specification

A hose insertion task

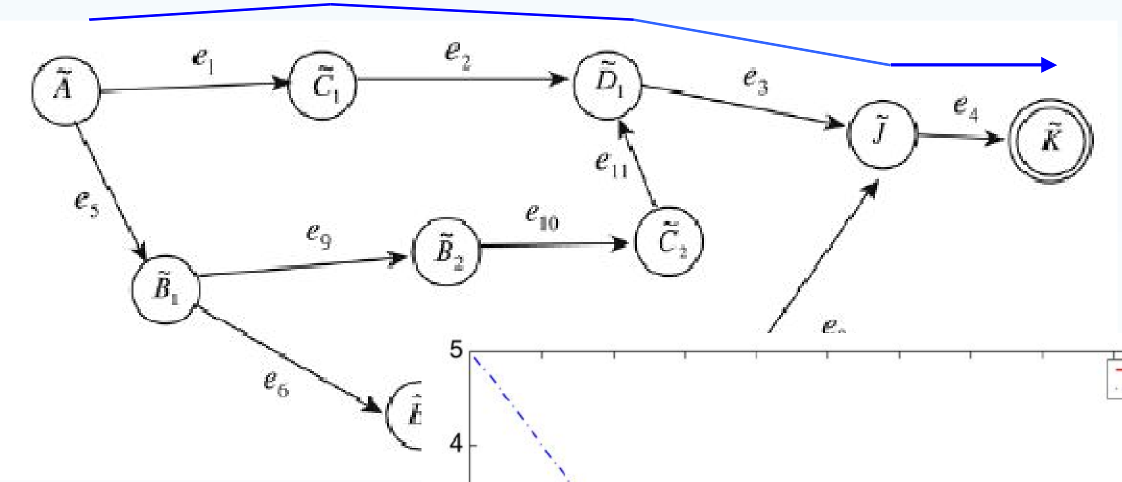
Two continuous states
x1 and *x2* represent
the positions of the fingertip.

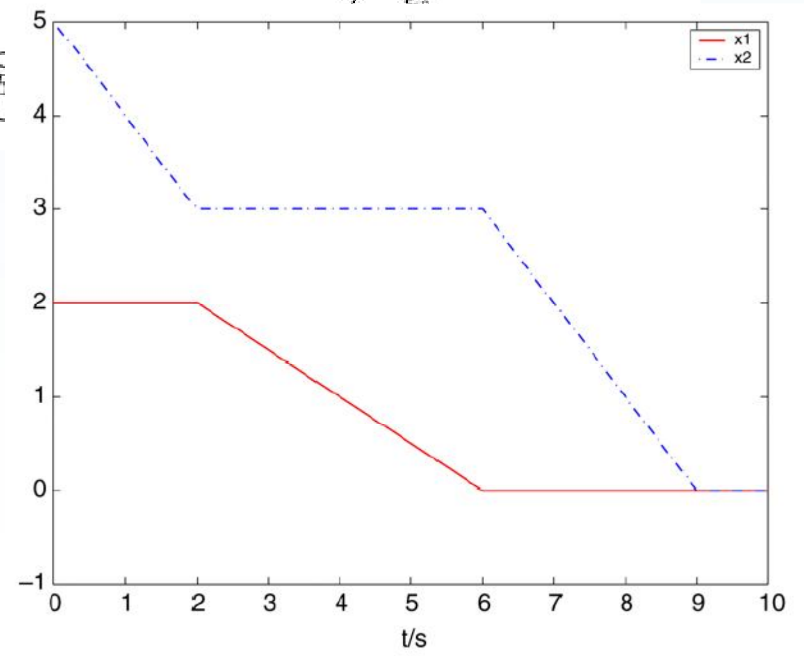


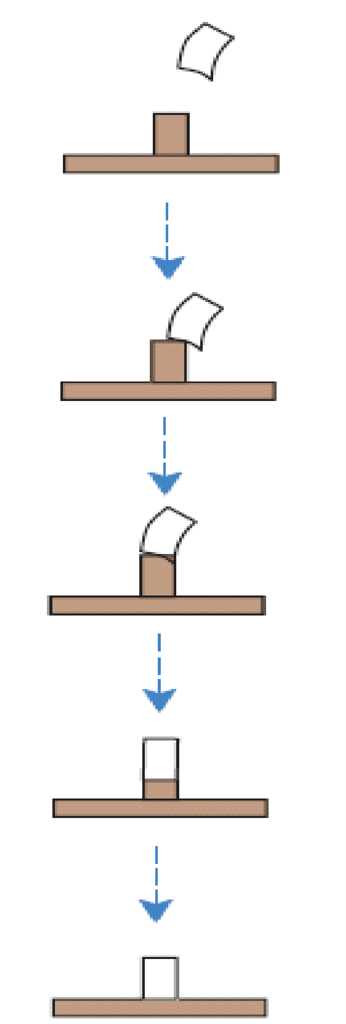
FTC for discrete specification



FTC for discrete specification

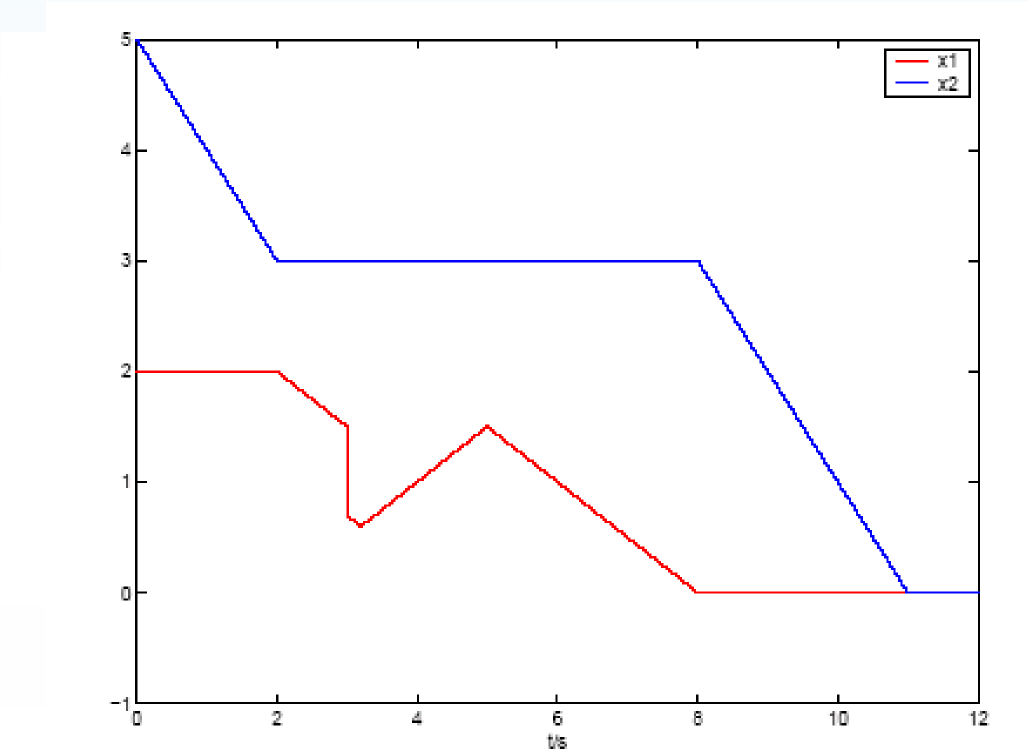


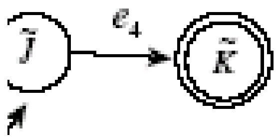


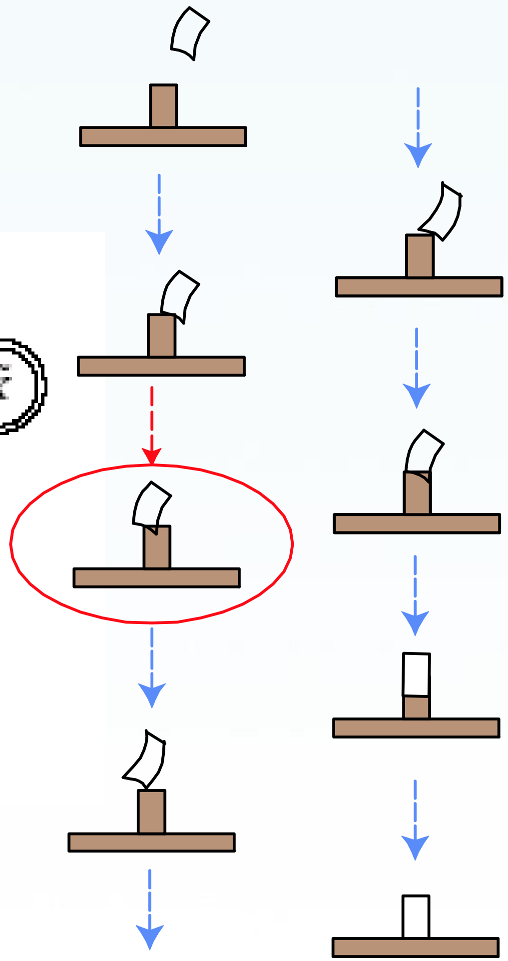


FTC for discrete specification

Abrupt changes of the fingertip's position due to physical faults of the robot arm.

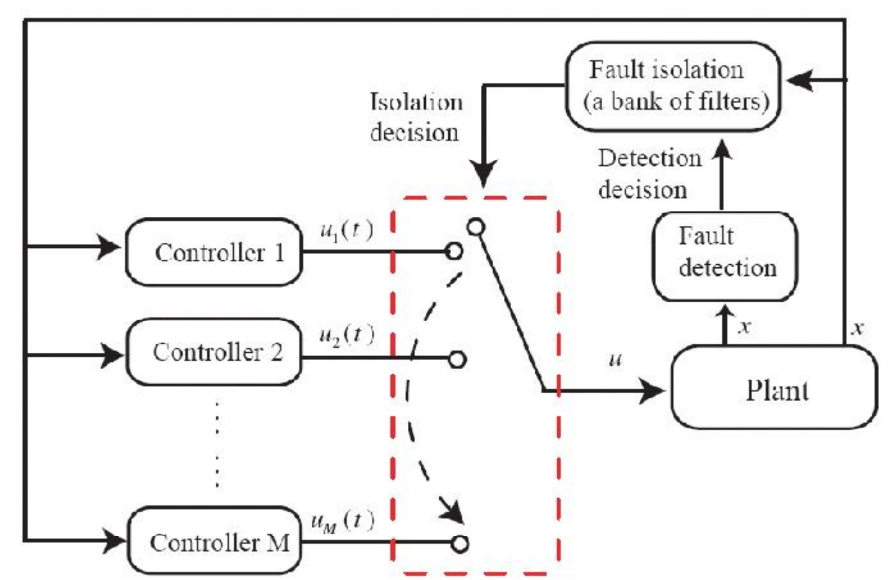






The potential faults in a system often reside within a very large region. A single controller is hard to stabilize all faulty situations.

Hybrid FTC tries to maintain system's goals by switching between a pre-specified family of FTC laws.

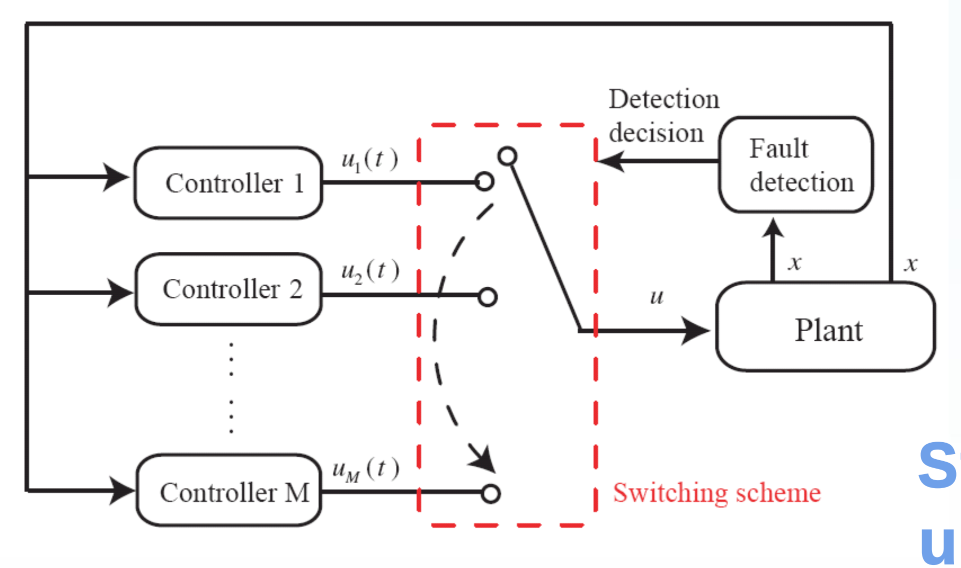


Classic supervisory FTC

Shortcoming

1) A bank of filters/models.
2) Fault isolation may end with a false isolation.
3) During FDI/FTC delay, the faulty system is controlled using an inappropriate controller.

- Supervisory FTC with integrated fault isolation



The proposed supervisory FTC

No fault isolation scheme

Step:

1) The fault is detected.
2) A sequence of controllers are switched until the correct one is found.

Stability results of HS with unstable modes (P.18)

FTC idea 2 : research directly the stability of HS without reconfiguring the controller in each unstable faulty mode

- Dwell time scheme

  The dwell time of faulty modes is short enough compared with that of stable modes.

  Hao Yang et al, IJC 2009, in press. IJRNC 2009, in press

- Energy analysis

  Global energy dissipativity, i.e., the *total energy* stored by the HS is less than the *total energy* supplied from the outside.

  Hao Yang et al, IEEE CSII , 55(12), 1279-1283, 2008

- Gain technique

  The function's gain of faulty modes is small enough compared with that of stable modes.

  Hao Yang et al, Systems and Control Letters, revised

Conclusion

- FTC for HS with continuous performance
  (stability, output tracking)
- FTC for HS with discrete specification
  (finite state machine, Petri net)
- Supervisory FTC via hybrid system approaches

Further research

- To consider optimality as a FTC goal besides the continuous stability and the discrete specification.
- To relax the structure of HS, e.g., the stability at non-zero equilibriums; The time-variant continuous vector fields.
- To combine continuous system theories with DES ones such that an integrated FTC framework can be provided.

Thanks

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