Robust Fault detection for uncertain switched systems: $\mathcal{H}_\infty$ approach

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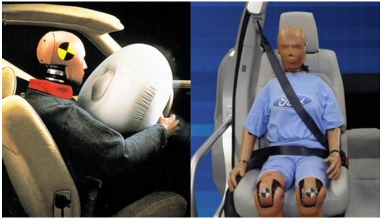
Motivations

Passive security systems

- Airbag
- Security belt

Active security and comfort systems

- ABS : Anti-lock braking system
- ESP : Electronic stability program
- ACC : Autonomous cruise control
- Suspension control



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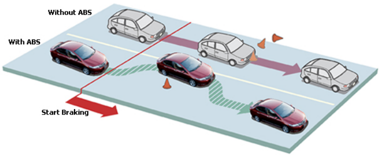

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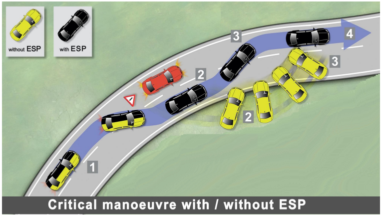

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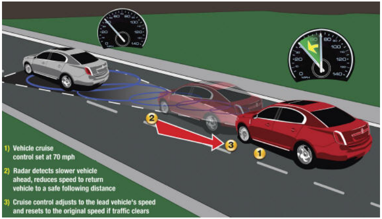

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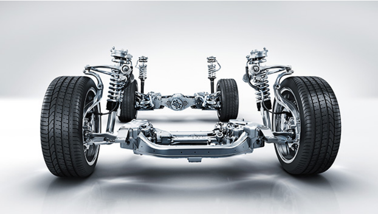

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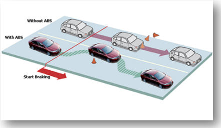

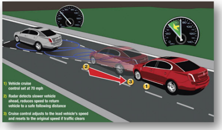

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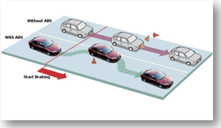

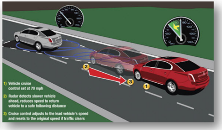

Problematic

What happens in case of sensors/actuators failure?

Could we avoid dramatic situation by detecting/estimating these faults?

Outline

Backgrounds

- A common fault detection technique is to calculate a residual signal.
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- In this study, we consider the model based techniques, and a focus on $\mathcal{H}_\infty$ fault detection problem
- For some problems, when fault frequency range is known (low frequencies, offsets...), the fault detection problem can be transformed into fault estimation problem.
- Switched uncertain approach may offer an interesting framework to model non-linear plants with parameters variation
- Working in the switched uncertain system framework can be used for the FD problem in lateral vehicle control

Multi-objective design

A reliable fault detection filter must meet several specification:

- (a) perturbation (and unknown inputs, noises) rejection,
- (b) sensitivity toward faults,
- (c) and robustness toward uncertainties
- (d) correct time response for fault detection/estimation.

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In our approach, a discrete time fault detection and estimation filter design is developed. The steps of the design are as follows:

- Express an $\mathcal{H}_\infty$ performance, useful to ensure the residual robustness to unknown inputs for switched systems.

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- Extend the proposed method to consider the uncertainties in the system.
- Finally, add loop shaping via weighting filters, useful to tune the transient response and steady state response, and enhance the overall fault detection.

Problem formulation

A linear switched system

A state space representation of switched linear time invariant system is:

$$\begin{cases} x_{k+1} = A_{\alpha(k)} x_k + E_{d,\alpha(k)} d_k + E_{f,\alpha(k)} f_k \\ y_k = C_{\alpha(k)} x_k + F_{d,\alpha(k)} d_k + F_{f,\alpha(k)} f_k \end{cases} \tag{1}$$

- $x \in \mathbb{R}^n$ is the state vector,
- $y \in \mathbb{R}^p$ is the measurement output vector,
- $\alpha(k)$ is the switching signal
- $d \in \mathbb{R}^{n_d}$ is the disturbance vector,
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Notation

When $\alpha(k) = i$, it means that the i^{th} subsystem is activated. Moreover, at the switching time $k : i = \alpha(k) \neq \alpha(k+1) = j$.

A class of uncertain switched system

Consider the state space representation of uncertain switched system :

$$\begin{cases} x_{k+1} = \bar{A}_{\alpha(k)} x_k + E_{d,\alpha(k)} d_k + E_{f,\alpha(k)} f_k \\ y_k = C_{\alpha(k)} x_k + F_{d,\alpha(k)} d_k + F_{f,\alpha(k)} f_k \end{cases} \tag{2}$$

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In this study, an additive uncertainties form is considered:

$$\bar{A}_{\alpha(k)} = A_{\alpha(k)} + H_{A,\alpha(k)}\Delta_{A,\alpha(k)}N_{A,\alpha(k)} \quad (3)$$

with $H_{A,\alpha(k)} \in \mathbb{R}^{n\times n_h}$ and $N_{A,\alpha(k)} \in \mathbb{R}^{n\times n_a}$

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Assumption 4

The state uncertainty matrix $\Delta_{A,\alpha}$ is assumed bounded:

$$\Delta_{A,\alpha}^T\Delta_{A,\alpha} \leq I$$

Problem formulation

We propose to design a switched fault estimation filter $K_{f,i}(q^{-1})$ with the following realization:

$$\begin{cases} x_{k+1}^K & = & A_i^K x_k^K + B_i^K y_k \\ \hat{f}_k & = & C_i^K x_k^K + D_i^K y_k \end{cases} \tag{4}$$

where $x^K \in \mathbb{R}_k^n$ is the filter's state vector and $\hat{f} \in \mathbb{R}^{n_f}$ is the vector of the estimated faults.

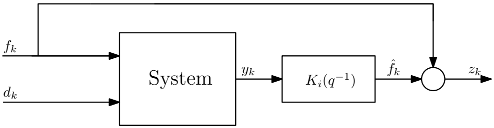

Figure: Fault estimation schemes

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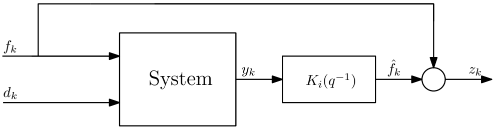

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Define $z_k = f_k - \hat{f}_k$ and $x^a_k = \begin{bmatrix} x^T_k & x^{KT}_k \end{bmatrix}^T$ then:

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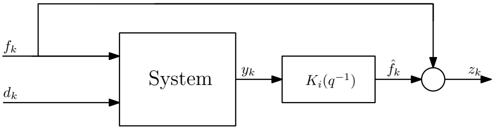

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Where $A^a_i = \begin{bmatrix} A_i & 0 \\ B^K_i C_i & A^K_i \end{bmatrix}$, $E^a_i = \begin{bmatrix} E_{w,i} \\ B^K_i F_{w,i} \end{bmatrix}$, $C^a_i = \begin{bmatrix} D^K_i C_i + \tilde{C}_i & C^K_i \end{bmatrix}$,
$F^a_i = D^K_i F_{w,i} + \tilde{F}_i$, $\tilde{C}_i = \begin{bmatrix} 0 \end{bmatrix}$, and $\tilde{F}_i = \begin{bmatrix} 0 & -I \end{bmatrix}$,

Problem formulation

Define $T_{zw}(q^{-1})$ as the transfer from w_k to the fault estimation error z_k:

$$T_{zw}(q^{-1}) = C_i^a(q^{-1}I - A_i^a)^{-1}E_i^a + F_i^a \tag{6}$$

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The $\mathcal{L}_2$-gain from the estimation error z_k to the vector w_k must be bounded by positive scalar γ_i i.e.

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The problem is formulated as following: Find a discrete filter $K(q^{-1})$ such that the augmented system is stable and the equation (8) is satisfied.

Filter design

$\mathcal{H}_\infty$ for switched system

Theorem (Bounded real lemma for switched system)

For a given switched linear system under arbitrary switching, if there exist matrices P_i and a positive definite matrices P_i $\forall i, j \in \{1..N\}$ such that:

$$\begin{bmatrix} -P_j^{-1} & A_i^a & E_i^a & 0 \\ A_i^{aT} & -P_i & 0 & C_i^{aT} \\ E_i^{aT} & 0 & -\gamma_i^2 I & F_i^{aT} \\ 0 & C_i^a & F_i^a & -I \end{bmatrix} < 0 \tag{9}$$

Then the switched discrete time fault estimation filter (SDTFEF) can be designed where the condition (8) is guaranteed.

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Proof

Using the switched Lyapunov function $V_k = x_k^{aT} P_k x_k^a > 0$ that must be decreasing for all k, solve $x_{k+1}^a P_{k+1} x_{k+1}^a - x_k^{aT} P_k x_k^a + z_k^T z_k - \gamma_i^2 w_k^T w_k < 0$. After some calculation and with Shur complement, (9) is easily obtained.

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Non linearities

This inequation is bilinear in P_i and P_j. Linearization is needed.

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2. Impose some structure on P, for example: $P_i = \begin{bmatrix} P_{1,i} & 0 \\ 0 & P_{2,i} \end{bmatrix}$

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3. In our study, no assumption on P_i is made, we simply denote:First, P_i is defined as follows:

$$P_i = \begin{bmatrix} S_i & N_i \\ N_i^T & U_i \end{bmatrix} = \begin{bmatrix} R_i & M_i \\ M_i^T & V_i \end{bmatrix}^{-1} > 0 \tag{10}$$

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2. Impose some structure on P, for example: $P_i = \begin{bmatrix} P_{1,i} & 0 \\ 0 & P_{2,i} \end{bmatrix}$

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On the structure of P_i and BMI linearization

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1. Multiply the BMI by some full rank matrices with zeros, and then proceed to a change of variable
2. Use the projection lemma to eliminate the unknown terms K_i, and then solve two sets of LMI, ones after another. This is explained in the next slides

BMI linearization

Lemma (Projection lemma)

Given a symmetric matrix Ψ *and the matrices* N*,*θ *and* N *with appropriate dimensions :*
$\Phi + M^T\theta^T N + N^T\theta M < 0$
Denote by W_X *any matrices whose columns form bases of the null spaces of* X*. The above equation is solvable for* θ *if and only if* $W_M^T\Psi W_M < 0$ *and* $W_N^T\Psi W_N < 0$

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Re-write (9) as:

$$\underbrace{\begin{bmatrix} -P_j^{-1} & A_i^0 & E_i^0 & 0 \\ A_i^{0\,T} & -P_i & 0 & C_i^{0\,T} \\ E_i^{0\,T} & 0 & -\gamma_i^2 I & F_i^{0\,T} \\ 0 & C_i^0 & F_i^0 & -I \end{bmatrix}}_{\Phi_i} + \underbrace{\begin{bmatrix} \mathcal{B}_i \\ 0 \\ 0 \\ \mathcal{D}_i \end{bmatrix}}_{\mathcal{M}_i^T} \mathcal{K}_i \underbrace{\begin{bmatrix} 0 & \mathcal{C}_i & \mathcal{F}_i & 0 \end{bmatrix}}_{\mathcal{N}_i} + [\star] < 0 \qquad (11)$$

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where:

$$A_i^0 = \begin{bmatrix} A_i & 0 \\ 0 & 0 \end{bmatrix}, E_i^0 = \begin{bmatrix} E_i \\ 0 \end{bmatrix}, C_i^0 = \begin{bmatrix} \tilde{C}_i & 0 \end{bmatrix}, F_i^0 = \begin{bmatrix} \tilde{F}_i \end{bmatrix}, \qquad \text{and } \mathcal{K}_i = \begin{bmatrix} D_i^K & C_i^K \\ B_i^K & A_i^K \end{bmatrix}$$

$$\mathcal{B}_i = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, \mathcal{D}_i = \begin{bmatrix} I & 0 \end{bmatrix}, \mathcal{C}_i = \begin{bmatrix} C_i & 0 \\ 0 & I \end{bmatrix}, \mathcal{F}_i = \begin{bmatrix} F_i \\ 0 \end{bmatrix}$$

Eliminate the $\mathcal{K}$-term

First apply the projection lemma on (11), it yields to the following inequalities:

$$W_{\mathcal{M}_i}^T \Phi_i W_{\mathcal{M}_i} < 0 \qquad (12a)$$

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Then compute $W_{\mathcal{M}_i}$ and $W_{\mathcal{N}_i}$, and the definition of P_i the following theorem is deduced

Theorem (Suboptimal discrete-time fault filter)

The suboptimal H_∞ fault estimation problem is solvable if and only if there exist positive definite matrices $R_{i/j}$ and $S_{i/j}$ $\forall i,j \in \{1..N\}$ such that:

$$\begin{bmatrix} -R_j + A_i R_i A_i^T & E_i \\ E_i^T & -\gamma_i^2 I \end{bmatrix} < 0 \tag{13a}$$

$$\begin{bmatrix} W_{CF_i}^T & 0 \\ 0 & I_{n_f} \end{bmatrix} \begin{bmatrix} -S_i + A_i^T S_j A_i & A_i^T S_j E_i & \tilde{C}_i^T \\ E_i^T S_j A_i & -\gamma_i^2 I + E_i^T S_j E_i & \tilde{F}_i^T \\ \tilde{C}_i^T & \tilde{F}_i & -I \end{bmatrix} \begin{bmatrix} W_{CF_i} & 0 \\ 0 & I_{n_f} \end{bmatrix} < 0 \tag{13b}$$

Where W_{CF_i} are basis of null spaces of $\begin{bmatrix} C_i & F_i \end{bmatrix}$

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Controller reconstruction: find P (part 1)

Recall P_i, defined as :

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From $P_i P_i^{-1} = I$, we infer:

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Product of two unknown matrices! $\Rightarrow$ Singular value decomposition

Controller reconstruction: find P (part 2)

From (16a): denote $X_i = I - S_i R_i$, the singular value decomposition of X_i is:

$$X_i = \Sigma_i \Lambda_i \Gamma_i^T \tag{17}$$

where Σ_i is an unitary orthogonal matrix, Λ_i is a diagonal matrix , and Γ_i is the transpose of an unitary orthogonal matrix.

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Then a solution of M_i and N_i (16a) is:

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Remark 1: One degree of freedom

The choice of matrix G_i offers one degree of freedom of the filter design.

Extension for uncertain switched LTI systems

Robust fault estimation for switched uncertain system

Lemma (Majoration lemma)

If $\Delta^T\Delta < Q_\Delta$, then for any $\alpha > 0$:

$$X^T\Delta Y + Y^T\Delta X \leq \alpha X^T X + \frac{1}{\alpha} Y^T Q_\Delta Y \tag{20}$$

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Theorem (Bounded real lemma for uncertain switched system)

For a given switched linear uncertain system under arbitrary switching, jf there exist matrices P_i and a positive definite matrices P_i $\forall i, j \in \{1..N\}$ such that:

$$\begin{bmatrix} -P_j^{-1} & A_i^a & E_i^a & 0 & H_{A_i}^a & 0 \\ A_i^{aT} & -P_i & 0 & C_i^{aT} & 0 & N_{A_i}^{aT} \\ E_i^{aT} & 0 & -\gamma_i^2 I & F_i^{aT} & 0 & 0 \\ 0 & C_i^a & F_i^a & -I & 0 & 0 \\ H_{A_i}^{aT} & 0 & 0 & 0 & -\alpha_i I^{-1} & 0 \\ 0 & N_{A_i}^a & 0 & 0 & 0 & -\alpha_i I \end{bmatrix} < 0 \tag{21}$$

Then a robust switched discrete time fault estimation filter (RSDTFEF) can be designed where the condition (8) is guaranteed.

Proof of previous theorem

For the uncertain switched system, the BRL gives:

$$\begin{bmatrix} -P_j^{-1} & \bar{A}_i^a & E_i^a & 0 \\ \bar{A}_i^{aT} & -P_i & 0 & C_i^{aT} \\ E_i^{aT} & 0 & -\gamma_i^2 I & F_i^{aT} \\ 0 & C_i^a & F_i^a & -I \end{bmatrix} < 0 \tag{22}$$

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With: $\bar{A}_i^a = \begin{bmatrix} \bar{A}_i & 0 \\ B_i^K C_i & A_i^K \end{bmatrix} = \begin{bmatrix} A_i + H_{A,i}\Delta_{A,i}N_{A,i} & 0 \\ B_i^K C_i & A_i^K \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ B_i^K C_i & A_i^K \end{bmatrix} + H_{A,i}^a \Delta_{A,i} N_{A,i}^a$,
where $H_{A,i}^a = \begin{bmatrix} H_{A,i} \\ 0 \end{bmatrix}$ and $N_{A,i}^a = \begin{bmatrix} N_{A,i} & 0 \end{bmatrix}$.

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where $H_{A,i}^a = \begin{bmatrix} H_{A,i} \\ 0 \end{bmatrix}$ and $N_{A,i}^a = \begin{bmatrix} N_{A,i} & 0 \end{bmatrix}$.
Following the same of calculation as in the previous part, the BMI to be solved is:

$$\Phi_i + \begin{bmatrix} H_{A_i}^{aT} & 0 & 0 & 0 \end{bmatrix}^T \Delta_{A,i} \begin{bmatrix} 0 & N_{A_i}^a & 0 & 0 \end{bmatrix} + [\star] < 0 \tag{23}$$

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Apply *Lemma 4* with $X = \begin{bmatrix} H_{A_i}^{aT} & 0 & 0 & 0 \end{bmatrix}^T$, $Y = \begin{bmatrix} 0 & N_{A_i}^a & 0 & 0 \end{bmatrix}$ and $Q_\Delta = I$.

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And finally with two Shur complement, the BMI (21) is deduced.

Suboptimal RSDTFEF

Theorem (Suboptimal robust discrete-time fault estimation filter)

The suboptimal H_∞ fault estimation problem is solvable if and only if there exist positive definite matrices R_i and S_i $\forall i \in \{1..N\}$ such that:

$$\begin{bmatrix} -R_j + A_i R_i A_i^T & E_i & H_{A,i} & A_i R_i N_{A,i}^T \\ E_i^T & -\gamma_i^2 I & 0 & 0 \\ H_{A,i}^T & 0 & -\alpha_i^{-1} I & 0 \\ N_{A,i} R_i A_i^T & 0 & 0 & -\alpha_i I + N_{A,i} R_i N_{A,i}^T \end{bmatrix} < 0 \tag{24a}$$

$$\mathcal{W}_{CF_i}^T \begin{bmatrix} A_i^T S_j A_i - S_i & A_i^T S_j E_i & \tilde{C}_i^T & A_i^T S_j H_{A,i} & N_{A,i}^T \\ E_i^T S_j A_i & E_i^T S_j E_i - \gamma_i^2 I & \tilde{F}_i^T & E_i^T S_j H_{A,i} & 0 \\ \tilde{C}_i & \tilde{F}_i & -I & 0 & 0 \\ H_{A,i}^T S_j A_i & H_{A,i}^T S_j E_i & 0 & -\alpha_i^{-1} I + H_{A,i}^T S_j H_{A,i} & 0 \\ N_{A,i} & 0 & 0 & 0 & -\alpha_i I \end{bmatrix} \mathcal{W}_{CF_i} < 0 \tag{24b}$$

Where $\mathcal{W}_{CF_i} = \begin{bmatrix} W_{CF_i} & 0 \\ 0 & I_{n_f+n_h+n_a} \end{bmatrix}$, α_i are free design scalars and W_{CF_i} are basis of null spaces of $\begin{bmatrix} C_i & F_i \end{bmatrix}$

Improved Robust fault estimation

Dynamical filters can be introduced in the design procedure:

- The filter $W_d(q^{-1})$ imposes robustness toward the disturbances in a specified frequency ranges
- and $W_f(q^{-1})$ is introduced to shape the desired response of $\hat{f}_k$ to the fault.

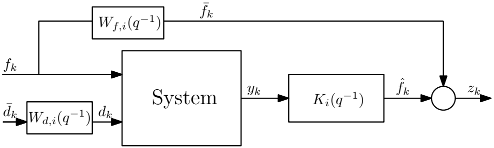

Figure: Fault estimation schemes

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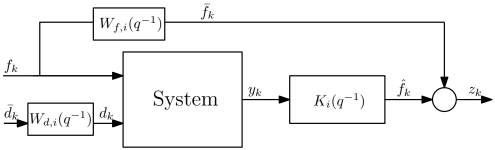

Figure: Fault estimation schemes

The dynamical filters $W_{d,i}$ and $W_{f,i}$ can be defined as switched systems driven by the same switching signal of the system, they have the realization:

$$W_{f,i}(q^{-1}) : \begin{cases} x_{k+1}^F = A_i^F x_k^F + B_i^F f_k \\ \bar{f}_k = C_i^F x_k^F + D_i^F f_k \end{cases} \quad (25)$$

$$W_{d,i}(q^{-1}) : \begin{cases} x_{k+1}^D = A_i^D x_k^D + B_i^D \bar{d}_k \\ d_k = C_i^D x_k^D + D_i^D \bar{d}_k \end{cases} \quad (26)$$

where $x_k^F \in \mathbb{R}^{n_F}$ and $x_k^D \in \mathbb{R}^{n_D}$, and it is assumed that $dim(f_k) = dim(\bar{f}_k)$ and $dim(d_k) = dim(\bar{d}_k)$.

Augmented system with weighting filters

The new state of the augmented system is now defined by $x_k^a = \begin{bmatrix} x_k^T & x_k^{FT} & x_k^{DT} & x_k^{KT} \end{bmatrix}^T$:

$$\left\{ \begin{array}{ccc} x_{k+1}^a & = & A_i^a x_{k+1}^a + E_i^a w_k \\ z_k & = & C_i^a x_k^a + F_i^a w_k \end{array} \right. \tag{27}$$

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Where $A_i^a = \begin{bmatrix} \breve{A}_i & 0 \\ B_i^K \check{C}_i & A_i^K \end{bmatrix}$, $E_i^a = \begin{bmatrix} \breve{E}_{w,i} \\ B_i^K \check{F}_{w,i} \end{bmatrix}$, $C_i^a = \begin{bmatrix} D_i^K \check{C}_i + \check{C}_i & C_i^K \end{bmatrix}$,
$F_i^a = D_i^K \check{F}_{w,i} + \check{F}_i$, $\breve{A}_i = \begin{bmatrix} A_i & 0 & E_{d,i} C_i^D \\ 0 & A_i^F & 0 \\ 0 & 0 & A_i^D \end{bmatrix}$, $\breve{E}_{w,i} = \begin{bmatrix} E_{d,i} D_i^D & E_{f,i} \\ 0 & B_i^F \\ B_i^D & 0 \end{bmatrix}$,
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Using the new notation, *theorem 2* can be applied on the augmented system.

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Remark 2: Post scaling for low frequencies estimation

In order to get an exact estimate of the fault signal in low frequency range, an a posteriori scaling factor can be added on the output of the K_i filter.

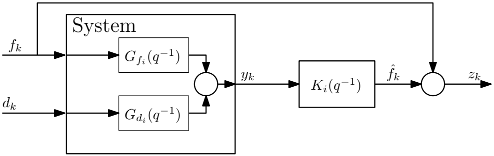
Numerical example for two design methods

Consider the uncertain switched LTI system :

$$\begin{cases} x_{k+1} = (A_{\alpha(k)} + H_{A,\alpha(k)}\Delta_{A,\alpha(k)}N_{A,\alpha(k)})x_k + E_{d,\alpha(k)}d_k + E_{f,\alpha(k)}f_k \\ y_k = C_{\alpha(k)}x_k + F_{d,\alpha(k)}d_k + F_{f,\alpha(k)}f_k \end{cases} \quad (28)$$

with the following nominal matrices:

$A_1 = \begin{bmatrix} 1 & 0.1 \\ -0.1 & 0.89 \end{bmatrix}$,
$A_2 = \begin{bmatrix} 1 & 0.07 \\ 0.1 & 0.91 \end{bmatrix}$,
$E_{d_1} = E_{d,2} = \begin{bmatrix} 0.01 \\ 0.02 \end{bmatrix}$, $E_{f_1} = \begin{bmatrix} 0.02 \\ 0.01 \end{bmatrix}$,
$E_{f,2} = \begin{bmatrix} 0.022 \\ 0.011 \end{bmatrix}$, $C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$,
$F_{d,1} = F_{d,2} = \begin{bmatrix} 0.01 \end{bmatrix}$,
$F_{f,1} = F_{f,2} = \begin{bmatrix} 1.6 \end{bmatrix}$.



And for the uncertainties directions:
$H_{A,1} = H_{A,2} = \begin{bmatrix} 0.2 & 0.002 \end{bmatrix}^T$ $N_{A,1} = \begin{bmatrix} 0.22 & .128 \end{bmatrix}$ and $N_{A,2} = \begin{bmatrix} 0.22 & .13 \end{bmatrix}$

Numerical example for two design methods: design procedure

The system is observable, and the matrices dimensions meet the assumptions. A RSDTFEF can be designed, using two previous methods.

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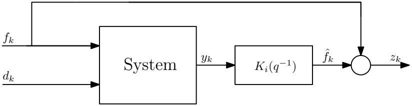

Figure: First approach: Fault estimation filter

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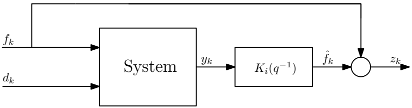

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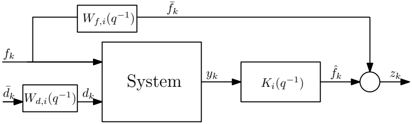

Figure: Second approach: improved filtering

First, define the loop shaping matrices and weighting filter W_f and W_d.

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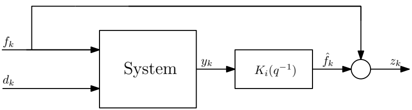

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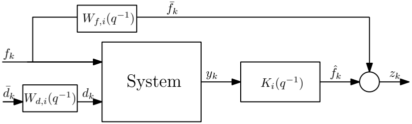

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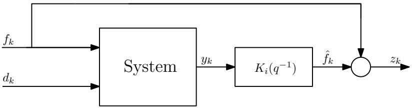

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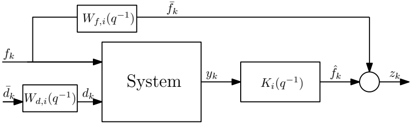

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The post filters scheduling factors ξ_i can be calculated and added to K_i according to *remark 2*: $\tilde{K}_i(q^{-1}) = \xi_i K_i(q^{-1})$

Numerical example for two design methods: results

In this example:

- The considered perturbation is a white noise.
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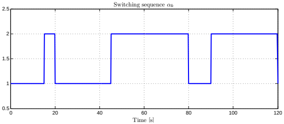
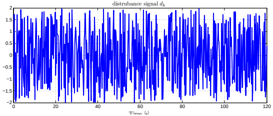


Figure: Switching sequence



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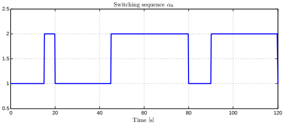

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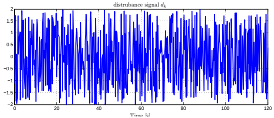

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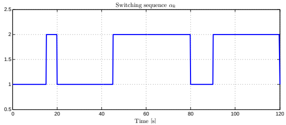

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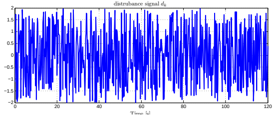

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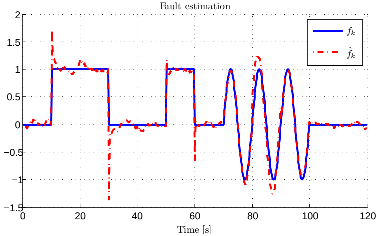

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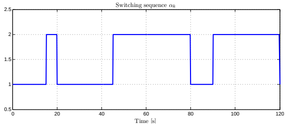

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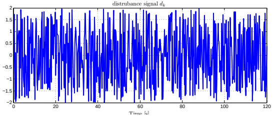

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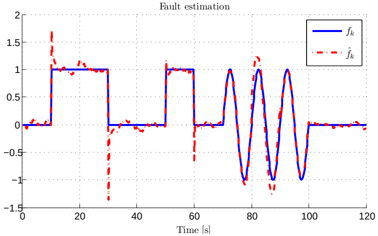

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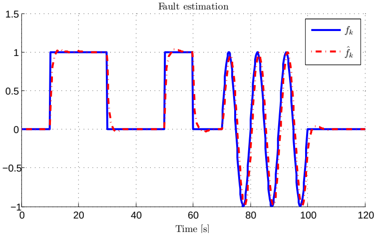

Figure: Fault estimation with filtering

Vehicle fault detection

Test campaign

- Collaboration with MIPS (Mulhouse), CAOR (Mines Paris-tech) and SOBEN, in the ANR Project INOVE.
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Uncertain bicycle model

Consider the non linear bicycle model of the vehicle for lateral control:

$$\begin{bmatrix}\dot{\beta}(t)\\ \ddot{\psi}(t)\end{bmatrix} = \begin{bmatrix} -\frac{c_r+c_f}{mv(t)} & \frac{c_r l_r - c_f l_f}{mv^2(t)} - 1 \\ \frac{c_r l_r - c_f l_f}{I_z} & -\frac{c_r l_r^2 + c_f l_f^2}{I_z v(t)} \end{bmatrix} \begin{bmatrix}\beta(t)\\ \dot{\psi}(t)\end{bmatrix} + \begin{bmatrix}\frac{c_f}{mv(t)}\\ \frac{c_r l_f}{I_z}\end{bmatrix} u_L(t) + \begin{bmatrix}\frac{1}{mv(t)}\\ \frac{l_w}{I_z}\end{bmatrix} F_w(t)$$

$$y = \begin{bmatrix} -\frac{c_r+c_f}{m} & c_f l_f - c_r l_r \end{bmatrix} \begin{bmatrix}\beta(t)\\ \dot{\psi}(t)\end{bmatrix} + \begin{bmatrix}\frac{c_f}{m}\end{bmatrix} u_L(t) \quad (29)$$

- states : side slip angle β and the yaw rate $\dot{\psi}$
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Using a Taylor expansion around the points v_α:

$$\frac{1}{v}|_{v=v_\alpha} = \frac{1}{v_\alpha} - \frac{1}{v_\alpha^2}(v - v_\alpha) + \mathcal{O}(\frac{1}{v^2}) \quad (30)$$

$$\frac{1}{v^2}|_{v=v_\alpha} = \frac{1}{v_\alpha^2} - \frac{2}{v_\alpha^3}(v - v_\alpha) + \mathcal{O}(\frac{1}{v^3}) \quad (31)$$

Then $A = \underbrace{A_0 + \frac{1}{v_\alpha}A_1 + \frac{1}{v_\alpha^2}A_2}_{A_\alpha} + \underbrace{(-\frac{1}{v^2}A_1 - \frac{2}{v_\alpha^3}A_2)}_{H_{A,\alpha}}\underbrace{(v - v_\alpha)}_{\Delta_{x,\alpha}}$

Experimental data

Consider the scenario of moose test (avoidance test):

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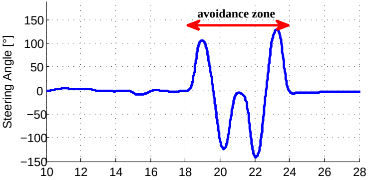

Figure: Steering angle [°]

Experimental data

Consider the scenario of moose test (avoidance test):

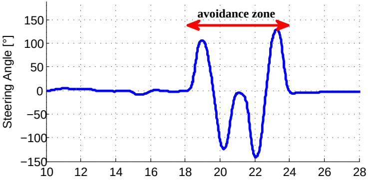

Figure: Steering angle [°]

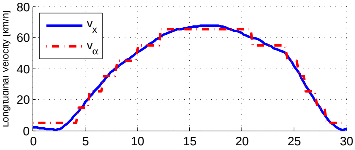

Figure: Longitudinal velocity [km/h] and switching rule

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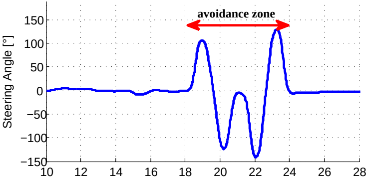

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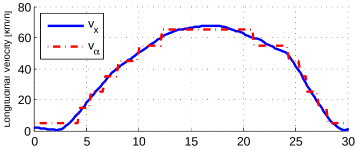

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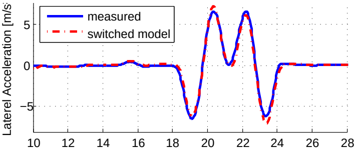

Figure: Lateral acceleration $[m.s^{-2}]$

Framework

The vehicle model can be put on the form (using the Taylor expansion):

$$\begin{cases} x_{k+1} & = & (A_{\alpha(k)} + H_{A,\alpha(k)}\Delta_{\alpha(k)}N_{A,\alpha(k)})x_k \\ & & +(E_{d,\alpha(k)} + H_{D,\alpha(k)}\Delta_{\alpha(k)}N_{D,\alpha(k)})d_k \\ & & +(E_{f,\alpha(k)} + H_{F,\alpha(k)}\Delta_{\alpha(k)}N_{F,\alpha(k)})f_k \\ y_k & = & C_{\alpha(k)}x_k + F_{d,\alpha(k)}d_k + F_{f,\alpha(k)}f_k \end{cases} \tag{32}$$

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- Extended this method for uncertain switched system
- Illustrated these two approches with numerical example

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- and maybe descriptor uncertain switched system.

Thank you for your attention

Any questions?